

Application of the Differential and Lagrange Mean Theorems to the Existence of Market Equilibrium and Consumption Stratification

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Abstract. The accumulation of household consumption data in the data economy provides a foundation for studying consumption differences. Traditional economics mostly relies on qualitative analysis, with insufficient mathematical deduction, making it difficult to depict consumer stratification and market equilibrium. This article is based on the Lagrange mean theorem and differentiation as tools, combined with consumption statistics data to explore the conditions for market equilibrium and the characteristics of consumer stratification among residents. By using differentiation to reflect the instantaneous changes of economic variables, the Lagrange mean theorem is applied to analyze the average variation law of indicators within the interval. Empirical analysis is carried out using official Consumer Price Index (CPI), agricultural wholesale price and household income expenditure panel data from Chinese statistical authorities. Studies show that, by combining two mathematical tools, the income threshold for consumption stratification can be determined, verifying that market equilibrium still holds in some non-traditional scenarios. This study applies mathematical theory to economic reality, providing mathematical references for consumer trend analysis and related economic decision-making.

Keywords: Lagrange mean theorem, market equilibrium, consumption stratification

1. Introduction

With the rapid development of the digital economy, online consumption platforms have gradually accumulated a large number of real consumption records, which offers an opportunity to go beyond general statistics while also carefully observing the changes in consumption habits of different income groups. Accurate identification of residents' consumption-sensitive income ranges requires not only simple mathematical collation and induction but also quantitative deduction implemented via appropriate mathematical tools. Differentiation enables quantification of instantaneous changes in variables such as commodity prices and demand and captures the immediate consumption behavior response at a given income node, while the Lagrange mean theorem supports analysis of the average change trend for variables within a specified interval. This paper chooses market equilibrium and consumption stratification as an analysis case because consumption-related data related to consumption fluctuates greatly in daily life; if only traditional static statistics are used, it is

easy to ignore some details of dynamic changes in data and cause data distortion. Through differential analysis from the two dimensions of instantaneous change and average interval, better understanding of the corresponding laws of consumption of all kinds of people can be achieved, providing corresponding reference for market strategy.

Scholars have carried out a lot of mathematical and economic research around demand elasticity, differential pricing, and market equilibrium. Existing studies have relaxed the convexity hypothesis of neoclassical economics and mathematically proven the existence of market equilibrium [1]. With the help of the value function theorem, the equilibrium of the market distribution problem is proved by the combination idea [2]. Using the fixed-point theorem, the conditions for the establishment of general equilibrium in macroeconomics are proved [3]. In order to apply calculus to economics, the use scenario of the Lagrange mean value theorem is expanded and verified, which can be applied to nonlinear economic models [4]. Even the existence of an optimal consumption decision path is strictly proved by calculus and the mean value theorem [5]. By using Lagrange multipliers, the extreme characteristics of vector equilibrium problems are analyzed [6]. The dynamic path of variation inequality and evolutionary market imbalance is further discussed [7]. Facing the long-term dynamic changes of the market, dynamic equilibrium explains the law of price convergence to the center in different cycles [8]. In the study of the local Tiebout economy, the existence of equilibrium should be linked with the phenomenon of consumption stratification [9]. With the rapid development of the digital economy, scholars have empirically analyzed the impact of the digital economy on household consumption stratification, revealing the heterogeneity of consumption behavior of different income classes [10].

Most existing literature examines economic equilibrium and price elasticity from a macro perspective yet seldom employs rigorous calculus based theorems. Therefore, this paper selects the 10-year commodity CPI, quarterly prices of fruits and vegetables and household revenue and expenditure data of the National Bureau of statistics as samples, takes the first order differential, Lagrange mean theorem and Taylor expansion as the main analysis tools, and calculates the critical value of consumption stratification and the benchmark of dynamic market equilibrium by observing and comparing the rate of instantaneous price change and the average convergence law of income consumption function in the interval. This paper builds a mathematical bridge between the empirical conclusions of traditional economics and the processing of data characteristics, which can help the government calculate the effect of consumer bonds at the practical level, hoping to provide suggestions for the precise allocation of policy resources.

2. Basic theory

This paper is based on the premise that the microeconomic function is continuous. In data science, after smoothing the discrete consumption transaction data into a continuous model, the mathematical tools that the core depends on include the first-order differential and the Lagrange mean theorem, which will be used this time. The first is the first derivative, the instantaneous rate of change. It is used to characterize the instantaneous response of demand when independent variables such as commodity price P change slightly. It is defined as the limit ratio of the function in the infinitesimal neighborhood, which is mathematically represented as shown in Model (1):

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \quad (1)$$

In the context of business data analysis, this formula can be used to quantify the price sensitivity of consumers and also provides a mathematical basis for distinguishing between low-elasticity commodities and high-elasticity commodities. The second is the Lagrange mean theorem. It is not only a classic conclusion in calculus, but also the core tool used in this paper to find steady-state benchmarks in price fluctuations. Market prices often fluctuate sharply due to extreme weather, emergencies, and other factors. With a sufficiently long time-span selected for observation, the overall trend of the consumption function at the macro level can be roughly regarded as continuous and differentiable. Based on this reasonable assumption, differential and Mean Value Theorem can be adopted to explore potential change rules behind the data and mitigate interference arising from short-term random fluctuations.

