

Model Uncertainty in Asset Pricing Models Based on Bayesian Methods

Mingyue Xue

*China School of Banking and Finance, University of International Business and Economics,
Beijing, China
202342037@uibe.edu.cn*

Abstract. Traditional asset pricing theory assumes that investors are aware of the "single true model", and thus fails to explain the widespread phenomenon of model uncertainty in the financial market. To investigate how asset prices are determined in a multi-model environment, this paper constructs a system that integrates Bayesian belief updating with the transmission of beliefs to asset prices. Using the Capital Asset Pricing Model (CAPM) and the Fama-French three-factor model (FF3) as competing benchmark models, and in conjunction with the Gordon Growth Model, this paper derives the evolution of asset prices after new information shocks. Therefore, based on the above results, investors use Bayes' theorem to adjust the weight given to different asset pricing models according to new information. The above non-linear changes in subjective beliefs alter the overall discount rate of the asset and thus have dynamic effects on its risk premium and absolute valuation. Based on the above derivation, it can be seen that a shift in investors' model beliefs is an internal source of excess volatility in asset prices, and this paper offers a new theoretical perspective on price formation under multiple models.

Keywords: asset pricing, model uncertainty, Bayesian updating

1. Introduction

The Problem of Asset Pricing in Modern Financial Research. With the development of the Capital Asset Pricing Model (CAPM), scholars have expanded the set of priced risk factors and built multi-factor models to better explain cross-sectional differences in asset returns. Traditional asset pricing theory generally assumes that investors have full information about the actual structure of the market, and thus there is no uncertainty at the model level.

Due to the complexity of the financial market in practice, there is no single model that can be regarded as the sole and absolute truth of the market. All the asset pricing models can explain some of the patterns in returns under certain circumstances, but they are not the same. Heterogeneity in the attributes of investors, for example, leads to multiple competing models selected by different investors simultaneously. Model uncertainty directly impacts the formation of investors' expectations and, thus, asset risk premia and valuations. Therefore, the way in which model uncertainty is transmitted has not been studied in the existing literature.

Given the reasons above, this paper will try to answer the following three questions: When multiple asset pricing models are available, how do investors form their initial opinions of these models? How do investors change the weight of the model dynamically after the release of new information? How do changes in model beliefs affect asset risk premia and valuation?

This paper has both theoretical and practical aims. From an academic standpoint, this paper does not conform to the previous assumption of a "single true model" in rational expectations theory, introduces Bayesian learning for asset pricing analysis, and sets up a framework of dynamic belief updates under multiple models to offer a simpler theoretical transmission path. The framework shows how a change in model confidence can lead to a change in required return and valuation in practice, but it has not been empirically verified.

The rest of this paper is organised as follows. Section 2 reviews the literature on the development of asset pricing models, the application of Bayesian methods in asset pricing and research on model uncertainty; Section 3 introduces the theoretical foundation and key concepts and general settings; Section 4 constructs the model and analyzes its underlying mechanism, derives the updating process for model weights, and examines the impact on risk premium and valuation; Section 5 summarizes and proposes future research directions.

2. Literature review

2.1. From a single asset pricing model to model uncertainty

The first studies in modern asset pricing mainly focused on relatively well-defined pricing models. Sharpe put forward the Capital Asset Pricing Model (CAPM) to explain expected returns as a function of risk premia for market risk [1]. Subsequently, Fama and French added other factors, such as size and book-to-market ratio, to develop the Fama-French three-factor model (FF3) and improved the explanatory power of asset pricing models for differences in stock returns [2]. As the number of candidate factors and models has been continuously increasing, the quantity of candidate explanatory variables in asset pricing is now very large, and variable selection has become an urgent problem [3]. The appearance of the concept of a "factor zoo" in recent years is another example of this.

Therefore, it can be seen that the idea of "uncertainty" in the older asset pricing model needs to be adjusted. Even after the investors have selected a specific asset pricing model, its parameters and pricing errors still need to be estimated from sample data, thus introducing parameter or estimation uncertainty in the model. At the same time, if investors are not sure whether to use CAPM or another specification of the Fama-French model, they face a higher-level problem of model uncertainty. Pástor and Stambaugh were among the first to use the Bayesian portfolio approach, assigned prior probabilities to the different asset pricing models, and then determined which models were more suitable for investment decisions based on sample data [4].

