

Diversification Beyond Stocks and Bonds: A Mean-Variance Assessment of Gold and Real Estate in a Multi-Asset Portfolio

Jingdun Li

*International Business School Suzhou, Xi'an Jiaotong-Liverpool University, Suzhou, China
Jingdun.Li23@student.xjtlu.edu.cn*

Abstract. Modern portfolio construction increasingly extends beyond the traditional combination of equities and fixed income to include alternative assets such as gold and real estate. This study evaluates whether the inclusion of these alternatives improves the risk-return efficiency of a conventional two-asset portfolio. Using representative long-run annualized estimates of returns, volatilities, and correlations for five asset classes, including domestic equities, international equities, investment-grade bonds, gold, as well as real estate investment trusts, the study uses a mean-variance optimization framework to construct efficient frontiers, global minimum-variance portfolios, and tangency portfolios under a long-only constraint. The results show that alternative assets provide different diversification benefits. Gold receives an allocation of approximately 15% in the optimal risky portfolio despite its low standalone Sharpe ratio, as its low correlation with equities reduces portfolio risk. In contrast, the allocations to real estate and international equities remain limited due to high correlation with domestic equities. At the portfolio level, the expanded opportunity set increases the maximum Sharpe ratio by approximately 4-5% and lowers the volatility required for a given return target. The results confirm that diversification benefits depend more on covariance with existing holdings than on standalone performance.

Keywords: Portfolio diversification, Mean-variance optimization, Efficient frontier, Asset allocation, Alternative assets

1. Introduction

The allocation of capital across asset classes is a central problem in investment management. In this context, the mean-variance framework introduced by Markowitz provides the theoretical foundation by formalizing the trade-off between expected return and risk and by showing that portfolio risk depends not only on asset volatility but on correlations among assets [1]. In practice, a long-standing benchmark is the balanced portfolio combining domestic equities with high-quality bonds, commonly represented by a sixty-forty allocation. Its appeal derives from the historically low correlation between the two asset classes, which allows bonds to cushion equity drawdowns. Recent market conditions have increased interest in expanding the opportunity set beyond stocks and bonds. Simultaneous declines in equity and bond markets have challenged the traditional two-asset portfolio framework, increasing attention to alternative assets such as gold and real estate that may

provide distinct risk exposures [2]. Gold has been widely examined as a potential hedge and safe-haven asset, whereas real estate investment trusts offer liquid exposure to property markets through exchange-traded instruments [3-5]. This study examines whether adding gold and real estate to a traditional equity-bond portfolio improves risk-return efficiency. In particular, the analysis applies mean-variance optimization to five asset classes, constructs efficient frontiers, and identifies portfolios that minimize variance and maximize the Sharpe ratio. Using this framework, the study identifies which alternative assets provide incremental diversification benefits and how their contributions differ.

2. Literature review

2.1. Mean-variance framework and portfolio theory

The theoretical foundation of diversification originates from the mean-variance framework, in which investors select portfolio weights to minimize variance for a given level of expected return [1]. Its central insight is that combining imperfectly correlated assets can reduce portfolio variance below the weighted average of individual asset variances, making covariance relationships central to diversification benefits. Several subsequent developments extended this framework, including Tobin's separation theorem, which describes the combination of the risk-free asset with the tangency portfolio of risky assets [6]; Sharpe's capital asset pricing model, which establishes the equilibrium relationship between risk and return [7]; and Merton's analytical derivation of the efficient frontier [8]. The Sharpe ratio, defined as excess return per unit of volatility, remains a standard measure for risk-adjusted portfolio comparison [9].

2.2. Alternative assets in portfolio diversification

Empirical studies of gold distinguish between its role as a hedge, referring to an asset that is uncorrelated with equities on average, and as a safe haven, referring to an asset that remains uncorrelated or negatively correlated with equities during periods of market stress [3]. Cross-market evidence documents safe-haven behavior in major European and United States (U.S.) markets, although the effect is not uniform across countries [4]. Evidence further shows that gold's safe-haven role varies across market conditions, with protection during severe disruptions but weaker effectiveness during recovery periods and across crises [10-12]. In contrast, the evidence on securitized real estate presents a different pattern: REIT returns exhibit equity-like behavior over shorter horizons and become more aligned with direct property markets over longer horizons [5]. Related studies further indicate that the diversification benefits of real estate vary across markets and investment periods, while listed real estate often maintains strong co-movement with broader equity markets during periods of stress [13, 14]. Despite these findings, existing studies typically examine individual alternative assets separately or focus on specific market episodes. Therefore, a direct comparison of the marginal contribution of gold and real estate within a unified optimization framework remains limited, which motivates the present study.

2.3. Empirical limitations of mean-variance optimization

Despite its foundational role in portfolio theory, mean-variance optimization remains subject to several limitations, primarily arising from parameter uncertainty and estimation error. In particular, Michaud argues that mean-variance optimizers can amplify estimation errors because optimal

weights are highly sensitive to small changes in input assumptions [15]. Furthermore, empirical evidence also demonstrates that optimized portfolios do not consistently outperform simple equally weighted strategies out of sample once estimation uncertainty is considered [16]. To address these limitations, later studies have proposed several improvements, including the Black-Litterman model, estimation-risk-aware portfolio rules, and shrinkage estimators for covariance matrices [17-19]. These approaches improve the robustness of mean-variance optimization by reducing sensitivity to input uncertainty, while preserving the framework's core structure.

3. Methodology and data

3.1. Data and asset universe

The study examines five asset classes in a diversified portfolio: domestic equities, international equities, investment-grade bonds, gold, and listed real estate. These asset classes are represented by the S&P 500, MSCI EAFE, Bloomberg US Aggregate Bond Index, US-dollar spot gold price, and FTSE Nareit All Equity REITs Index, respectively.

