Proceedings of the 5th International Conference on Financial Technology and Business Analysis
DOI: 10.54254/2754-1169/2026.39175

From Linear Correlation to Tail Dependence: A Comparison and Selection of Methods for Measuring the Dependence of Financial Assets

Yixiang Gao

Beijing Technology and Business University, Beijing, China
2405020203@st.btbu.edu.cn

Abstract. Measures of dependence between financial assets are fundamental to portfolio risk assessment. This paper reviews the literature and summarises the mathematical foundations of Pearson's correlation coefficient, Spearman's rank correlation coefficient and Kendall's τ, highlighting their limitations in characterising non-linear dependence, heavy-tailed distributions and extreme co-movements. Building on Sklar's theorem, it introduces the key properties of copulas, commonly used copula families and tail dependence coefficients, and explains how copulas separate marginal distributions from the dependence structure. A comparison of conventional correlation measures and copula-based models highlights the latter's ability to represent asymmetric dependence and tail behaviour that scalar correlation measures may fail to capture. The paper also examines how correlation-based descriptions relate to copula models under elliptical assumptions. It then delineates the scope and limitations of both approaches, arguing that the choice of dependence measure or model should reflect the specific risk objective, rather than rely solely on statistical fit. Finally, it identifies promising directions for further research, including high-dimensional and time-varying copulas, with a view to improving the assessment of complex and evolving dependence in financial markets.

Keywords: financial assets, correlation coefficients, copula functions, tail dependence, portfolio risk

1. Introduction

Financial assets are a vital component of the modern financial system. A diverse range of financial asset classes, mature market participants, and sound asset regulatory and trading frameworks form the foundation for building an efficient financial system; they also serve as crucial safeguards and pillars for optimising resource allocation and maintaining stable economic operations. In the field of quantitative financial analysis, the measurement of asset correlation is fundamental to exploratory financial data analysis and portfolio risk assessment; the accuracy of such measurements directly determines the effectiveness of financial investment risk assessment and return forecasting. Furthermore, the interdependence structure among financial assets forms the core foundation for assessing portfolio risk and managing asset risk. Any inaccuracies in characterising the correlations

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DOI: 10.54254/2754-1169/2026.39175

between assets can directly lead to practical issues such as the underestimation of portfolio risk. The interdependence of asset prices in financial markets is complex, and return series generally exhibit characteristics of non-linearity, fat tails and clustered volatility; recent years have further demonstrated that tail-risk interdependence among assets is particularly pronounced during extreme market conditions. To date, correlation models, typified by the Pearson correlation coefficient, have been widely used due to their computational simplicity. However, such models suffer from inherent mathematical flaws; they rely strictly on the assumptions of linear relationships and normally distributed data, and are unable to capture the non-linear correlations and tail-mode co-movement characteristics of financial assets. This can easily lead to issues such as the underestimation of portfolio risk, making them ill-suited to the analytical demands of complex financial markets [1]. Copula functions, by utilising Sklar's theorem to separate marginal distributions from dependence structures, provide a novel mathematical approach for characterising non-linear financial dependencies and have gradually become an important analytical tool in the field of financial statistics. Currently, academic research on these two categories of correlation methods remains rather fragmented, lacking systematic mathematical comparisons and a clear definition of their applicable scenarios. Therefore, a systematic review of the mathematical methods for measuring correlation—ranging from the correlation coefficient to the copula function—and an analysis of the applicability limits of each method hold theoretical value for portfolio risk analysis.

With regard to measures of correlation, Zhang Xiaomei and Zhu Jiaming analysed the conditions under which the Pearson correlation coefficient should be used, pointing out that this metric is only applicable to linear relationships and is prone to producing misleading conclusions when applied to non-normal financial data; Zhao Xia et al. compared the practical differences between the Pearson correlation coefficient and Kendall's τ in investment portfolios, revealing the different aspects of interdependence characterised by the two measures; Chen Rong and Chen Miaoqiong systematically discussed the statistical properties of Kendall's τ and its transformation relationship with Copula parameters, providing a bridge for the transition from rank-correlation measures to the Copula framework [1-3]. In terms of copula theory, Zhang Yaoting was the first to introduce copula methods into domestic financial risk research; Lin Yu et al. compared the practical effectiveness of different copula models in characterising the interdependence structure of financial assets, drawing on typical phenomena in financial markets; in recent years, Sahamkhadam and Stephan have further applied the Vine copula to portfolio optimisation and risk measurement, verifying the practical value of this method under conditions of financial turbulence [4-6].