2.2. From a single model to multiple competing models

With the expansion of the scope of research from a single model to several candidate models, a new problem has arisen: which model is more suitable for investors? Avramov studied model uncertainty in stock return prediction and, based on different combinations of predictors, constructed multiple forecasting models; then, using Bayesian Model Averaging (BMA), assigned different weights to the forecasts of these models according to their posterior probabilities rather than assuming that any single model was correct [5]. Based on the above results, when selecting a model is not clear, model

averaging can provide more stable out-of-sample forecasts; otherwise, if model uncertainty is not addressed, a serious loss in investment decisions may occur.

Later, some studies have begun to compare the performance of the above models directly. Barillas and Shanken propose a Bayesian asset pricing test framework with closed-form calculations that computes the posterior probabilities of different competing models in a model space consisting of subsets of candidate tradable factors, and thus compares the extent to which each model is supported by the data [6]. Barillas and Shanken's framework does not combine the forecasts of multiple models, as BMA does; instead, they select a model based on the calculation of the posterior model probability to determine how much weight investors should give to each competing model according to the data.

More recently, some scholars have also been studying this problem. Huang and Shi develop a Bayesian framework to first calculate the posterior probability that each candidate model is the "true model" based on the data, and then use the entropy of these posterior probabilities to quantify model uncertainty [7]. A high entropy indicates that the posterior probabilities of the models are more uniformly distributed, and it is harder for investors to determine which model is better. They also show that even when the set of candidate models omits some true factors, the measure is still within a controllable range and does not generate a spurious signal of "extreme uncertainty". Therefore, the research results suggest that model uncertainty is no longer merely a side effect of model testing but has gradually become a subject of study in itself in asset pricing research.

2.3. From static model comparison to dynamic belief updating

Model comparison addresses the question of which model should be given more weight at present. As new information continuously enters the financial market, investors' models of the world are also changing. Bayesian learning provides a natural analytical framework for this process: investors first form prior probabilities of all models, and when new information, such as returns and factor realizations, is available, they update their beliefs based on the conditional probability that each model explains the information to obtain new posterior model probabilities.

It also falls under the general category of research on investor beliefs. Chen, Hansen and Hansen do not directly compare the competing models; instead, they employ forward-looking information in asset prices and asset pricing models to derive investors' actual beliefs from market data [8]. The framework does not meet the requirements of full rationality for investors and, therefore, will be subject to statistical errors in identification. The above studies have shown that investor beliefs can be used as an independent subject of analysis for asset pricing, rather than being an implicit assumption in a model, and thus expanded the scope of research on asset pricing.

2.4. From model beliefs to risk premium and asset valuation

Model uncertainty will have an economic cost because the different models show different expected returns and risk-reward trade-offs. When new information is released, investors first adjust their beliefs about the various models; as a result, their assessment of the model-average expected return changes, and thus the risk premium and required return are altered.

Hansen selects an alternative linear factor model from an out-of-sample economic perspective, based on actual portfolio performance, and adds model mispricing uncertainty in a Bayesian framework [9]. According to the above results, although factor models perform well in-sample, they generally do not outperform simple historical benchmark models out-of-sample; their advantages are mainly limited to short-term industry portfolio allocation. Carrasco and Hansen have also examined

the portfolio allocation choices of Bayesian mean-variance optimisers by explicitly adding uncertainty in model mispricing to the asset allocation framework [10]. The study assumes that investors have a zero-centered prior for model pricing errors (alpha) while remaining uncertain about whether true pricing deviations exist. Based on the above empirical results, accounting for mispricing uncertainty expands the point estimates of portfolio returns and significantly reduces their statistical significance. At the same time, the investment value of the models is re-evaluated, and some factor models that appeared superior under the conventional evaluation are no longer so after considering the uncertainty of pricing errors.

A change in expected returns is generally reflected in the price of assets as a lower discount rate. Therefore, a change in model beliefs will affect asset prices by changing the expected return and, thus, the discount rate.

2.5. Literature review summary

In short, the existing studies have shown a general trend. The first studies focused on models for pricing individual assets and parameter uncertainty; later, several competing models and the development of approaches such as model averaging, direct model comparison, and model probabilities gradually appeared. Recently, more research has been carried out on how changes in investors' beliefs about new information affect asset returns and prices as a result of these shifts in belief. Researchers have achieved various results in the past, but they have not all focused on the same problems of model evaluation, portfolio selection and investor beliefs.