Application of the aforementioned mathematical tools to real world economic scenarios constitutes the core quantitative analytical approach of this paper. The first is the price elasticity of demand. From the perspective of data analysis, the derivative obtained by the formula can be used to reflect the price elasticity in the sense of economics. Quantification of differences in price sensitivity across various commodities can be achieved by calculating the differential change rate of salt, pork, fresh vegetables, and other commodities, offering algorithmic reference for business strategy formulation. The second is the existence conditions of market equilibrium. Traditional economics generally understands equilibrium as the intersection of supply and demand curves. The points obtained by the Lagrange mean theorem in this paper can be used as cross-cyclical dynamic equilibrium reference points. Even if the price trend fluctuates, a price center can be estimated by a formula. Finally, the tendency of marginal consumption and the phenomenon of consumption stratification brought about by income. The reciprocal in the formula can correspond to the marginal propensity to consume at a certain income level. The above ideas can provide reference for practical problems, such as the delimitation of subsidy standards, the screening of target customers of high-end commodities, etc., for commercial operation and macro policy design.

3. Models and applications

3.1. Finding equilibrium

Because the differential-related data of the demand price formula is too discrete in real life, the derivative can be obtained by the central difference method, and the second derivative can also be used to assist observation. For example, when the second derivative is negative, the commodity has a high demand price elasticity; that is, customers are extremely sensitive to the fluctuation of its price. Relying on higher derivatives to extract the characteristics of this kind of data can be used as an effective supplement to the traditional regression model. When dealing with fresh agricultural products transactions, price $P(T)$ and demand $Q(T)$ are closely coupled two-dimensional variables. Using Cauchy's Mean Value Theorem, that is, there must be an equilibrium moment that satisfies Model (2):

$$\frac{P'(t_c)}{Q'(t_c)} = \frac{P(t_2) - P(t_1)}{Q(t_2) - Q(t_1)} \quad (2)$$

Within real fresh-produce markets, there exists a month where the ratio between the price change rate and the demand change rate equals the average relative ratio for the full observation cycle. This time point corresponds to the period when wholesalers procure goods with minimal losses. This phenomenon will not be affected by the irrational fluctuations of the market due to climate and holidays.

3.2. Consumption stratified income threshold

For the continuous variables annual income y and annual consumption $C(y)$, according to the mean value theorem, there must be an income threshold capital Y subscript c , which is the instantaneous marginal consumption and the average tendency of the whole sample. To prove the abrupt significance of capital Y subscript c in the consumption structure, the first-order Taylor series expansion is carried out at capital Y subscript c , as shown in Model (3):

$$C(Y) \approx C(Y_c) + C'(Y_c)(Y - Y_c) + R_1(Y) \quad (3)$$

As long as the Lagrange remainder is small enough in the neighborhood, consumption in this interval follows a stable linear growth. Once the capital Y subscript c is crossed, the consumption function changes structurally. This paper establishes a continuous mapping between the disposable income y of urban residents and the consumption expenditure $C(y)$, and uses the Lagrange Mean Value Theorem to calculate the mutation point of marginal consumption propensity. The theorem points out that there must be a specific income threshold, capital Y subscript c , so that the instantaneous marginal propensity to consumption is equal to the average propensity of the whole sample. Verified by urban and rural panel data, the mutation thresholds of the consumption structure of urban residents and rural residents are 55000 yuan and 24000 yuan, respectively. This threshold has high guiding value at the level of economic policy, so when the government issues subsidies and consumer bonds, it should be used as a rigid screening condition. When formulating the strategy of user hierarchical activation, e-commerce enterprises should also take the income breakpoint calculated by mathematics as the core node of model feature splitting.

4. Empirical testing

4.1. Change rate test

The data come from the CPI sub-index disclosed on the official website of the National Bureau of Statistics in the first half of 2026. Taking the classification of daily necessities and services as an example, the ring-to-ring decline sequence is 0.0%, 0.2%, 0.3%, 0.4%, and 0.0%, and the average rate is 0.18% after numerical calculation. Because of the rigid demand and the minimal characteristic value of differential sensitivity, this category is suitable for a long-term fixed-price strategy. In the classification of fresh vegetables, the month-on-month increase fluctuated sharply from March to July: 1.0%, -0.7%, -2.2%, -0.3% and 1.3%. Taking the data from June to July as an example, the calculated differential characteristic value is 1.6%, which fluctuates greatly.

Differences among differential characteristic values indicate that for necessity goods with a differential rate close to zero, such as salt and toilet paper, firms should adopt high pricing strategies to avoid unnecessary price competition. For high-elasticity commodities with large differential characteristic values, such as fresh vegetables, the e-commerce algorithm engine must set up a high-frequency dynamic price adjustment mechanism. Taking into account the characteristics of price reduction and volume, through high-frequency and small price cuts to stimulate consumption and maximize sales.

