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Black–Scholes Pricing and Market Mispricing of XSP European Index Options

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Abstract. This study compares Black–Scholes prices with 6,414 XSP European option quotes from August 7, 2026. The calculation uses the XSP index level, a three-month U.S. Treasury yield, an annualized dividend yield, and 20-day historical volatility from S&P 500 log returns. Signed, absolute, and absolute relative pricing errors (ARPE) are examined by option type, moneyness, maturity, and liquidity. Model prices broadly follow market midquotes, but calls have a larger negative bias and larger absolute errors than puts. Errors vary most clearly with moneyness, with near-at-the-money calls showing the largest absolute deviations. Maturity patterns depend on the error measure, while spreads and volume show no simple monotonic relationship with errors. Excluding low-priced contracts reduces ARPE substantially and weakens its maturity pattern, but the main absolute-error and moneyness findings remain. Black–Scholes provides a useful benchmark for this sample, although one historical-volatility estimate cannot reproduce price differences across all contracts. The evidence is descriptive and comes from a single trading day, so its generality requires further testing.

Keywords: Black–Scholes model, XSP options, option pricing, pricing errors, moneyness

1. Introduction

Index options provide broad-market exposure and a way to hedge systematic risk. Comparing their quoted prices with model values helps assess how well a pricing model describes observed market conditions. Black–Scholes remains a common benchmark for European options [1], despite its assumptions of constant volatility and interest rates, geometric Brownian motion, and frictionless trading. Empirical tests have documented departures from its predicted prices [2].

Merton extended the continuous-time pricing framework [3], while later work examined the volatility smile and skew across strikes and maturities [4]. Stochastic-volatility and local-volatility models address some of these patterns [5, 6]. Pricing comparisons also depend on spreads, dividends, transaction costs, and data alignment; mismatched index prices and option quotes can distort the results [7].

Much of the empirical literature concerns large, liquid index-option markets, where more flexible models can improve pricing fit [8]. Evidence on Mini-SPX (XSP) options is more limited. Cboe documents variation in XSP implied volatility across moneyness and expiration [9], but this leaves the distribution of Black–Scholes pricing errors across contract groups to be examined.

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XSP is approximately one-tenth the size of standard S&P 500 Index (SPX) options and retains European exercise and cash settlement. Its smaller contract size lowers the capital required per contract. Option prices reflect expectations of market uncertainty [10], making XSP a useful setting for examining how a common historical-volatility estimate compares with information in market quotes.

This paper compares Black–Scholes values with XSP midquotes, then groups the errors by option type, moneyness, maturity, and liquidity. The aim is to identify where this benchmark fits the sample and where larger differences remain.

2. Data and methodology

2.1. Data sources, data cleaning, and sample construction

The sample contains Cboe Mini-SPX Index Option (XSP) quotes observed on August 7, 2026, obtained from the Cboe XSP Quotes Dashboard. The analysis uses option type, expiration, strike, bid, ask, and trading volume. XSP is a European-style, cash-settled option with an index level approximately one-tenth of SPX [11].

Quotes are converted to contract-level records, numeric fields and dates are standardized, and records with missing or invalid bid, ask, strike, expiration, or volume are removed. Contracts with non-positive strikes or time to maturity are also excluded.

Market price is measured by the bid–ask midpoint in equation (1), and the spread is calculated in equation (2):

$$P_{Market,i} = \frac{Bid_i + Ask_i}{2} \tag{1}$$

$$Spread_i = Ask_i - Bid_i \tag{2}$$

Here, Bid_i and Ask_i are the bid and ask quotes for contract i, $P_{Market,i}$ is its midpoint, and $Spread_i$ is its quoted spread. The midpoint avoids relying on a potentially stale last trade. The spread is used as a liquidity indicator [12].

Spread groups use the final sample's 33.3rd and 66.7th percentiles: Narrow includes values at or below the first cut-off, Medium includes values above the first and at or below the second, and Wide includes the remainder. Volume is classified as zero or positive because many contracts have no recorded trades.

The S&P 500 closing level on the observation date is 7,757.64 in the series used here. Dividing by ten gives an XSP input of 775.764, consistent with the quoted level of about 775.76. Daily closes for volatility estimation come from the Federal Reserve Economic Data (FRED) SP500 series, sourced from S&P Dow Jones Indices LLC [13].