Based on the theoretical developments presented above, this paper investigates the mechanism of Bayesian updating of model beliefs in light of new information under a set of simplified assumptions, including two competing linear factor models, normal conditional likelihoods, exogenous cash flows and growth rates, and the Gordon growth model. Specifically, the study derives how new information changes the posterior model weights and uses local comparative statics to examine how alterations in these weights affect the model-averaged expected return and the resulting valuation under the Gordon growth model.

3. Theoretical foundation

This chapter will introduce the main theoretical foundation, key concepts and basic assumptions of the study in an organised manner to provide a solid theoretical basis for constructing a belief-updating model with multiple competing pricing models and examining how model uncertainty affects asset prices. The first part of this chapter introduces the general ideas of Bayesian inference, then presents the specific forms of two well-known models for asset pricing, the Capital Asset Pricing Model (CAPM) and the Fama-French three-factor model (FF3), and finally builds the expected return system. Finally, the Gordon Growth Model provides the theoretical basis for examining changes in asset values.

It should be pointed out that the Bayesian belief-updating framework built in this study is general in form and can handle any number of competing models. Besides the two benchmark models mentioned above, other well-known factor pricing models can also be included in the framework, such as Merton's Intertemporal Capital Asset Pricing Model (ICAPM), Breeden's Consumption Capital Asset Pricing Model (CCAPM), and Fama and French's five-factor model (FF5) [11-13]. A framework can also be used to build a non-factor characteristic-based pricing model, such as the firm-characteristics-based pricing model put forward by Daniel and Titman [14]. Adding these alternative models only requires replacing the corresponding return-generating processes and

conditional expected return specifications; the core mechanism of belief-weight updating and the subsequent asset pricing mechanism developed in this study will remain the same.

3.1. Bayesian inference and belief updating

Bayesian methods are the foundation of statistics under uncertainty. The first is to change a person's previous beliefs based on new data; thus, new beliefs are formed and existing beliefs can be modified when new information is acquired. Bayes' Theorem can be written as follows:

$$P(M|D) = \frac{P(D|M)P(M)}{P(D)} \quad (1)$$

where $P(M)$ is the prior probability, representing the decision-maker's belief in model M before new information is observed. $P(D|M)$ is the conditional likelihood of observing data D under model M. $P(M|D)$ is the posterior probability, representing the updated belief in model M after data D are observed.

Given the uncertainty in the asset pricing model, Bayes' theorem can be employed to determine how an investor's belief in such a model changes after acquiring new information. When new information, such as realised returns and prices, is released in the market, investors will change their views according to which pricing model can better explain this new information. A model with a smaller prediction error and stronger explanatory power for the new data will be given a higher subsequent belief weight; otherwise, this weight will decrease as its relative explanatory performance declines.

3.2. Specifications of the competing asset pricing models

First, the purpose of asset pricing theory is to determine how much risk should be added to the expected return. The two representative competing models with rich theory and empirical support in this study are the classical asset pricing models. These are the Capital Asset Pricing Model (CAPM) and the Fama-French three-factor model (FF3). Both models describe the generation of asset excess returns through a linear factor structure and assume normally distributed disturbance terms, thus providing an analytically tractable basis for the following Bayesian likelihood calculations.

Model A is the Capital Asset Pricing Model (CAPM).

The CAPM assumes that the expected return of an asset is determined by the risk-free rate and its exposure to the market portfolio, as measured by its market beta, i.e.,

$$R_t = R_f + \beta_A R_{M,t} + \varepsilon_{A,t} \quad (2)$$

where R_t denotes the asset's realized return in period t. R_f denotes the market risk-free rate. β_A denotes the asset's market beta, which measures the sensitivity of the asset's return to market risk. $R_{M,t}$ denotes the excess return on the market portfolio in period t. $\varepsilon_{A,t}$ denotes the random disturbance term, assumed to follow a zero-mean normal distribution $\varepsilon_{A,t} \sim N(0, \sigma_A^2)$, where σ_A^2 denotes the residual variance of the CAPM.

Assume that F_t denotes the investor's information set at time t . The conditional expected return under Model A is given by:

$$R_t^A = E[R_t | M_A, F_{t-1}] = R_f + \beta_A E[R_{M,t} | F_{t-1}] \quad (3)$$

Model B is the Fama-French three-factor model (FF3).

