

Dynamic Pricing of Credit Derivatives under Market Uncertainty: A Default-Risk Adjustment Model

Xinci Lin

*School of Economics and Finance, City University of Macau, Macau, China
F23090103128@cityu.edu.mo*

Abstract. The static pricing model of credit default swaps (CDS) may not reflect the actual changes in the market due to capital flows and geopolitical conflicts at present, and thus will have a large valuation error. This paper extends a simplified reduced-form model by adding a Knightian uncertainty index to a jump-diffusion model of the default intensity and then builds a CDS pricing model with dynamic evolution of default risk. No-arbitrage pricing is used to find the equilibrium CDS spread, and the transmission from Knightian uncertainty to default intensity, expected default losses and CDS spread is explained. Based on the above theoretical analysis, a higher level of uncertainty will lead to a higher long-run default intensity, an increased expected loss from default, and thus a wider CDS spread. As the analysis is theoretical, the results of the reduction in pricing errors by the model should be regarded as a hypothesis. The following research will focus on the calibration and testing of the model with market data.

Keywords: credit default swaps (CDS), knightian uncertainty, jump-diffusion process, dynamic pricing

1. Introduction

With the continuous development and deep-seated changes in the global credit derivatives market, credit default swaps (CDS) have gradually become one of the necessary instruments for financial institutions to manage credit risk and optimise the asset-liability structure; they are employed for risk hedging and price discovery [1, 2]. Recently, due to frequent geopolitical conflicts, a tight liquidity environment, and multiple intersecting factors that have caused repeated disruptions from unforeseen events, capital markets have been under continuous pressure and market uncertainty has risen [3, 4]. The old pricing model for Credit Default Swaps (CDS) generally uses fixed-parameter default probability settings and cannot be adjusted easily to reflect changes in the market now. Therefore, there is a long-standing and considerable discrepancy between the theoretical quotes provided by the model and the actual transaction prices in the secondary market [5-10]. According to data from an industry report, with an increase in market volatility, the calculation errors of a fixed-price model will also rise, and its ability to reflect actual credit risk accurately will be significantly reduced. Therefore, the operating space for financial institutions in dynamic risk hedging and setting reasonable transaction prices is also restricted [11-18]. In short, the pricing mismatches of the traditional static model in a volatile market have become increasingly severe, and it is difficult to

meet the actual demand of financial institutions for precise risk control and reasonable pricing. Given the unstable market conditions, the method of dynamic calculation for default risk needs to be adjusted, and the CDS pricing formula should be revised according to the actual situation.

The following are the main contents of this paper: through which paths does market uncertainty affect corporate default behaviour and what are the specific values of default probabilities; how can uncertainty factors be incorporated into the existing classical pricing model to build a default risk model with dynamic adjustment capabilities and correct structural deviations in traditional CDS pricing; and finally, what is the internal mechanism by which uncertainty is transmitted to CDS prices, and how to achieve reasonable optimisation and logically self-consistent interpretation of the formula at the theoretical level [1, 6, 9, 18].

Most of the existing studies have theoretically included uncertainty in their models as a static exogenous variable, and although the model can improve pricing accuracy, it has not been empirically tested under uncertain conditions [3, 9, 12]. This way will extend the scope of research for credit derivatives pricing and offer some references for subsequent related studies. The optimised dynamic pricing model will reduce the gap between the theoretical value and the actual market price in practice, help banks and asset management companies determine the cost of credit risk more accurately, and improve the practical effectiveness of credit derivatives for risk hedging.

The five parts of the whole are developed sequentially in the sections below. The first part of this paper introduces the background and research objectives; the second part reviews related studies at home and abroad; the third part presents the theoretical foundation for the study; the fourth part builds a pricing model and demonstrates the internal transmission path of uncertainty affecting CDS prices; and the fifth part summarizes the research results and discusses the limitations and future directions of the study.