Existing studies have examined correlation coefficients and copula functions in considerable detail, but several gaps remain. Few studies compare Pearson and rank correlations with alternative copula models within a unified mathematical framework, and many empirical studies give limited attention to model assumptions and domains of applicability. This review therefore explains the mathematical foundations of these measures, compares their strengths and limitations in portfolio risk management, and identifies priorities for future research.

2. A theoretical framework for measuring the interdependence of financial assets

2.1. Correlation and dependence

Let F be the joint distribution function of the bivariate random vector (x, y), and let $F_1(x)$ and $F_2(y)$denote its marginal distribution functions. If, for all x and y,

$$F(x, y) = F_1(x)F_2(y) \tag{1}$$

DOI: 10.54254/2754-1169/2026.39175

Then X and Y are independent. If there exist values of x and y for which the equality does not hold, the variables are dependent. Independence is therefore a qualitative property: it indicates whether the joint distribution equals the product of the marginal distributions.

Correlation refers to a class of numerical measures that summarise particular features of dependence as scalars. Pearson correlation and the Spearman and Kendall rank correlations capture different aspects of that dependence.

2.2. Pearson's linear correlation

The Pearson correlation coefficient is the most fundamental and commonly used parametric measure in the analysis of correlations between financial assets. Let X and Y be random variables. Their Pearson correlation coefficient is the covariance divided by the product of their standard deviations:

$$\rho_{X,Y} = \frac{\operatorname{Cov}(X,Y)}{\sqrt{\operatorname{Var}(X)}\sqrt{\operatorname{Var}(Y)}} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y} \tag{2}$$

Here, μ_X and μ_Y are the means of X and Y, and σ_X and σ_Y are their standard deviations. The coefficient measures the extent to which the variables move together around their means. It ranges from -1 to 1; a larger absolute value indicates stronger linear association, and the sign indicates direction. The coefficient is symmetric and invariant under positive affine transformations.

Pearson correlation is most informative when the association is approximately linear and can be sensitive to outliers and heavy tails. Joint normality is not required to compute the coefficient, although it supports familiar inferential interpretations and allows the coefficient to characterise dependence within the Gaussian family. Non-stationarity can also make a single sample estimate misleading.

2.3. Spearman's and Kendall's rank correlations

Pearson correlation uses observed values, whereas rank correlations depend only on their ordering and are non-parametric. Zhang notes that rank correlations do not require a specified marginal distribution and are therefore useful when the assumptions underlying linear correlation are unsuitable [7]. Wei et al. likewise treat rank correlations as basic tools for analysing financial dependence [8].

Spearman's rank correlation coefficient, denoted by ρ_S, is the Pearson correlation between the two uniform variables obtained through probability integral transforms:

$$\rho_S = \operatorname{Corr}\left(F_X\left(X\right), F_Y\left(Y\right)\right) \tag{3}$$

Here, F_X and F_Y are the marginal distribution functions of X and Y, respectively. The expression can also be written as:

$$\rho_S = 12E\left[F_X\left(X\right)F_Y\left(Y\right)\right] - 3 \tag{4}$$

For a sample of n paired observations, let R_i and S_i denote the ranks of R and S. If there are no ties, Spearman's ρ can be computed using the simplified formula:

$$\widehat{\rho}_S = 1 - \frac{6\sum_{i=1}^{n} {d_i}^2}{n(n^2-1)} \tag{5}$$

DOI: 10.54254/2754-1169/2026.39175

where $d_i = R_i - S_i$ is the difference between the ranks of the ith observation pair.

Ji explains that Spearman's ρ measures whether two asset prices tend to move in the same ranked direction [9]. It ranges from −1 to 1 and is invariant under strictly increasing transformations. Because it uses ranks rather than observed values, it is less sensitive to outliers than Pearson correlation.