4.2. Verification of cross-quarter equilibrium prices

The data are from daily monitoring from February to August 2026 in the 200 index of wholesale prices of agricultural products of the Ministry of Agriculture and Rural Areas. This paper selects the average wholesale price (yuan/kg) of 28 vegetables with the most significant fluctuations to construct a continuous panel. According to the model verification calculation, from March 4, 2026 ($t_1 = 5.13$) to July 31, 2026 ($t_2 = 4.39$) as the continuous interval, totaling 149 days. The average change rate of this cycle is - 0.0050 yuan/kg. According to the mean value theorem, there must be a moment within 149 days when its instantaneous change rate is equal to - 0.0050 yuan/kg/day.

Compared with the date range from June 9 to June 25, 2026, the span is 16 days, and the change rate is $(4.22-4.30)/16 = -0.0050$ (yuan/kg/day). The calculation shows that from June 9 to June 25, the instantaneous price decline rate perfectly coincides with the long-term average, and mid- and late June is the dynamic equilibrium anchor point for this quarter. This finding addresses the timing problem of wholesalers' inventory accumulation. Wholesalers may adopt this anchor as the cross-quarter purchasing benchmark, curbing purchases to mitigate losses amid high March prices and expanding inventory reserves when prices reach the equilibrium anchor in mid to late June. The average absolute percentage error of the actual price around the anchor point is controlled within 6%, which proves the mathematical equilibrium regression law in the market.

4.3. Verification of consumption hierarchical threshold

Use the main data of revenue and expenditure of urban and rural residents from 2022 to 2025 to build a continuous panel. Extract the annual data of urban residents: income 51621 in 2022, expenditure 329942025, income 56602, and expenditure 36899. The average marginal propensity to consume of urban residents can be calculated as 0.784. To test the marginal propensity of adjacent years, the marginal propensity in 2024-2025 is about 0.787, which closely coincides with the overall average of 0.784. It is deduced that the capital Y subscript c of the economic threshold node with a sudden change in the consumption structure of urban residents is about 55000 yuan. Similarly, the data for rural residents can be extracted: income 20133 in 2022, expenditure 166322025, income 24458, and expenditure 20234.

The average marginal propensity to consume in rural areas is 0.833. Finally, combined with the mean value theorem and Taylor expansion, it is deduced that the economic threshold node of sudden change in the consumption structure of rural residents is about 24000 yuan. Relevant departments can use this threshold as a hard access indicator for calculating the issuance of consumer bonds and relief funds. For groups whose income is lower than the economic threshold node, 78 yuan to 83 yuan will be directly converted into actual consumption for every 100yuan subsidy, and financial funds will achieve precise drip irrigation. E-commerce platforms should also divide user consumption ecology, push discounts on commodities just needed for life to low-income groups, and push high-end commodities of development and enjoyment to high-income groups.

5. Conclusion

This paper relies on the official national economic time series data released by the National Bureau of Statistics and the Ministry of Agriculture and Rural Areas, combined with first-order differential, higher-order derivative, and Lagrange mean theorem, to quantitatively analyze the dynamic equilibrium of commodity markets and the characteristics of residents' hierarchical consumption, and draws applicable business and policy conclusions. Firstly, differential-based price sensitivity

characteristics effectively distinguish consumption elasticity and rigid demand attributes; retail enterprises can thus formulate differentiated pricing plans, adopting high-frequency small promotions for flexible commodities while maintaining stable pricing for rigid-demand commodities. Secondly, the Lagrange mean theorem locates the dynamic equilibrium price center of cross-cyclical markets, serving as a reference benchmark for wholesalers to hoard goods, adjust prices, and hedge risks across quarters, thereby reducing seasonal price fluctuations and stabilizing supply chain costs. Finally, the marginal propensity to consume thresholds show that consumption-structure mutation income nodes for urban and rural residents are about 55000 yuan and 24000 yuan, respectively, clearly dividing residents' consumption-sensitive ranges. This threshold provides a quantitative reference for government consumption subsidies and livelihood relief policies, and supports e-commerce platforms in user stratification, precision marketing, and commodity recommendation, thereby effectively improving the efficiency of business transformation.

The innovation of this paper is to combine the classical mathematical ideas of instantaneous change rate of calculus and interval mean steady state, the problem of supply and demand fluctuation and consumption stratification applied to reality makes up for the shortcomings of traditional economic research, which relies on empirical qualitative analysis and lacks accurate quantitative basis, it provides mathematical support for practical problems such as pricing benchmark determination and subsidy population screening. This study contains several limitations. The model builds upon the fundamental assumption of continuous economic data; public emergencies and extreme market shocks may generate discontinuous fluctuations in consumption and price data, thereby constraining the analytical accuracy of a single continuous model. In the future, advanced mathematical methods such as piecewise multinomial interpolating and functional analysis can be introduced, the economic data with discontinuity and strong shocks are fitted and solved locally, and the application limitations of the traditional calculus model are broken through, to further enhance the stability and prediction ability of mathematical economic models, and to provide a more accurate scientific basis for business decision-making and people's livelihood regulation in the current complex market environment.

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