Moneyness, M_i, is the ratio of the underlying XSP level S to contract i's strike K_i, as shown in equation (3):

$$M_i = \frac{S}{K_i} \tag{3}$$

For calls, S/K > 1 means in the money; for puts, the interpretation is reversed. Contracts are grouped as low moneyness (S/K < 0.95), near at the money (ATM, 0.95–1.05), and high moneyness (S/K > 1.05).

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Time to maturity, T_i, is measured in years from the calendar-day difference between observation and expiration, as shown in equation (4):

$$T_i = \frac{\text{ExpirationDate}_i - \text{ObservationDate}}{365} \tag{4}$$

Maturity groups also use the sample's 33.3rd and 66.7th percentiles. Short includes observations at or below the first cut-off, Medium those above the first and at or below the second, and Long those above the second.

The cleaned sample contains 6,414 observations: 3,207 calls and 3,207 puts. Table 1 reports descriptive statistics.

Table 1. Descriptive statistics of the XSP option sample

Variable	N	Mean	Std.Dev.	Min	Median	Max
Market Price (Midquote)	6414	44.00	71.14	0.005	6.22	415.08
Strike Price	6414	719.14	103.90	360.00	733.00	1000.00
Moneyness	6414	1.105	0.189	0.776	1.058	2.155
Time to Maturity	6414	0.0330	0.0178	0.0082	0.0329	0.0658
Bid–Ask Spread	6414	1.95	4.52	0.01	0.08	23.88
Trading Volume	6414	9.26	72.85	0	0	2944
Historical Volatility	6414	0.1437	0.0000	0.1437	0.1437	0.1437

Note: Prices and spreads are in index-option price units; moneyness is S/K; maturity is in years. Historical volatility is annualized from 20 trading days of S&P 500 log returns.

2.2. Black–Scholes pricing framework and model inputs

Black–Scholes values are calculated for each European-style contract [1]. With a continuous dividend yield q, equations (5) and (6) give the call and put prices, while equations (7) and (8) define d_1 and d_2:

$$C_{BS} = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2) \tag{5}$$

$$P_{BS} = Ke^{-rT}N(-d_2) - Se^{-qT}N(-d_1) \tag{6}$$

$$d_1 = \frac{\ln(S/K) + (r - q + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \tag{7}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{8}$$

Here, C_{BS} and P_{BS} denote the model call and put prices; d_1 and d_2 are the standardized terms used in the valuation. S is the XSP level, K the strike, T maturity in years, r the risk-free rate, q the continuous dividend yield, σ volatility, and N(·) the standard normal cumulative distribution function. The calculation uses the processed index level, strike, and maturity, with an annualized three-month U.S. Treasury yield of 3.87% and a dividend-yield input of 1.04%.

Volatility is estimated from historical returns. Using each contract's own implied volatility would feed its observed price back into the pricing equation and mechanically improve the fit.

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Daily log returns are calculated from S&P 500 closes using equation (9):

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right) \tag{9}$$

In equation (9), r_t is the daily log return and S_t and S_{t-1} are the S&P 500 closing levels on trading days t and t − 1. These symbols differ from the risk-free rate r and XSP level S used above. Equation (10) annualizes the standard deviation of the most recent 20 trading-day returns using 252 trading days:

$$\sigma_t^{HV} = \mathrm{Std}\left(r_{t-19}, \ldots, r_t\right)\sqrt{252} \tag{10}$$

Here, σ_t^{HV} denotes annualized historical volatility, and Std denotes the standard deviation of the 20 daily log returns. Pre-observation-date data give an annualized volatility of 0.1437 (14.37%). This input is shared by all 6,414 contracts because they are observed on one date, explaining the zero cross-sectional standard deviation in Table 1. Implied volatility is retained in the source data but is not a pricing input.

2.3. Pricing error measurement

Let $P_{Market,i}$ be contract i's market midquote and $P_{BS,i}$ its model price. Equation (11) defines signed pricing error, PE_i:

$$PE_i = P_{Market,i} - P_{BS,i} \tag{11}$$

A positive error means the midquote exceeds the model price; a negative error means the model price is higher.

Absolute pricing error (APE) avoids cancellation between positive and negative errors, as defined in equation (12):

$$APE_i = \left|P_{Market,i} - P_{BS,i}\right| \tag{12}$$

APE measures the discrepancy in option price units.