Kendall's rank correlation coefficient, denoted by τ, is based on concordance between pairs of observations. Let (X_1, Y_1) and (X_2, Y_2) be two independent copies of (X, Y). Then:

$$\tau = E\left[\text{sign}\left((X_1 - X_2)(Y_1 - Y_2)\right)\right] \tag{6}$$

Equivalently, τ is the probability of concordance minus the probability of discordance:

$$\tau = P\left((X_1 - X_2)(Y_1 - Y_2) > 0\right) - P\left((X_1 - X_2)(Y_1 - Y_2) < 0\right) \tag{7}$$

When estimating a sample, if the number of consistent pairs is C and the number of inconsistent pairs is D, then:

$$\widehat{\tau} = \frac{C - D}{\frac{1}{2}n(n-1)} \tag{8}$$

Kendall's τ also ranges from −1 to 1. Compared with Spearman's ρ, it has a direct probabilistic interpretation as the difference between concordance and discordance probabilities [9]. It equals 1 for perfectly concordant orderings, −1 for perfectly discordant orderings, and 0 under independence.

2.4. Copulas and Sklar's theorem

Let X and Y have joint distribution function F and marginal distribution functions $F_X(x)$ and $F_Y(y)$. If there is a bivariate function C such that $C\left(F_X(x), F_Y(y)\right) = F_{X,Y}(x, y)$ for all x and y, then C is a copula of X and Y. A bivariate copula is a distribution function on $[0,1]^2$ with uniform marginal distributions [10].

For a continuous random vector (X, Y) with marginal distributions F_X and F_Y, respectively, its joint probability distribution can be expressed as:

$$P(X \le x, Y \le y) = C\left[F_X(x), F_Y(y); \theta\right] = C(u_1, u_2) \tag{9}$$

where C is the copula with dependence parameter θ, and u_1 and u_2 are the marginal probabilities [10]. The copula formulation separates the marginal distributions from the dependence structure.

Bivariate copulas have the following key properties [10]. First, they have uniform marginal distributions on $[0,1]$, satisfying $C(u, 1) = u$ and $C(1, v) = v$, This allows the dependence structure to be described separately from the marginal distributions, regardless of the variables' measurement units. Second, they satisfy the Fréchet–Hoeffding bounds: every bivariate copula lies between the lower and upper bounds, which correspond to perfect negative and perfect positive dependence, respectively. Third, a copula is a 2-increasing distribution function on $[0,1]^2$, meaning that it assigns non-negative probability mass to every rectangle in the unit square. This requirement is stronger than monotonicity in each argument. For estimation, the probability integral transform maps variables with continuous marginal distributions to variables that are uniformly distributed on $[0,1]$. The copula family and its parameters can then be estimated using the transformed data. For

DOI: 10.54254/2754-1169/2026.39175

financial time series, pseudo-observations are commonly constructed by applying the empirical distribution functions to standardised residuals from fitted marginal models.

Sklar's theorem states that, for any d-dimensional distribution function H with univariate margins $F_1, \ldots, F_d$, there exists a d-dimensional copula C such that, for all $x \in R^d$,

$$H(x) = C(F_1(x_1), \ldots, F_d(x_d)) \tag{10}$$

Sklar's theorem decomposes a joint distribution into the marginal distribution of each variable and their dependence structure. For two financial-asset returns, y_1 and y_2, the joint distribution can be written as:

$$F(y_1, y_2) = C_\theta(F_1(y_1), F_2(y_2)) \tag{11}$$

The joint distribution is therefore determined by the marginal cumulative distribution functions, $F_1(y_1)$ and $F_2(y_2)$, and the copula parameter θ. In practice, the margins are typically estimated first, followed by the copula parameters.

2.5. Tail dependence and its financial implications

Within the copula framework, the joint behaviour of financial assets under extreme market conditions can be quantified using tail dependence coefficients. Let (U, V) have copula C. The upper-tail and lower-tail dependence coefficients are defined as follows:

$$\lambda_U = \lim_{u \to 1^-} P(U > u | V > u) = \lim_{u \to 1^-} \frac{[1 - 2u + C(u,u)]}{(1-u)} \tag{12}$$

$$\lambda_L = \lim_{u \to 0^+} P(U \leq u | V \leq u) = \lim_{u \to 0^+} \frac{C(u,u)}{u} \tag{13}$$

The coefficients λ_U and λ_L measure the limiting conditional probability that one variable is extreme given that the other is extreme in the same tail. Values range from 0 to 1: zero indicates asymptotic independence in that tail, whereas a positive value indicates tail dependence. Because the coefficients depend only on the copula, they are invariant to the marginal distributions.