In addition to the market factor, the FF3 model also includes the size and value factors to capture the size and value premia in the stock market, respectively:

$$R_t = R_f + \beta_B R_{M,t} + s_B SMB_t + h_B HML_t + \varepsilon_{B,t} \quad (4)$$

where β_B, s_B, h_B denote the asset's risk exposures to the market, size, and value factors, respectively. SMB_t denotes the size factor return in period t , constructed as the return difference between small-cap and large-cap portfolios, thereby capturing the size premium. HML_t denotes the value factor return in period t , constructed as the return difference between portfolios with high and low book-to-market ratios and capturing the associated value premium. $\varepsilon_{B,t}$ denotes the random disturbance term, which is assumed to follow a normal distribution with mean zero $\varepsilon_{B,t} \sim N(0, \sigma_B^2)$, where σ_B^2 denotes the residual variance of the FF3 model.

The conditional expected return under Model B is also given by:

$$R_t^B = E[R_t | M_B, F_{t-1}] = R_f + \beta_B E[R_{M,t} | F_{t-1}] + s_B E[SMB_t | F_{t-1}] + h_B E[HML_t | F_{t-1}] \quad (5)$$

3.3. Model uncertainty and the belief weighting framework

Model uncertainty refers to the lack of information investors have on the source of the return. The above models can all explain the experimental results, and investors' opinions on their effectiveness vary from person to person. The probability weights of the competing models in this study are considered to be the investors' beliefs.

Suppose that the market contains n competing models, denoted by $M_1, M_2, M_3, \dots, M_n$. Let $w_{i,t}$ denote the investor's belief weight assigned to model at time t . This weight represents the investor's subjective probability that model M_i is the true model, and the weights satisfy: $\sum_{i=1}^n w_{i,t} = 1$, $w_{i,t} \in [0, 1]$.

Given n competing models, the investor's model-averaged expected excess return $E[R_t]$ is constructed as the weighted average of the conditional expected excess returns implied by the competing models, where the weights correspond to the investor's belief probabilities:

$$E[R_t | F_{t-1}] = \sum_{i=1}^n w_{i,t} \bullet E[R_t | M_i, \mathcal{F}_{t-1}] \quad (6)$$

3.4. Asset valuation benchmark: the Gordon growth model

To analyze how changes in model beliefs affect asset valuation, this study adopts the Gordon growth model as the theoretical benchmark for changes in expected cash flows. The model is based on the perpetual cash flow discounting framework and assumes that the asset's future cash flows grow at a constant rate g . Its core pricing equation is:

$$P_t = \frac{cashflow_t(1+g)}{r_t - g} \quad (7)$$

where P_t denotes the intrinsic value of the asset at time t , $cashflow_t$ denotes the current cash flow. r_t denotes the investor's required return, i.e., the discount rate. g denotes the perpetual growth rate, which must satisfy $r_t > g$ to ensure valuation convergence.

As the discount rate rises, the asset value implied by the Gordon Growth Model decreases. When investors alter their assessments of the competing models, the model-averaged expected return will be modified accordingly; thus, a different discount rate will be applied to future cash flows and the implied asset value will change.

4. Model construction and mechanism analysis

4.1. Construction of expected returns and bayesian belief updating

Before examining the dynamic updating mechanism, the analysis characterizes the investor's information set and belief state. Building on the generalized framework developed in Section 3.3, and to maintain both analytical tractability and an intuitive derivation, Model A (CAPM) and Model B (FF3) are treated as the two competing benchmarks. Suppose that at the end of period $t - 1$, which corresponds to the beginning of period t , the investor faces model uncertainty and assigns Model A a subjective belief weight, or prior probability, of w_t . Because only two competing models are considered, Model B is assigned a corresponding belief weight of $1 - w_t$.

In this setting with multiple competing models, the investor's expectation for the asset in period t is not determined by any single model but instead is formed as a belief-weighted aggregate across the competing models. The model-averaged expected return of the asset in period t is given by:

$$ER_t = w_t R_t^A + (1 - w_t) R_t^B \quad (8)$$

where R_t^A and R_t^B denote the conditional expected returns implied by the CAPM and FF3 factor models, respectively, at the end of period $t - 1$.