Absolute relative pricing error (ARPE) scales the discrepancy by the market midquote, as defined in equation (13):

$$ARPE_i = \frac{\left|P_{Market,i} - P_{BS,i}\right|}{P_{Market,i}} \times 100\% \tag{13}$$

ARPE is expressed as a percentage. Zero midquotes must be excluded from ARPE calculations, though they can be used for other statistics. Following empirical model comparisons [8], the three measures are reported together and grouped by contract characteristics.

3. Results and discussion

3.1. Overall pricing performance

Table 2 reports a full-sample mean signed error of -0.4561 and a median of -0.3901, indicating that model prices are higher on average. Mean absolute error is 0.6629 and the median is 0.6351. In

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Tables 2–5, SE denotes signed error, APE absolute pricing error, and ARPE absolute relative pricing error.

Table 2. Overall Black–Scholes pricing errors

Sample	N	MeanSE	MedianSE	MeanAPE	MedianAPE	MeanARPE (%)	MedianARPE (%)
All	6414	-0.4561	-0.3901	0.6629	0.6351	58.4712	18.9228
Call	3207	-0.8737	-0.7668	0.8885	0.7680	56.9136	1.5796
Put	3207	-0.0384	0.0300	0.4372	0.1850	60.0288	79.5407

Calls account for most of the negative bias: their mean signed and absolute errors are -0.8737 and 0.8885, compared with -0.0384 and 0.4372 for puts. This difference is specific to the sample and volatility input. At the same S/K, calls and puts occupy different payoff regions, while market-implied volatility can vary across strikes.

The put median signed error is positive (0.0300) despite the slightly negative mean, showing how opposite errors can offset in the average.

In Figure 1, most observations lie near the 45-degree line, particularly at higher prices. The broad price range, however, can obscure differences between contract groups.

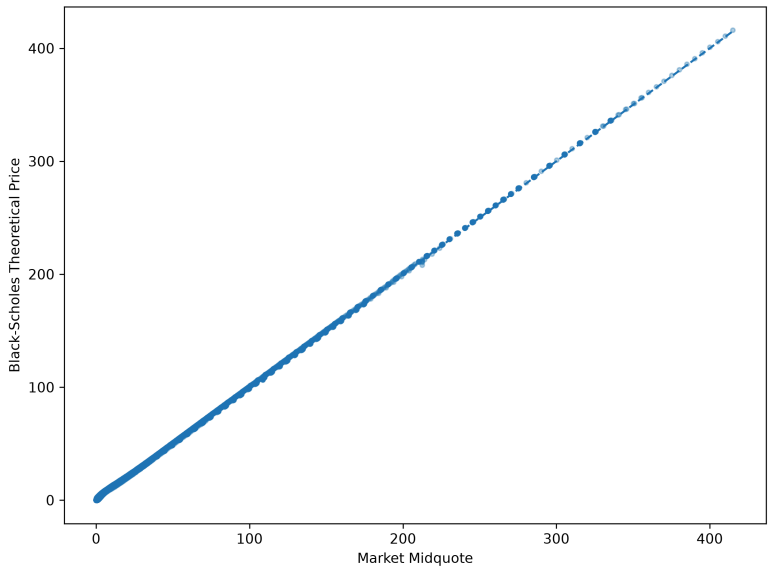


Figure 1. Market midquotes and Black–Scholes theoretical prices

Mean ARPE is 58.47%, compared with a median of 18.92%; for calls, these values are 56.91% and 1.58%. Small midquotes can turn modest price differences into large percentage errors, so ARPE is considered alongside signed and absolute errors.

3.2. Cross-sectional patterns in pricing errors

Moneyness shows the clearest group differences (Table 3). For calls, mean absolute error rises from 0.0280 at low moneyness to 1.4260 near ATM, then falls to 0.7162 at high moneyness. The corresponding signed errors are -0.0011, -1.4257, and -0.6944.

One possible explanation is Vega: option prices are generally more sensitive to volatility near the money. A common volatility input may therefore create larger discrepancies there. These descriptive

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results do not establish Vega as the sole cause.