In finance, tail dependence measures whether assets tend to experience extreme gains or losses together. Empirical studies often find stronger lower-tail than upper-tail dependence in equity markets; in such cases, symmetric models may understate portfolio risk during crises. Tail dependence is therefore relevant to stress testing and systemic-risk analysis.

2.6. Summary

The measures discussed in this section describe different aspects of dependence between financial assets. Pearson correlation summarises linear association as a scalar, whereas Spearman's ρ and Kendall's τ use ranks and require fewer distributional assumptions. Copulas separate marginal distributions from dependence and represent the dependence structure as a function. Tail dependence coefficients focus specifically on co-movement under extreme market conditions. It is thus evident that this chapter has achieved a shift in perspective from 'summarising correlation with a single number' to 'characterising dependence through overall structure', thereby establishing a common

DOI: 10.54254/2754-1169/2026.39175

theoretical framework for the subsequent comparison of the patterns of dependence across various copula models and the discussion of metric selection.

Table 1. Comparison of traditional correlation and copula dependence measures

Measure	Measured object and core idea	Main assumptions	Main advantages	Main limitations
Pearson correlation coefficient	Degree of linear co-movement	Linear relationship, approximate normality, no extreme values	Simple to compute, intuitive	Sensitive to nonlinearity, heavy tails and extreme values
Spearman rank correlation	Rank consistency	No distributional assumption	Captures monotone nonlinearity, robust to extreme values	Reduces to a single scalar, losing structural information
Kendall's τ	Difference between concordant and discordant probabilities	No distributional assumption	Direct probabilistic interpretation, robust	Single scalar, information somewhat compressed
Copula (Sklar's theorem)	Complete dependence structure (a function)	Marginal distributions known	Separates margins from dependence, captures nonlinearity and tails	Requires parameter estimation and model selection
Tail dependence coefficient	Joint co-movement in extreme market conditions	Limit exists	Focuses on tail risk, independent of marginal distributions	Reflects only the tails, not overall dependence

3. Major copula models and their dependence structures

3.1. Gaussian copula

The Gaussian copula is derived from the multivariate normal distribution and is the most basic type within the elliptical family. Let Φ denote the standard normal distribution function and Φ_ρ the bivariate standard normal distribution function with correlation parameter ρ. For transformed marginal probabilities u and v, the Gaussian copula is:

$$C^{\mathrm{Ga}}(u,v;\rho)=\Phi_\rho\left(\Phi^{-1}(u),\Phi^{-1}(v)\right) \tag{14}$$

Here, $\rho\in(-1,1)$ is a dependence parameter, and Φ^{-1} is the inverse of the standard normal distribution function. The Gaussian copula is symmetric and has zero upper- and lower-tail dependence for $|\rho|<1$. As ρ approaches 1 or −1, it approaches the Fréchet–Hoeffding upper or lower bound, respectively; when $\rho=0$, it reduces to the independence copula. Its simplicity is useful, but its asymptotic tail independence can be restrictive for heavy-tailed or asymmetric financial data [10].

3.2. t-Copula

The t-copula is derived from the multivariate t-distribution. Let t_ν denote the univariate Student's t distribution with ν degrees of freedom and $t_{\rho,\nu}$ the corresponding bivariate distribution with correlation parameter ρ. The t-copula is then:

$$C^{t}(u,v;\rho,\nu)=t_{\rho,\nu}\left(t_\nu^{-1}(u),t_\nu^{-1}(v)\right) \tag{15}$$

Both the t-copula and Gaussian copula belong to the elliptical family and impose symmetric dependence. The t-copula additionally includes the degrees-of-freedom parameter ν, which controls

DOI: 10.54254/2754-1169/2026.39175

tail thickness: smaller values of ν imply heavier tails and stronger tail dependence. Its upper- and lower-tail dependence coefficients are equal and positive for ρ > −1:

$$\lambda_U = \lambda_L = 2t_{\nu+1}\left(-\sqrt{\frac{(\nu+1)(1-\rho)}{(1+\rho)}}\right) \tag{16}$$

The t-copula can therefore capture symmetric tail dependence. As ν approaches infinity, it converges to the Gaussian copula; therefore, the Gaussian copula may be regarded as the limit case of the t-copula as the degrees of freedom approach infinity.