2. Literature review

2.1. Research and evolution of CDS pricing methods

The first two paths for the early credit pricing framework were a structural model and a reduced-form model. The former assumes that enterprise asset values follow geometric Brownian motion and considers the asset value falling below the face value of debt as the trigger for default, thus building the earliest credit pricing system. However, this model is based on the assumption of a complete-information market and fails to consider the actual features of financial markets. The reduced-form model does not consider the internal asset structure of enterprises anymore; instead, it uses a random intensity function to describe the occurrence process of default events and is thus much simpler computationally. Therefore, it has become one of the most popular basic models for CDS pricing [5, 8]. Frequent occurrences of market extremes have shown the obvious deficiencies of traditional models with fixed parameters. Therefore, to address this problem, many studies have proposed jump-diffusion processes and Lévy random processes in the pricing model to account for the abrupt jump feature of default events [6, 7, 9]. Other research has introduced machine learning algorithms, such as LSTM and GNN, into the pricing process to improve the prediction accuracy and in-sample fit of CDS prices using high-frequency trading data, and promoted the development of dynamic pricing methods from static ones [13, 14].

2.2. Research on market uncertainty and default risk

Knightian uncertainty cannot be expressed in terms of probability, and thus continuously impacts the operating cash flow and solvency of enterprises, causing fluctuations in default risk over time [3, 12]. Based on the above analysis of moral hazard and information asymmetry, it can be seen that market uncertainty increases the risk of default for enterprises and raises CDS credit spreads. Based on the above quantitative analysis, some studies have found a positive correlation between the market uncertainty index and CDS spreads; that is to say, credit derivative prices generally rise during periods of market panic [2, 4, 15, 16]. Most of the existing studies fix the degree of uncertainty as an exogenous constant, and few have constructed a dynamic default parameter system that can be adjusted in real time according to market fluctuations; thus, this area remains unexplored in related research [6, 9, 18].

2.3. Supplementary research on credit volatility from a behavioral finance perspective

In addition to the inherent uncertainty of the objective data, irrational behaviour by market participants can also cause a deviation of credit prices from the theoretical value at the micro level [1, 10]. Herd behaviour and panic selling can spread external shocks in the credit market more rapidly, increase the clustering of default events, and thus indirectly lead to a sharp drop in CDS prices [15, 17]. Behavioural-level studies offer the foundation for investigating how uncertainty at the micro-level affects credit risk. This paper will add this idea to the mechanism discussion and thus strengthen the theoretical support for behavioural theory and make the whole logical system more solid.

2.4. Research review

Existing research has achieved some results in the theory of CDS pricing and uncertainty risk measurement [5, 8, 14]. However, two problems have not been fully solved: first, most dynamic pricing models do not systematically incorporate the multi-level market volatility characteristics into the real-time update process of default parameters; second, discussions on the intermediary paths through which uncertainty transmits to credit risk are still mainly at the macro level and lack detailed mechanistic analysis [3, 6, 9, 10]. Given the above deficiencies, this paper constructs a dynamically adjustable default risk model in an uncertain market environment, optimises the CDS pricing formula accordingly, and dissects the entire path through which uncertainty transmits to credit derivative prices to provide a theoretical extension of previous research.

3. Theoretical basis

3.1. Operating mechanism of CDS products

Credit Default Swaps (CDS) are derivative contracts employed to manage credit risk in the over-the-counter (OTC) market, and they have two sides: a protection buyer and a protection seller. The buyer pays a fixed premium to the seller on a fixed schedule, and in the event of a credit event, such as the bankruptcy or debt default of the underlying entity, the seller bears the corresponding debt loss of the buyer as stipulated in the contract [8, 16]. The price of a CDS reflects the market's demand for compensation against the risk of default by the underlying party, and a higher probability of default will be associated with a larger CDS spread; thus, this study will use the CDS as the pricing benchmark [2, 14].

3.2. Basic theory of classic CDS pricing

At the beginning of the model is a stochastic model for firm asset value; a scenario is constructed in which the market value of debt falls below its face value to trigger a default, thus establishing an initial pricing paradigm [5]. Conversely, the reduced-form model avoids directly observing a firm's internal financial conditions and, instead, introduces a Poisson process to represent external default rates; thus, it has shown good computational efficiency and is now a widely used pricing system in practice [8]. Both foundation models assume that the market is stable and the default intensity is constant; however, external disturbances still occur in practice, and thus the parameters change accordingly. The disparity between these assumptions and reality serves as the entry point for model refinement in this paper [6, 9].

3.3. Knight's uncertainty theory

The absence of a probability distribution derived from historical data is the characteristic feature of Knightian uncertainty and differs from general risk [3]. In practice, the policy has not anticipated sudden changes in policy or systemic liquidity crises, and such events are thus classified as Knightian uncertainty [13]. Knightian uncertainty constantly affects the operational stability and fluctuations in the level of default probability. This paper will take the above theoretical dimension as the main variable in the entire framework of pricing analysis [12].