Table 3. Black–Scholes pricing errors by moneyness

Type	Moneyness Group	N	MeanSE	MedianSE	MeanAPE	MedianAPE	MeanARPE (%)	MedianARPE (%)
Call	Low(<0.95)	373	-0.0011	0.0100	0.0280	0.0100	100.4681	99.9998
Call	Near ATM(0.95-1.05)	1140	-1.4257	-1.3217	1.4260	1.3217	126.0861	32.6102
Call	High(>1.05)	1694	-0.6944	-0.7274	0.7162	0.7295	0.7729	0.6029
Put	Low (<0.95)	373	0.8858	0.8708	0.8858	0.8708	0.9943	0.8953
Put	Near ATM (0.95-1.05)	1140	-0.5598	-0.4846	0.7783	0.6886	26.9682	15.5574
Put	High (>1.05)	1694	0.1090	0.0350	0.1090	0.0350	95.2761	100

Put mean absolute errors are 0.8858, 0.7783, and 0.1090 across the same groups, while signed errors are 0.8858, -0.5598, and 0.1090. Put errors thus change direction across strike regions.

Figure 2 shows these differences at the contract level: call errors become strongly negative around S/K = 1, while put errors change sign.

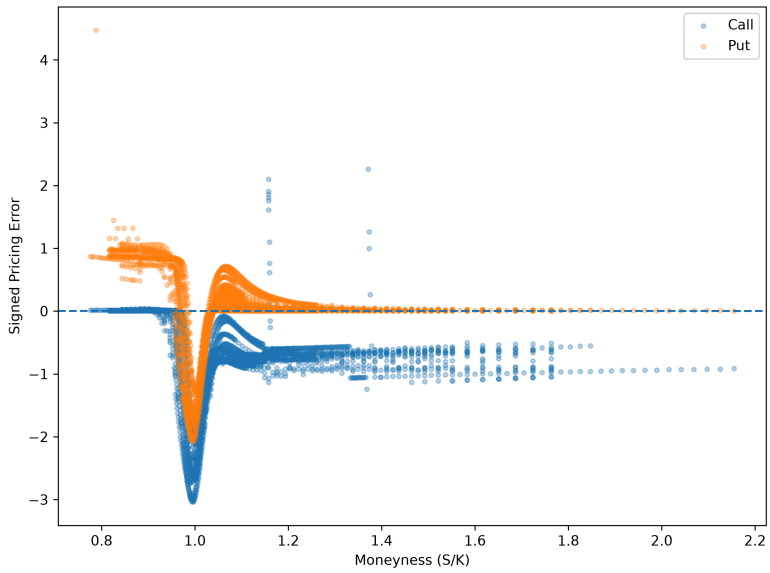


Figure 2. Signed pricing errors across moneyness

Figure 3 summarizes the absolute errors. Near-ATM calls have the largest mean error; low- and near-moneyness puts have larger errors than high-moneyness puts. These group differences are obscured by the aggregate fit in Figure 1.

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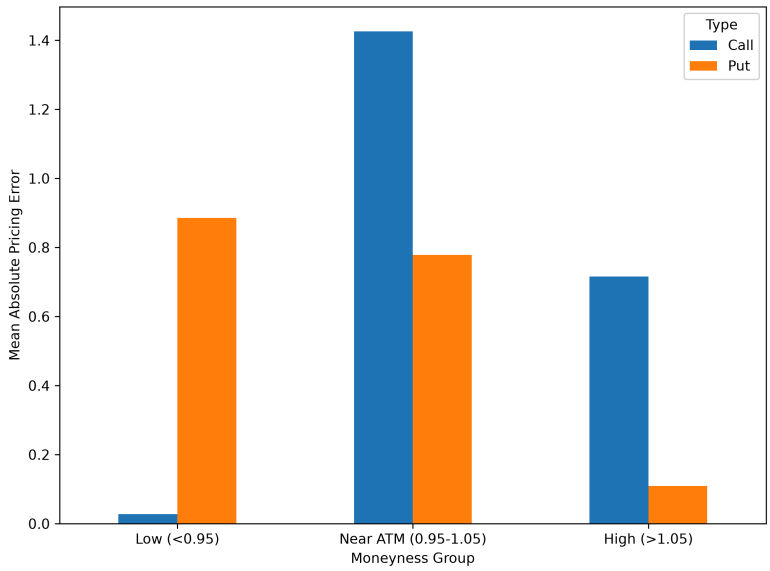


Figure 3. Mean absolute pricing errors by moneyness group

Absolute errors increase with maturity in Table 4. From short to medium to long maturity, call mean errors rise from 0.6962 to 0.9519 and 1.0369; put errors rise from 0.3122 to 0.4202 and 0.5980.