3.3. Clayton, Gumbel and Frank copulas

An Archimedean copula is constructed from a generator φ that is continuous, strictly decreasing and convex on [0, 1]. Let $\varphi(1) = 0$ and φ^{-1} denote its pseudo-inverse. The copula then has the form:

$$C(u, v) = \varphi^{-1}(\varphi(u) + \varphi(v)) \tag{17}$$

The Clayton, Gumbel and Frank copulas are among the most widely used Archimedean copulas. Their generators and distribution functions are given below.

The generator of the Clayton Copula is:

$$\varphi(t) = \frac{(t^{-\theta}-1)}{\theta}\,(\theta > 0) \tag{18}$$

The distribution function is:

$$C(u, v; \theta) = \left(u^{-\theta} + v^{-\theta} - 1\right)^{\frac{-1}{\theta}} \tag{19}$$

The Clayton copula exhibits lower-tail but no upper-tail dependence, making it suitable for modelling joint downside movements [11]. As θ approaches zero, it converges to the independence copula; as θ approaches infinity, it approaches perfect positive dependence.

$$\lambda_L = 2^{\frac{-1}{\theta}}, \lambda_U = 0 \tag{20}$$

The generator of the Gumbel copula is:

$$\varphi(t) = (-lnt)^{\theta}(\theta \geq 1) \tag{21}$$

The distribution function is:

$$C(u, v; \theta) = \exp\{-[(-\ln u)^{\theta} + (-\ln v)^{\theta}]^{\frac{1}{\theta}}\} \tag{22}$$

The Gumbel copula exhibits upper-tail but no lower-tail dependence, making it suitable for modelling joint extreme gains. When θ = 1, it reduces to the independence copula.

$$\lambda_U = 2 - 2^{\frac{1}{\theta}}, \lambda_L = 0 \tag{23}$$

The generator of the Frank copula is:

DOI: 10.54254/2754-1169/2026.39175

$$\varphi(t) = -ln(\frac{(e^{-\theta t}-1)}{(e^{-\theta}-1)})(\theta \neq 0) \tag{24}$$

The distribution function is:

$$C(u,v;\theta) = -\frac{1}{\theta}\ln(1+\frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{(e^{-\theta}-1)}) \tag{25}$$

The Frank copula has zero upper-tail and lower-tail dependence and is therefore asymptotically independent in both tails. It nevertheless captures symmetric positive or negative dependence: θ > 0 indicates positive dependence, whereas θ < 0 indicates negative dependence.

3.4. Copula model identification, estimation and selection

Model identification begins with an assessment of the sample dependence structure. Rank correlations such as Kendall's τ and Spearman's ρ_S are calculated, and scatter plots of pseudo-observations are inspected for central and tail clustering. These diagnostics help determine whether the dependence is symmetric or asymmetric and whether tail dependence should be modelled. For several Archimedean families, Kendall's τ has a one-to-one relationship with θ; for the Clayton and Gumbel families, respectively:

$$\tau = \frac{\theta}{\theta+2}, \tau = 1-\frac{1}{\theta} \tag{26}$$

On this basis, the parameters can be directly inverted from the sample rank correlation, providing initial values for parameter estimation.

Copula parameters can be estimated in three main ways. Full maximum likelihood jointly estimates the marginal and copula parameters but can be computationally demanding in high dimensions. Two-stage maximum likelihood estimates the margins first and the copula second; it is simpler, although errors may propagate between stages [4]. Semi-parametric estimation replaces parametric margins with empirical distribution functions, reducing sensitivity to marginal misspecification.

After several candidate copulas have been estimated, model selection typically combines information criteria with goodness-of-fit tests. Models with lower AIC and BIC values are preferred, all else being equal:

$$\mathrm{AIC} = -2\ln L + 2k, \mathrm{BIC} = -2\ln L + k\ln n \tag{27}$$

Here, L represents the value of the likelihood function, k is the number of parameters, and n is the sample size. Goodness-of-fit tests, including the Cramér–von Mises and Kolmogorov–Smirnov tests, can also be used to assess each candidate and exclude poorly fitting models.

3.5. Summary

These copula families represent different dependence structures: the elliptical copulas are symmetric, Clayton exhibits lower-tail dependence, Gumbel exhibits upper-tail dependence, and Frank is symmetric but tail independent. These differences are relevant to model selection. The Clayton copula may be appropriate for modelling joint downside risk, whereas the Gumbel copula may be suitable for joint upper-tail movements. When symmetric dependence is the main concern, Gaussian, t or Frank copulas may be considered, depending on the required tail properties. Model

DOI: 10.54254/2754-1169/2026.39175

selection should therefore not rely solely on information criteria such as AIC and BIC, but should also account for the risk objective and the dependence features relevant to that objective.