3.4. Jump-diffusion random process theory

A jump term can be added to basic Brownian motion to build a jump-diffusion process that simulates market phenomena such as sudden default outbreaks and sharp increases in credit spreads, which are in line with the abrupt change characteristics of credit risk under uncertainty [9, 14]. Based on the above mathematical theory, this paper derives and builds the dynamic equation for default probability.

4. Model construction and mechanism analysis

4.1. Basic assumptions of the model

Given the research environment, the following prerequisites are assumed: first, the term of the CDS contract is fixed, and it is assumed that premiums are paid continuously until default or maturity; Second, the risk-free interest rate is stable, and fluctuations in interest rates are disregarded for now; third, market uncertainty is mainly transmitted through changes in default intensity; finally, upon the occurrence of a default event, a standard cash settlement method is employed for compensation [1]. The purpose of setting up these assumptions is to exclude abstract factors from the additional sources of variation, focus the research scope on the core interrelationships among uncertainty, default risk and CDS prices, and reduce the complexity of model derivation [15].

4.2. Derivation of the pricing model based on dynamic adjustment of default risk

To make the default intensity responsive to changes in market uncertainty, it should not be fixed; therefore, the Knight uncertainty correction coefficient needs to be added to the original default intensity formula of the reduced-form model, which is the main improvement of the basic model in

this paper [8]. Based on the simplified CDS pricing model initially, this paper introduces the dual features of credit risk—"continuous fluctuation + sudden jump"—via a jump-diffusion process:

$$d\lambda_t = \mu(\theta - \lambda_t)dt + \sigma\lambda_t dW_t + \lambda_{t-}dJ_t \quad (1)$$

Here, λ_t denotes the instantaneous default intensity of the enterprise at time t , representing the conditional probability of default per unit time; μ signifies the mean reversion speed of default intensity, reflecting the rate of return to steady state after default risk deviates from the equilibrium level; θ represents the long-term equilibrium default intensity of the enterprise in the absence of external shocks, determined by the enterprise's fundamentals and industry attributes; σ denotes the continuous volatility of default intensity; W_t is the standard Brownian motion, depicting sustained credit volatility induced by business disturbances in a stable market; J_t is a Poisson jump process, capturing sudden changes in default risk triggered by extreme events such as debt crises and liquidity shortages; λ_{t-} indicates the default intensity immediately preceding the jump. From an economic perspective, Brownian motion characterizes conventional market risks with quantifiable probabilities, while Poisson jumps capture tail extreme risks. However, the benchmark model possesses inherent limitations: the long-term equilibrium default intensity θ is a constant parameter, implying the assumption of a stable market credit risk center and failing to characterize the reality where fuzzy shocks like Knightian uncertainty, difficult to quantify with probability distributions, continuously reshape the market pricing center.

A jump-diffusion process is employed to add a jump term to continuous fluctuations, and two market states are identified: regular fluctuations in a stable period and sudden tail shocks in an extreme market environment [9]. To correct the shortcomings of the benchmark model, this paper modifies the mean reversion term, adds the Knight uncertainty index U_t , and builds an extended dynamic default intensity equation:

$$d\lambda_t = \mu(\theta(1 + \alpha U_t) - \lambda_t)dt + \sigma\lambda_t dW_t + \lambda_{t-}dJ_t \quad (2)$$

Among them, U_t represents the Knight uncertainty index, which measures non-probabilistic shocks that cannot be quantified by probability distributions, such as geopolitical conflicts, market panic, and ambiguous policies. The coefficient $\alpha > 0$ is the uncertainty sensitivity coefficient, reflecting the responsiveness of corporate default risk to ambiguity-related market shocks. This improvement embodies a clear economic transmission logic: when Knight uncertainty U_t increases, the long-term equilibrium default risk center $\theta(1 + \alpha U_t)$ of firms rises accordingly. Relying on the mean-reversion mechanism, this drives a continuous increase in the current default intensity, reflecting the practical phenomenon where external ambiguous shocks weaken firms' long-term debt repayment expectations and elevate credit risk. Conversely, when uncertainty subsides, the risk center declines, and the default risk gradually diminishes. Brownian motion W_t captures conventional fluctuations during periods of stable market operation, while the Poisson jump process J_t describes sudden default shocks [14]. Based on this expanded intensity setting, the survival probability of the underlying entity at each time point can be further derived as follows:

$$P(0, t) = \mathbb{E}[\exp(-\int_0^t \lambda_s ds)] \quad (3)$$

$P(0, t)$ denotes the risk-neutral survival probability from time 0 to t under the risk-neutral measure. An increase in Knight uncertainty elevates the default intensity λ_s within the interval,

leading to a rise in cumulative default risk and a decline in firm survival probability, which directly enhances the hedging value of CDS protection contracts.