Table 4. Black–Scholes pricing errors by time to maturity

Type	MaturityGroup	N	MeanSE	MedianSE	MeanAPE	MedianAPE	MeanARPE (%)	MedianARPE (%)
Call	Short	1125	-0.6929	-0.6945	0.6962	0.6945	74.2345	1.7316
Call	Medium	1091	-0.9353	-0.8293	0.9519	0.8307	60.7031	1.6505
Call	Long	991	-1.0112	-0.7216	1.0369	0.7271	33.0789	1.2445
Put	Short	1125	0.0746	0.0150	0.3122	0.0307	67.9069	99.7573
Put	Medium	1091	-0.0468	0.0350	0.4202	0.1297	58.0198	73.5995
Put	Long	991	-0.1574	0.1100	0.5980	0.3896	53.2971	47.4824

Mean ARPE falls across these groups: from 74.23% to 60.70% and 33.08% for calls, and from 67.91% to 58.02% and 53.30% for puts (Figure 4).

These directions are compatible. Longer-dated options generally have more time value and greater Vega, so a volatility mismatch can create larger errors in price units. Short-dated options can have small midquotes, which inflate percentage errors. The maturity result therefore depends on the metric.

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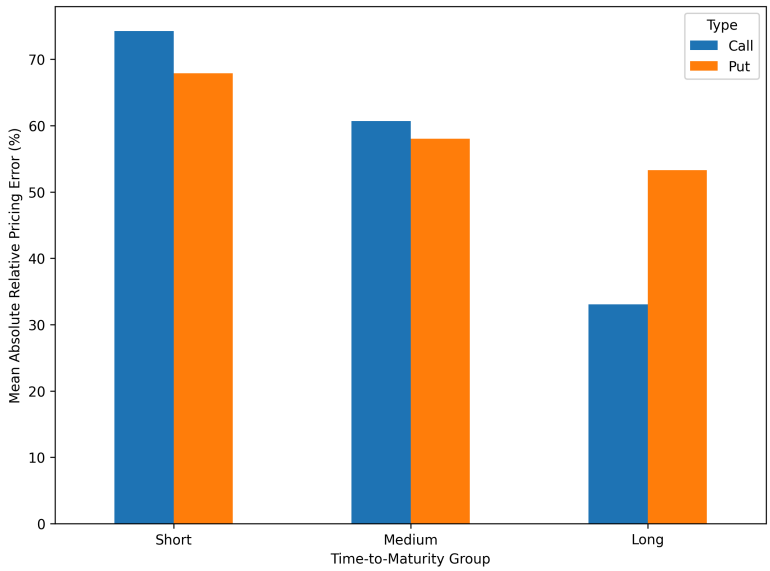


Figure 4. Mean absolute relative pricing errors by time-to-maturity group

3.3. Liquidity and pricing errors

Table 5 shows no uniform increase in errors with wider spreads or lower volume. For calls, mean absolute error is 0.4941, 1.4823, and 0.7089 across narrow, medium, and wide spreads; signed errors are -0.4845, -1.4774, and -0.6868. The largest discrepancy occurs in the medium group.

Put mean absolute error rises from 0.1623 to 0.7472 and 0.7566 across the spread groups. This differs from the call pattern, with little difference between medium and wide spreads.

Table 5. Black–Scholes pricing errors across liquidity groups

Liquidity Measure	Type	Liquidity Group	N	MeanSE	MedianSE	MeanAPE	MedianAPE	MeanARPE (%)	MedianARPE (%)
Bid-Ask Spread	Call	Medium Spread	918	-1.4774	-1.3682	1.4823	1.3682	35.5398	13.9029
Bid-Ask Spread	Call	Narrow Spread	624	-0.4845	-0.0472	0.4941	0.0472	237.9734	105.7171
Bid-Ask Spread	Call	Wide Spread	1665	-0.6868	-0.7068	0.7089	0.7100	0.8415	0.5870
Bid-Ask Spread	Put	Medium Spread	1033	-0.5263	-0.2492	0.7472	0.8048	43.9990	24.3828
Bid-Ask Spread	Put	Narrow Spread	1707	0.0687	0.0250	0.1623	0.0680	85.8133	99.8389
Bid-Ask Spread	Put	Wide Spread	467	0.6496	0.8285	0.7566	0.8436	1.2375	1.0100
Trading Volume	Call	Positive Volume	684	-1.4679	-1.4603	1.4683	1.4603	130.8294	35.6406
Trading Volume	Call	Zero Volume	2523	-0.7127	-0.7162	0.7313	0.7178	36.8746	1.0945
Trading Volume	Put	Positive Volume	903	-0.2498	0.0150	0.4999	0.2948	59.8506	49.7688