Table 2. Comparison of dependence structure characteristics of the main copula models

Copula model	Family	Dependence structure	Tail dependence characteristics	Applicable scenarios and financial implications
Gaussian	Elliptical	Symmetric	Asymptotically independent in both tails	Symmetric dependence with no tail co-movement
t	Elliptical	Symmetric, heavy-tailed	Equal tail dependence in both tails	Symmetric extreme co-movement risk
Clayton	Archimedean	Asymmetric	Lower-tail dependence only	Captures downside co-movement risk
Gumbel	Archimedean	Asymmetric	Upper-tail dependence only	Captures upside co-movement spillovers
Frank	Archimedean	Symmetric	Asymptotically independent in both tails	Symmetric but with no tail co-movement

4. A systematic comparison of correlation measures and copula methods

4.1. Differences between the objects being measured

The central difference concerns what each method represents. A correlation coefficient summarises one aspect of dependence as a scalar, whereas a copula represents the dependence component of a joint distribution as a function [10]. A scalar captures only a projection of dependence; a copula can represent a broader range of structures. For elliptical distributions, correlation parameters and copulas are closely linked, but outside that setting identical correlations can correspond to different dependence structures.

4.2. Dependence on marginal distributions

There is also a fundamental difference between the two types of methods in terms of their dependence on the marginal distribution. Pearson correlation depends on both the margins and the dependence structure and uniquely identifies dependence only within restricted families such as elliptical distributions. Spearman's ρ, Kendall's τ and tail dependence coefficients are copula-based and invariant under strictly increasing transformations [9]. Thus, two asset pairs can have the same Pearson correlation but markedly different tail behaviour, so correlation alone may not describe extreme co-movement.

4.3. Identification of nonlinear and asymmetric dependence

When it comes to identifying non-linear and asymmetric dependencies, the differences in capability between the two types of methods become even more pronounced. Pearson correlation measures only linear association. Rank correlations capture monotonic nonlinear relationships but still summarise dependence as a single number. Copulas can represent asymmetric lower- and upper-tail dependence through families such as Clayton and Gumbel and through rotated variants. In essence, traditional measures can only answer questions such as 'Is there a correlation?', 'What is its direction?', and 'How strong is it?', whereas copulas can go further to answer 'Under what market conditions?' and 'In what manner?'.

DOI: 10.54254/2754-1169/2026.39175

Empirical evidence indicates that dependence is often asymmetric. Cao et al. report stronger dependence in falling than in rising markets, implying that linear correlation may overstate diversification benefits [12]. Sahamkhadam and Stephan show that incorporating nonlinear and asymmetric dependence can improve the robustness of portfolio allocations during crises [6].

4.4. Identification of tail risks

A key advantage of copulas is their ability to represent tail risk explicitly. Linear and rank correlations summarise overall association, whereas tail dependence measures the limiting probability of joint extremes. Assets with weak overall correlation may still exhibit strong lower-tail dependence. TailCoR separates the linear and nonlinear components associated with extreme co-movements, showing that tail dependence is not simply correlation calculated on extreme observations [13].

Several empirical studies document tail dependence in China's financial markets. Evidence has been reported for energy and equity markets, for A-, B- and H-share markets across changing conditions [14], and for the effect of investor sentiment on tail risk across asset classes. These findings suggest that tail dependence can persist outside crises and intensify during periods of stress.

Similar patterns appear across countries and markets. For example, green bonds and conventional financial markets exhibit extreme-risk dependence that standard correlation coefficients may not capture [15]. When the research question concerns extreme losses, systemic risk or contagion, relying only on correlation coefficients may therefore understate tail dependence.