The two main modules are the present value of premium cash flows and the present value of compensation cash flows. The expression of the present value of CDS premiums is:

$$PV_{premium} = s \int_0^T P(0, t) e^{-rt} dt \quad (4)$$

$PV_{premium}$ denotes the total expected discounted premium expenditure by the protection buyer over the contract duration. Premiums are paid continuously only if the reference entity has not defaulted, hence the survival probability $P(0, t)$ is embedded in the formula. e^{-rt} serves as the risk-free interest rate discount factor to convert future cash flows into their present values, and s represents the equilibrium CDS spread to be determined.

The expression of the expected present value of default losses is given by:

$$PV_{loss} = \mathbb{E}[\int_0^T (1 - R) \lambda_t \exp(-rt - \int_0^t \lambda_s ds) dt] \quad (5)$$

PV_{loss} depicts the total discounted expected default losses borne by the protection seller, with $1 - R$ representing the loss rate on claims following a default. An increase in market Knight uncertainty comprehensively elevates the default intensity λ_t , thereby expanding the scale of expected compensation.

According to the principle of no-arbitrage pricing, the present value of premiums equals the present value of compensation in equilibrium, i.e., $PV_{premium} = PV_{loss}$. Rearranging yields the equilibrium CDS spread pricing formula:

$$s = \frac{\mathbb{E}[\int_0^T (1 - R) \lambda_t e^{-rt - \int_0^t \lambda_s ds} dt]}{\int_0^T P(0, t) e^{-rt} dt} \quad (6)$$

The numerator is the total discounted expected default losses over the contract period, and thus there will be an increase in CDS spreads. The denominator is the total discounted premium cash flow for a unit spread. The above derivation shows that Knight's uncertainty increases the long-term default risk of a firm, thereby raising its instantaneous default intensity; as a result, the probability of survival declines, expected default losses rise, and finally, CDS equilibrium spreads increase. Traditional static models set a fixed default intensity and do not link it to credit premiums; therefore, they fail to account for changes in risk caused by fluctuations in external uncertainty and are thus inaccurate in valuation. At the same time, this model dynamically adjusts the default intensity according to Knight's uncertainty, continuously updates the expected default loss, reduces pricing errors caused by fixed-parameter assumptions, and better aligns with price fluctuations in the actual CDS market.

4.3. Analysis of the impact mechanism of uncertainty

An increase in market uncertainty reduces the stability of corporate operating cash flows, causes a downturn in the market, raises the risk of default for the underlying entity, and thus increases the expected compensation cost of CDS contracts. Therefore, CDS protection sellers have increased the premium demanded to cover the higher risk and raised CDS prices [12]. At the same time, with a

lower risk of default, CDS spreads will also fall. The above dynamic model offers a theoretical path for transmission and a practical reference for pricing credit derivatives under uncertainty [15].

5. Conclusion

In this study, a theoretical framework for the CDS pricing is developed in which uncertainty in the sense of Knight changes the long run mean of the default intensity. The model suggests that a higher uncertainty increases the default intensity, decreases the probability of survival, and increases the equilibrium CDS spread. This is a theoretical implication not empirical finding. Future work should be carried out to define an observable proxy for U_t , calibrate the model and compare the pricing errors for out-of-sample data with that from benchmark models.

The model derivation in this paper is limited to the theoretical level and lacks empirical evidence from actual market data; this is the main deficiency of the current study and a direction for future research. Add real CDS market data to the analysis system, calibrate and validate model parameters, and then empirically test how applicable the system is.