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Table 5. (continued)

Trading Volume	Put	Zero Volume	2304	0.0444	0.0300	0.4127	0.1250	60.0986	98.3245

Figure 5 shows considerable dispersion: some narrow-spread contracts have noticeable errors, while many wider-spread contracts do not.

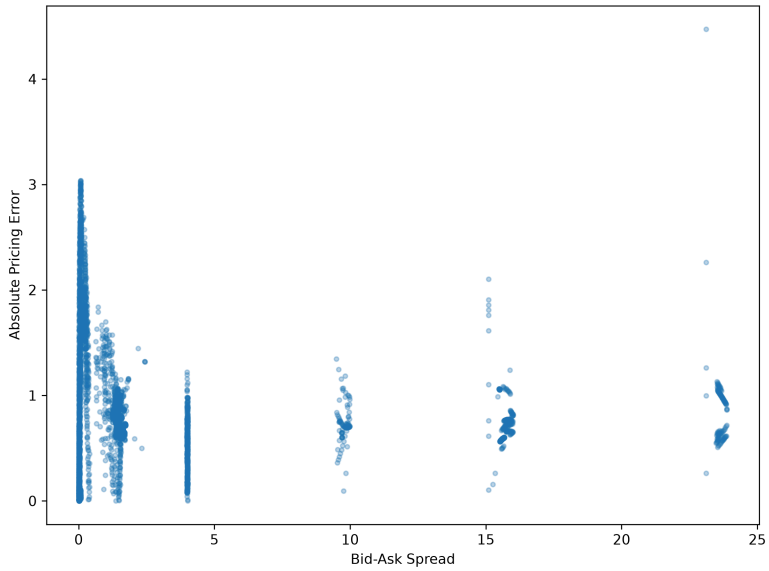


Figure 5. Absolute pricing error and bid–ask spread

Positive-volume contracts also have larger mean absolute errors than zero-volume contracts: 1.4683 versus 0.7313 for calls, and 0.4999 versus 0.4127 for puts. Active trading is not associated with smaller errors here.

These associations are descriptive. Liquidity overlaps with moneyness and maturity, and its pricing effects may differ across contract groups [14]. It provides a less consistent explanation of this sample's errors than moneyness.

3.4. Robustness check for low-priced options

To assess the effect of small denominators on ARPE, the analysis excludes midquotes below 0.10 and then below 0.50, keeping model inputs and error definitions unchanged (Table 6).

Table 6. Robustness check excluding low-priced options

Sample	N	MeanAPE	MeanARPE(%)	MedianARPE(%)
Full sample	6,414	0.6629	58.47	18.92
Midquote ≥ 0.10	4,698	0.8904	34.99	2.35
Midquote ≥ 0.50	4,089	0.9723	17.65	1.59

Mean ARPE falls from 58.47% to 34.99% and 17.65% under the two filters; median ARPE falls from 18.92% to 2.35% and 1.59%. Low-priced contracts account for much of the large full-sample percentage error.

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Under both filters, calls retain larger mean absolute errors than puts, and near-ATM calls retain the largest mean absolute error among call moneyness groups. These findings are not solely driven by very low prices.

The maturity pattern in ARPE is less stable. With Midquote ≥ 0.50, call mean ARPE is 11.94%, 18.30%, and 20.95% across short, medium, and long maturities; put values are 14.05%, 14.28%, and 24.77%. The full-sample decline with maturity disappears, suggesting that it partly reflects the distribution of low-priced contracts.

3.5. Discussion

The results describe Black–Scholes performance with a particular input choice: a common 20-day historical volatility of 14.37%. This estimate is independent of option quotes but backward-looking, whereas quoted prices reflect expectations over each contract's remaining life. The measured errors therefore reflect both the model and its inputs.