4.5. Static and dynamic dependence structures

The correlation coefficient typically assumes that the structure of interdependence remains constant over time, whereas in financial markets it often evolves with institutional and market conditions. For the mainland and Hong Kong stock markets, the strength of interdependence and the upper- and lower-tail structures changed markedly after the launch of the Shanghai–Hong Kong Stock Connect [16], and Wu Jilin et al. used a regime-switching hybrid copula to trace the evolution of tail dependence among A, B and H shares [14]. Zeng and Wu employed a DCC-EGARCH-t-copula and found that China–US correlation strengthened and remained positive during crises [17], while Xiao reported that the dependence structure of high-frequency agricultural commodity futures is likewise dynamic [18]. Copula models can incorporate time-varying parameters, regime switching and structural breaks, allowing dependence to evolve rather than remain fixed.

4.6. Applicability limits of different methods

Neither approach is uniformly more accurate; suitability depends on the data and the research objective. Correlation coefficients remain useful when dependence is approximately linear and the focus is overall association. Heavy-tailed or asymmetric data and questions about extreme loss call for copulas with appropriate tail dependence, while evolving dependence may require time-varying or regime-switching models.

Method choice also varies by asset class. Equity returns often exhibit heavy and asymmetric tails, bond markets require models of nonlinear default contagion, and leverage and high-frequency trading can amplify tail events in futures markets. The selected measure should therefore reflect both the data-generating features and the risk objective.

DOI: 10.54254/2754-1169/2026.39175

5. A framework for selecting dependence measures for financial assets

5.1. Method selection by risk-management objective

Method choice should begin with the risk question. Portfolio diversification often requires a summary of overall covariance, extreme-loss analysis requires measures of joint tail behaviour, and systemic-risk monitoring may also require direction, time variation and network structure. The model should be only as complex as the information required by the objective.

Pearson, Spearman and Kendall measures are widely used in portfolio analysis because they are simple and interpretable, but they may not be sufficient when dependence is asymmetric. Cao et al. find stronger dependence in bear than in bull markets, so ignoring asymmetry may overstate diversification benefits [12]. Sahamkhadam and Stephan likewise show that nonlinear and asymmetric dependence can improve allocation robustness during crises [6].

When the objective is to estimate Value at Risk (VaR), Expected Shortfall (ES) or stress losses, tail dependence becomes central to method selection. In such cases, reporting linear correlation alone is insufficient to assess joint losses under extreme market conditions; copula families such as Clayton and Gumbel, which can characterise lower- and upper-tail dependence separately, are better suited to these requirements. The TailCoR metric proposed by Babić et al., which measures the linear and non-linear components contributing to the co-occurrence of extremes separately, serves precisely this purpose [13]; estimates of tail dependence in the energy markets and in the Shanghai and Shenzhen stock markets likewise indicate that tail-risk measurement must rely on models capable of identifying lower-tail interdependence.

Systemic-risk and contagion analysis also requires dynamic and network information. Time-varying copula estimates can change the measured ranking of institutions' systemic contributions, while asymmetric tail dependence can distinguish system-to-institution from institution-to-system spillovers. Evidence that investor sentiment amplifies asset-price tail risk further suggests that systemic risk cannot be attributed solely to direct links between assets.

5.2. Method selection by data characteristics

Data characteristics constrain method choice. Financial returns often exhibit leptokurtosis and heavy tails and depart from multivariate normality, which can bias models based on linearity and normality. If overall covariance is sufficient, a conventional correlation measure may be adequate; if joint extremes matter, the method must represent tail dependence.

Asymmetry between the lower and upper tails is another important criterion. A symmetric model will understate joint downside risk when lower-tail dependence exceeds upper-tail dependence. In such cases, an asymmetric copula family or a rotated copula is more appropriate [12, 19].

The complexity of the method is further determined by whether the interdependence structure changes over time and by the dimensionality of the variables. When the interdependence structure evolves with institutional or market conditions, static models prove inadequate, so time-varying parameters or regime-switching settings must be introduced: Wu Jilin et al. employed a regime-switching hybrid copula to characterise the trend evolution of tail interdependence among A, B and H shares [14], whilst Zeng and Wu utilised a DCC-EGARCH-t-copula to capture the time-varying correlation between the Chinese and US stock markets [17]. Liu Wenqiong and Wang Ying found that the interdependence structure between the Shanghai and Hong Kong stock markets underwent significant changes before and after the launch of the Shanghai–Hong Kong Stock Connect [16]. In

DOI: 10.54254/2754-1169/2026.39175

high-dimensional settings, vine copulas mitigate the curse of dimensionality through pair-copula decompositions and provide a practical way to model the joint tails of multi-asset portfolios [6].