As time progresses from $t - 1$ to t , new information continuously arrives in the financial market. Let the Bayesian updating function be represented by f . When new information D_t arrives, a rational investor updates the existing model weights through Bayesian learning, yielding posterior belief weights w_{t+1} at the beginning of the next period. This mapping can be expressed as:

$$w_{t+1} = f(w_t, D_t) \quad (9)$$

In the theoretical framework developed here, new information D_t is explicitly defined as the latest realized return of the asset R_t in period t , together with the corresponding realizations of the pricing factors. To evaluate which model is closer to the true asset return-generating process, investors need to assess how well each model explains the new information, which is mathematically captured by the conditional likelihood function. Under the assumption in Section 3 that the random disturbance terms of both models follow normal distributions, the study derives the conditional likelihood density of the realized return under each model given historical information F_{t-1} :

The likelihood function of Model A (CAPM) is:

$$L(R_t|M_A, F_{t-1}) = \frac{1}{\sqrt{2\pi\sigma_A^2}} \exp\left(-\frac{(R_t - R_t^A)^2}{2\sigma_A^2}\right) \quad (10)$$

Model B (FF3) has the following likelihood function:

$$L(R_t|M_B, F_{t-1}) = \frac{1}{\sqrt{2\pi\sigma_B^2}} \exp\left(-\frac{(R_t - R_t^B)^2}{2\sigma_B^2}\right) \quad (11)$$

Bayes' theorem is used to find the posterior probability of Model A (i.e., its updated belief weight):

$$w_{t+1} = \frac{P(R_t|M_A, F_{t-1})P(M_A|F_{t-1})}{P(R_t|M_A, F_{t-1})P(M_A|F_{t-1}) + P(R_t|M_B, F_{t-1})P(M_B|F_{t-1})} \quad (12)$$

Substituting the initial prior probabilities w_t and $1 - w_t$ together with the two normal conditional likelihood functions derived above into Bayes' rule, the constant term $\frac{1}{\sqrt{2\pi}}$ cancels from both the numerator and denominator, yielding the following closed-form expression for the Bayesian belief-weight updating function:

$$w_{t+1} = f(w_t, D_t) = \frac{\frac{w_t}{\sigma_A} \exp\left(-\frac{(R_t - R_t^A)^2}{2\sigma_A^2}\right)}{\frac{w_t}{\sigma_A} \exp\left(-\frac{(R_t - R_t^A)^2}{2\sigma_A^2}\right) + \frac{1-w_t}{\sigma_B} \exp\left(-\frac{(R_t - R_t^B)^2}{2\sigma_B^2}\right)} \quad (13)$$

The analytical expression above admits a straightforward economic interpretation. First, when the return for the new period R_t is released, the squared prediction error, such as $(R_t - R_t^A)^2$, appearing in the exponential term serves as a penalty term in the Bayesian learning process. If the prediction error under CAPM (Model A) is substantially smaller than that under FF3 (Model B), indicating that Model A provides a closer account of the newly observed market information, its conditional likelihood will be relatively higher. In this case, Model A's contribution to the numerator

increases relative to the denominator, resulting in a substantial increase in its posterior belief weight relative to its prior weight.

Second, the conditional likelihood also depends on the residual variance of the model in addition to the current prediction error. Under the assumption of a normal conditional likelihood, the residual standard deviation appears in both the scale component of the likelihood and the penalty for prediction error. Therefore, the change in model weights is not necessarily increasing. Because the analysis focuses on belief updating via relative model likelihoods, the comparative statics with respect to the residual variances have not been explored in detail.

Finally, it should be pointed out that the one-period adjustment of model weights mentioned above only reflects the short-run response of beliefs to a new information shock. Whether the belief weights asymptotically converge to the "true model" in the long run cannot be determined by a single-period update process alone. Such convergence requires very strict theoretical conditions: the candidate model space must contain the true asset return-generating process (DGP), the model parameters must be fully identifiable, and sufficiently long time-series data need to be available. Therefore, the one-period shift in beliefs analysed in this study should not be considered evidence of convergence with the true model.