The errors across strikes and maturities are consistent with earlier evidence that volatility functions can capture variation missed by a constant-volatility specification [4]. The present benchmark cannot adjust to these differences, and the effect may be more visible near the money because of greater volatility sensitivity.

Stochastic-volatility and jump models offer ways to represent additional price variation [8, 15]. This study does not estimate those alternatives, so it cannot determine which would fit XSP best. Nor does the irregular liquidity pattern rule out liquidity effects [14]; spreads and volume may interact with other contract characteristics.

The low-price checks also show why model assessment needs more than one metric. Percentage errors are sensitive to small midquotes, while absolute errors retain information about the size of price differences. The evidence remains descriptive and limited to one day; a longer sample is needed to test whether these patterns persist across market conditions.

4. Conclusion

Using 6,414 XSP quotes from August 7, 2026, this study finds that Black–Scholes broadly tracks market price levels but leaves differences across contract groups. Calls have a larger negative bias and larger absolute errors than puts, with the clearest variation occurring across moneyness.

Maturity patterns depend on the error measure, and liquidity has no simple monotonic relationship with errors. Removing low-priced contracts sharply reduces ARPE and changes its maturity pattern, while the main absolute-error and moneyness findings remain.

Black–Scholes remains a useful benchmark for this sample, although one historical-volatility estimate cannot represent all strike- and maturity-specific information in quotes. The residual patterns support examining more flexible volatility specifications [16]. A multi-date study could compare historical, implied, and time-varying volatility inputs, together with stochastic-volatility, jump, or liquidity-based models.

References

[1] Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy, 81*(3), 637–654.

[2] Macbeth, J. D., & Merville, L. J. (1979). An empirical examination of the Black–Scholes call option pricing model. *The Journal of Finance, 34*(5), 1173–1186.

DOI: 10.54254/2754-1169/2026.37764

[3] Merton, R. C. (1973). Theory of rational option pricing. *Bell Journal of Economics and Management Science, 4*(1), 141–183.
[4] Dumas, B., Fleming, J., & Whaley, R. E. (1998). Implied volatility functions: Empirical tests. *The Journal of Finance, 53*(6), 2059–2106.
[5] Heston, S. L. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. *The Review of Financial Studies, 6*(2), 327–343.
[6] Dupire, B. (1994). Pricing with a smile. *Risk, 7*(1), 18–20.
[7] Wallmeier, M. (2024). Quality issues of implied volatilities of index and stock options in the OptionMetrics IvyDB database. *Journal of Futures Markets, 44*, 854–875.
[8] Bakshi, G., Cao, C., & Chen, Z. (1997). Empirical performance of alternative option pricing models. *The Journal of Finance, 52*(5), 2003–2049.
[9] Cboe Exchange, Inc. (2020). Options on Mini-SPX (XSP) index: Access the advantages of SPX options at 1/10 the contract size. *Cboe U.S. Options Research Publication*.
[10] Aït-Sahalia, Y., Li, C., & Li, C. X. (2021). Implied stochastic volatility models. *The Review of Financial Studies, 34*(1), 394–450.
[11] Cboe Global Markets. (2025). XSP options product specifications. *Cboe Global Markets*. https://www.cboe.com/tradable-products/sp-500/xsp-options/specifications (accessed September 14, 2026).
[12] Bollen, N. P. B., & Whaley, R. E. (2004). Does net buying pressure affect the shape of implied volatility functions? *The Journal of Finance, 59*(2), 711–753.
[13] S&P Dow Jones Indices LLC. (2026). S&P 500 [SP500]. *Federal Reserve Economic Data (FRED)*, Federal Reserve Bank of St. Louis. https://fred.stlouisfed.org/series/SP500 (accessed September 14, 2026).
[14] Feng, S.-P., Hung, M.-W., & Wang, Y.-H. (2016). The importance of stock liquidity on option pricing. *International Review of Economics & Finance, 43*, 457–467.
[15] Merton, R. C. (1976). The impact on option pricing of specification error in the underlying stock price returns. *The Journal of Finance, 31*(2), 333–350.
[16] Singh, V. K., & Kumar, P. (2024). Effectiveness of deterministic option pricing models: New evidence from Nifty and Bank Nifty Index options. *Journal of Asset Management, 25*(2), 172–189.