References

- [1] Weistroffer, C. (2009, December 21). Credit default swaps: Heading towards a more stable system. Deutsche Bank Research.
- [2] Bartram, S. M., Conrad, J., Lee, J., & Subrahmanyam, M. G. (2022). Credit default swaps around the world. *The Review of Financial Studies*, 35(5), 2464–2524. <https://doi.org/10.1093/rfs/hhab092>
- [3] Bouteska, A., & Mili, M. (2023). The role of investor sentiment and valuation uncertainty in the changes around analyst recommendations: Evidence from U.S. firms. *Journal of Behavioral Finance*, 24(1), 73–96. <https://doi.org/10.1080/15427560.2021.1948853>
- [4] Huang, Y.-S. (2025). When geopolitical risks hit the supply chain: Impacts on credit default swap market. *Finance Research Letters*, 82, 107622. <https://doi.org/10.1016/j.frl.2025.107622>
- [5] Zhou, C. (1997). A jump-diffusion approach to modeling credit risk and valuing defaultable securities. Finance and Economics Discussion Series 1997-15. Board of Governors of the Federal Reserve System. <https://doi.org/10.2139/ssrn.39800>
- [6] Huang, H., Jiang, M., Ning, Y., & Wang, S. (2025). A structural credit risk model with jumps based on uncertainty theory. *Mathematics*, 13(6), 897. <https://doi.org/10.3390/math13060897>
- [7] Livieri, G., Radi, D., & Smaniotto, E. (2024). Pricing transition risk with a jump-diffusion credit risk model: Evidences from the CDS market. *Review of Corporate Finance*, 4(12), 177–201. <https://doi.org/10.1561/114.00000064>
- [8] Akyildirim, E., Hekimoglu, A. A., Sensoy, A., & Fabozzi, F. J. (2023). Extending the Merton model with applications to credit value adjustment. *Annals of Operations Research*, 326(1), 27–65. <https://doi.org/10.1007/s10479-023-05289-3>
- [9] Huang, H., Ning, Y., & Chen, X. (2022). A structural credit risk model driven by the Lévy process under Knightian uncertainty. *Symmetry*, 14(5), 1041. <https://doi.org/10.3390/sym14051041>
- [10] Wu, L., & Hua, H. (2025). Modeling fuzzy moral hazard in credit default swap pricing: A reduced-form approach. *International Journal of Computational Intelligence Systems*, 18, 149. <https://doi.org/10.1007/s44196-025-00872-x>
- [11] Rutkowski, M., & Xu, H. (2026). Valuation of variable annuities with equity protection swaps under jumps and default risks. arXiv. <https://doi.org/10.48550/arXiv.2605.25450>
- [12] Gao, J., Bai, X., Tan, H., Fan, C., Mao, Y., & Xu, Z. (2025). Adapting to uncertainty: A quantitative investment decision model with investor sentiment and attention analysis. *Technological and Economic Development of Economy*, 31(2), 447–479. <https://doi.org/10.3846/tede.2024.21961>
- [13] Rao, G., Trinh, T. K., Chen, Y., Shu, M., & Zheng, S. (2024). Jump prediction in systemically important financial institutions' CDS prices. *Spectrum of Research*, 4(2).
- [14] Fu, X., Li, M. C., & Molyneux, P. (2021). Credit default swap spreads: Market conditions, firm performance, and the impact of the 2007–2009 financial crisis. *Empirical Economics*, 60(5), 2203–2225. <https://doi.org/10.1007/s00181-020-01852-0>

- [15] Mateev, M., & Marinova, E. (2019). Relation between credit default swap spreads and stock prices: A non-linear perspective. *Journal of Economics and Finance*, 43(1), 1–26. <https://doi.org/10.1007/s12197-017-9423-9>
- [16] Ertugrul, H. M., & Ozturk, H. (2013). The drivers of credit default swap prices: Evidence from selected emerging market countries. *Emerging Markets Finance and Trade*, 49(sup5), 228–249. <https://doi.org/10.2753/REE1540-496X4905S514>
- [17] Al Maruf, A., Kowsar, M. M., Mohiuddin, M., & Mohna, H. A. (2024). Behavioral factors in loan default prediction: A literature review on psychological and socioeconomic risk indicators. *American Journal of Advanced Technology and Engineering Solutions*, 4(1), 43–70. <https://doi.org/10.63125/0jwtn29>
- [18] Bednarek, P., & Franke, G. (2024). Dynamics of probabilities of default. *Deutsche Bundesbank Discussion Paper No. 32/2024*. <https://doi.org/10.2139/ssrn.4930492>