4.2. The impact of changes in model beliefs on risk premium

The first economic problem in this paper is how to determine whether changes in investor sentiment toward the competing models affect asset risk premia and valuations. The risk premium of an asset is the amount by which an investor's required rate of return exceeds the risk-free rate, as shown in the formula:

$$RP_t = ER_t - R_f = w_t R_t^A + (1 - w_t) R_t^B - R_f \quad (14)$$

To formally characterize how changes in model weights affect the risk premium, this paper derives the first-order partial derivative of the risk premium with respect to the model weight w_t :

$$\frac{\partial RP_t}{\partial w_t} = R_t^A - R_t^B \quad (15)$$

To facilitate a clear comparative static analysis, this study further considers the case where $E_t(R_t^B) > E_t(R_t^A)$. For an asset with positive exposures to both the size and value factors and positive corresponding factor risk premia, this condition has a straightforward economic interpretation. In this case, the partial derivative $\frac{\partial RP_t}{\partial w_t} < 0$. This implies that the direction of change in the model-averaged risk premium is inversely related to changes in investors' belief weight w_t assigned to the CAPM.

Importantly, shifts in belief weights are not determined directly by whether the market appears broadly "stable" or "volatile," but instead arise from the relative prediction errors and residual volatilities of the two models in the period under consideration. If returns are primarily driven by systematic market risk during a certain period, the CAPM may maintain a relatively small prediction error due to its simple structure. Under the Bayesian updating mechanism, this relative prediction-

error advantage leads investors to assign a higher belief weight to Model A (w_t increases), which in turn reduces the model-averaged market risk premium.

Conversely, when structural disturbances—such as extreme returns on value stocks—cause the CAPM's prediction error to increase substantially, while the multifactor structure of FF3 produces a smaller relative prediction error and a higher conditional likelihood, investors' beliefs shift toward FF3 (w_t decreases). In this case, investors' aversion to model uncertainty risk leads to a rapid increase in the model-averaged market risk premium. This mechanism indicates that when competing models differ in their ability to account for new information, changes in model beliefs can further amplify fluctuations in the risk premium.

4.3. Dynamic transmission of changes in model beliefs to asset valuation

The fundamental asset valuation equation is given by: $P_t = \frac{E_t(\text{cash flow})}{1+r_t}$. To characterize the transmission mechanism through which new information affects the discount rate r_t and, in turn, asset valuation, the corresponding transmission mechanism is derived below.

The classical Gordon growth model is adopted as the analytical valuation framework. Suppose that the asset's expected future cash flows grow at a constant rate g , that is:

$$E_t [CF_{t+k}] = CF_t(1+g)^k \quad (16)$$

The discount rate r_t represents the investor's model-averaged expected return required at time t , conditional on the latest belief weights w_{t+1} :

$$r_t = w_{t+1}R_{t+1}^A + (1 - w_{t+1})R_{t+1}^B \quad (17)$$

According to the perpetual cash flow discounting formula, when $r_t > g$, the asset value admits the following closed-form solution:

$$P_t = \frac{CF_t(1+g)}{r_t - g} \quad (18)$$

To characterize the effect of changes in model beliefs on asset valuation, applying the chain rule gives the sensitivity of asset value to the updated model weight w_{t+1} :

$$\frac{\partial P_t}{\partial w_{t+1}} = \frac{\partial P_t}{\partial r_t} \cdot \frac{\partial r_t}{\partial w_{t+1}} \quad (19)$$

where:

$$\frac{\partial P_t}{\partial r_t} = -\frac{CF_t(1+g)}{(r_t - g)^2} \quad (20)$$

$$\frac{\partial r_t}{\partial w_{t+1}} = R_{t+1}^A - R_{t+1}^B \quad (21)$$

It is important to note that the validity of this chain-rule derivation relies on a local comparative-static assumption. Specifically, it is assumed that, at the instant of a marginal change in the belief weight w_{t+1} , the asset's expected fundamental cash flows CF_t , perpetual growth rate g , risk-free rate r_f , and the conditional expected returns implied by each model (R_{t+1}^A and R_{t+1}^B) remain exogenous and unchanged. If this assumption is relaxed to allow model weights and expectations of macroeconomic factors to adjust jointly and endogenously, the resulting derivative expansion would contain additional cross terms.

Given the above local static assumption, multiplying the terms results in the final derivation of the valuation shock:

$$\frac{\partial P_t}{\partial w_{t+1}} = -\frac{CF_t(1+g)}{(r_t-g)^2} (R_{t+1}^A - R_{t+1}^B) \quad (22)$$

The formula shows that fluctuations in asset prices arise not only from changes in cash flows but are also driven by the denominator, namely, model beliefs. Suppose that $R_{t+1}^A < R_{t+1}^B$. When new information increases investors' belief weight assigned to Model A (w_{t+1} increases), the model-averaged discount rate r_t decreases, and the corresponding partial derivative $\frac{\partial P_t}{\partial w_{t+1}} > 0$ implies an increase in asset valuation P_t . Thus, under the assumptions of this study, changes in model beliefs can affect asset valuation through the model-averaged discount rate as an endogenous channel.