5.3. Applications in equity, futures and bond markets

Applying the above framework to specific asset classes provides a clearer picture of how the choice of methodology varies by asset class. A prominent feature of the stock market is its fat-tailed distribution with sharp peaks, clustered volatility and a correlation structure that evolves with institutional changes. Liu and Wang find that, after the launch of the Shanghai–Hong Kong Stock Connect, dependence between the two equity markets increased, lower-tail dependence strengthened and upper-tail dependence weakened [16]. This pattern of stronger joint declines than joint rises makes static symmetric models potentially misleading. Furthermore, ARMA-GARCH models can be employed to fit the marginal returns, whilst time-varying SJC copulas can be used to characterise the dynamic changes in the upper and lower tails; there are also marked differences in the clustering of upper and lower tails across sectors within the same market. Consequently, for equity markets, it is typically necessary to address the three features of asymmetry, tail distribution and time-varying behaviour simultaneously.

Method choice in futures markets is strongly influenced by trading mechanisms. Leverage, hedging and high-frequency trading can amplify tail events, whilst volatility often exhibits a leverage effect. A copula-EGARCH analysis of non-ferrous metal futures and the CSI 300 Index captures their tail-risk interdependence, with the EGARCH component modelling the asymmetry of volatility and the copula component characterising extreme co-movement. For government bond futures and the SSE 50, the copula correlation is significant over short horizons but becomes insignificant when the horizon reaches eight days or more. By contrast, the correlation with gold is accompanied by significant upper-tail dependence. A comparison of high-frequency agricultural commodity futures further shows that the t-copula and SJC-copula significantly outperform the other candidates [18]. It is therefore evident that, for futures markets, the choice of methodology must account for both the time horizon and the portfolio structure.

A distinctive feature of the bond market is the non-linear contagion of default risk. Approaches to modelling the interdependence of corporate defaults can be categorised into three types: copula, structural and intensity models. Among these, the copula approach directly constructs the dependence structure between default events and is well suited to capturing the joint risk of 'simultaneous defaults'. Huang Ran et al. superimposed a copula-based dependence structure on a tail-risk-driven network to trace contagion between sectoral credit spreads, showing that the bond market must account for both non-linear interdependence and the question of 'who transmits risk under extreme conditions' [20]. Karim et al.'s analysis of green bonds, meanwhile, suggests that extreme risk interdependence between emerging bond classes and established financial markets can only be reliably estimated with time-varying optimal copulas [15]. Across these asset classes, method selection should begin with the return characteristics and risk objective and then determine the required tail behaviour, dynamics and model complexity.

6. Conclusions and outlook

6.1. Conclusions

Choosing a measure of dependence determines which aspects of financial risk the analysis can represent. Correlation coefficients are widely used because they summarise linear association in a

DOI: 10.54254/2754-1169/2026.39175

simple scalar, but this compression can omit asymmetry, tail behaviour and state dependence. Copulas address these limitations by separating marginal distributions from the dependence structure and allowing alternative forms of dependence to be modelled explicitly.

This paper does not argue that copulas are universally superior to correlation coefficients. Instead, it places method selection within the risk-management objective. Overall correlation may be sufficient for some diversification questions, tail probabilities are necessary for extreme-loss analysis, and systemic-risk analysis may additionally require direction, time variation and network structure. The proposed framework therefore selects the object of measurement before choosing the model.

More generally, financial econometrics increasingly uses different dependence measures for different questions. Copulas complement rather than replace correlation coefficients by extending the analysis to dynamic, asymmetric and tail-specific dependence. This flexibility increases the demands of estimation, model identification and high-dimensional modelling, so method choice must balance information content, robustness and interpretability.

6.2. Outlook

This review has two main limitations. First, it focuses on bivariate settings and commonly used copula families and gives limited attention to high-dimensional, time-varying, non-parametric and extreme-value copulas. Second, its discussion of the link between dependence measures and investment decisions remains conceptual rather than operational.

Future research should extend the framework in four directions. First, vine, factor and network structures can model high-dimensional joint tails while controlling computational cost. Second, time-varying parameters, regime switching and structural breaks can capture non-stationarity. Third, non-parametric and semi-parametric methods, high-frequency data and extreme-value theory may improve the identification of joint extremes. Finally, these measures should be evaluated within portfolio optimisation, stress testing and regulatory applications.

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