5. Conclusion

5.1. Main results

Rather than confining the analysis to the conventional static framework of evaluating a single asset pricing model, this paper considers the widespread model uncertainty in financial markets and develops a theoretical system of competing models, such as CAPM and FF3. Based on the above mathematical foundation, this paper derives a dynamic asset pricing transmission mechanism using the Gordon growth model and Bayes' rule.

The results show that when new information, such as the asset's realised return in the current period, becomes available, investors start a Bayesian learning process by assessing the conditional likelihood of the competing models given the observed information, and thus dynamically adjust the weight of belief across models. Therefore, the main research question of this paper has been addressed: Fluctuations in asset prices are not solely due to shocks in fundamentals (represented by cash flows in the numerator); changes in investors' model beliefs can also affect risk premia and asset valuations through non-linear alterations in the model-averaged discount rate (the denominator). Based on the above analysis, under the simplified assumptions in this paper, changes in investors' model beliefs can affect asset risk premia and valuations through the model-averaged discount rate, thus providing a theoretical path for transmitting model uncertainty into fluctuations in asset prices.

5.2. Limitations and future research

Although this study has established an initial dynamic asset pricing framework under model uncertainty, the model specification is still subject to some constraints due to the demand for analytical tractability and an intuitive closed-form solution. Based on the derivations in this paper and the frontier issues of asset pricing, future research can be further extended in the following three directions.

First, extension to a continuous-time framework and analysis of convergence. The theoretical analysis in this paper is limited to a discrete-time system with regular intervals of information update. However, in a real-time continuous-trading environment, the information is not released at fixed times. In the future, one can expand the scope of the model's uncertainty treatment to a continuous-time system. Stochastic filtering theory and the Zakai equation can be used to study how investor beliefs are changed by continuous information arrivals in a Brownian motion process.

More importantly, it is unknown whether the investors' beliefs in the competing models converge to the same value as time approaches infinity. If the true data-generating process does not belong to the given candidate model space, will the Bayesian weights in continuous time converge to the suboptimal model with the smallest Kullback-Leibler divergence from the true DGP, or will they exhibit persistent cyclical fluctuations among multiple models? Research on this convergence problem will offer a more stable mathematical foundation for the long-term stability of asset prices.

Second, endogenous correlation among competing models and nonlinear weight specification. A single-linear-weighting scheme is used to construct the model-averaged expected return in this study. However, in the large-scale factor zoo, the competing pricing models are generally not mutually independent. For example, CAPM and FF3 are closely related models that both include the market risk factor. When the candidate models show significant cross-sectional dependence and factor collinearity, a simple linear Bayesian model averaging (BMA) scheme may effectively double-count common risk exposures in the valuation. In the future, research can be carried out to move away from the linear weighting assumption and use factor orthogonalisation techniques or build hierarchical Bayesian models to address complex dependence and covariance structures among multiple models. Future research should investigate how to construct consistent belief allocations in a non-independent model space to improve the generalizability of the framework.

Third, incorporating a behavioural finance perspective to characterize anomalies in subjective beliefs. Although this study includes the subjective beliefs of the competing models, the update mechanism is still in line with the fully rational Bayesian rule. However, behavioural finance suggests that investors' beliefs may be systematically distorted by cognitive biases. The characteristics of the investors will affect how the prior distribution and likelihood function are used in practice. For example, investors may be subject to confirmation bias and give more weight to information that supports their initial beliefs; they may also overreact to a recent large-scale increase in returns due to momentum and thus disrupt the smooth process of Bayesian updating. Future research can combine the traditional rational framework with behavioural finance to explore how subjective cognitive biases affect Bayesian belief updating under prospect theory or non-standard utility functions that incorporate ambiguity aversion. Thus, an extension may provide a new perspective on market anomalies, such as momentum crashes and overreactions, through the idea of "irrational learning".

Thus, model uncertainty is an endogeneity problem in asset pricing that can be viewed as a connection between rational mathematical finance and behavioural finance. Further development of

the transmission mechanism proposed in this study can provide more theoretical support for the study of price formation in complex financial markets.

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