The model inputs consist of calibrated long-run annualized parameters rather than estimates derived from a single sample period. Specifically, expected returns are defined as annualized arithmetic averages of nominal total returns, volatilities as annualized standard deviations of annual returns, and correlations as long-run pairwise correlations among asset classes. These parameters are calibrated using the multi-decade historical compilations of Damodaran, whose US market series extend back to 1928, and Dimson, Marsh, and Staunton [20, 21]. For asset-specific histories, gold parameters are based on the floating-price period since 1971, while real estate parameters reflect the development of US REIT indices since the early 1970s. The annual risk-free rate is set at 3.0%, consistent with long-run returns on short-term US government securities [20].

3.2. The mean-variance framework

For a portfolio with a vector of weights w across n assets, the expected return and variance are expressed in matrix form as follows [22].

$$E(R_p) = w^T \mu \quad (1)$$

$$\sigma_p^2 = w^T \Sigma w \quad (2)$$

where w , μ , and Σ denote the portfolio weight vector, expected return vector, and covariance matrix of asset returns, respectively.

The covariance matrix is constructed from the calibrated parameters as follows:

$$\Sigma = D \rho D \quad (3)$$

where D is the diagonal matrix of asset volatilities and ρ is the correlation matrix. The covariance matrix thus reflects the calibrated long-run volatility and correlation inputs rather than sample-based covariance estimates.

This approach uses the calibrated second moments rather than covariance estimates obtained from historical return samples. Accordingly, the efficient frontier is then defined as the set of portfolios that minimize variance for each feasible expected return. Specifically, the global

minimum-variance portfolio represents the feasible portfolio with the lowest variance, while the tangency portfolio is the risky portfolio that maximizes the Sharpe ratio:

$$S_p = \frac{E(R_p) - R_f}{\sigma_p} \quad (4)$$

When a risk-free asset is included, the capital market line is constructed by combining the risk-free asset with the tangency portfolio. The framework is applied to the specified investment opportunity sets.

3.3. Portfolio construction and diversification metrics

For portfolio comparison, two investment opportunity sets are constructed: a traditional opportunity set comprising domestic equities and investment-grade bonds, and an expanded opportunity set including all five asset classes. For each opportunity set, the efficient frontier, global minimum-variance portfolio, and tangency portfolio are obtained under a long-only constraint, where portfolio weights are restricted between zero and one and sum to one, excluding short selling and leverage.

Within this framework, the optimization procedures are implemented in Python using NumPy and SciPy. The portfolio problems are solved numerically with the Sequential Least Squares Programming (SLSQP) algorithm in the SciPy optimization module. Under these constraints, the global minimum-variance portfolio minimizes portfolio variance in Eq. (2) subject to the weight constraints. The efficient frontier is generated by extending this minimization across a range of target returns, while the tangency portfolio maximizes the Sharpe ratio in Eq. (4) under the same constraints.

Based on these portfolio characteristics, efficiency is evaluated using two pre-specified metrics: the change in the maximum attainable Sharpe ratio and the reduction in volatility required to achieve the expected return of the traditional tangency portfolio. An equally weighted five-asset portfolio is included as an unoptimized benchmark for reference.

4. Results

4.1. Asset-level risk-return profiles and correlation structure

As shown in Table 1, domestic equities exhibit the highest standalone Sharpe ratio at 0.45, followed by real estate investment trusts (REITs) at 0.34 and international equities at 0.32, with investment-grade bonds at 0.25 and gold at 0.22. Figure 1 presents the pairwise correlation structure among the five asset classes. Domestic equities show high correlations with international equities (0.80) and REITs (0.65), whereas bonds and gold exhibit much lower correlations with domestic equities (0.10 and 0.05, respectively). The highest correlation between gold and any other asset is only 0.25, observed with bonds. These results indicate that equity-like assets provide similar return exposures, while bonds and gold contribute stronger diversification potential.

Table 1. Standalone risk-return characteristics of each asset class (risk-free rate=3.0%)

Asset class	Expected return (%)	Volatility (%)	Sharpe ratio
US equities	10.0	15.5	0.45
International equities	8.5	17.0	0.32
Investment-grade bonds	4.5	6.0	0.25

Table 1. (continued)

Gold	6.5	16.0	0.22
Real estate (REITs)	9.5	19.0	0.34

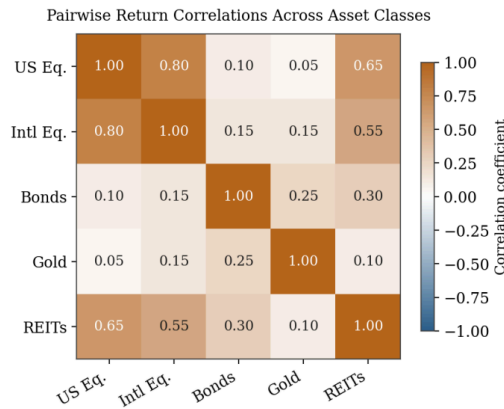


Figure 1. Pairwise return correlations across the five asset classes

4.2. Efficient frontier improvement and diversification effects

Figure 2 illustrates the efficient frontiers of the traditional and expanded opportunity sets, together with individual assets, optimal portfolios, and the capital market line. The expanded frontier is located slightly above and to the left of the traditional frontier across most of the feasible return range, with the improvement concentrated in the intermediate region where gold enhances diversification efficiency by reducing portfolio volatility. The two frontiers share the same upper bound, as the maximum return of 10.0 percent can only be achieved through a portfolio fully invested in domestic equities. They also exhibit similar behavior at the lower end, where bonds dominate the minimum-variance portfolios in both asset universes. The inclusion of additional asset classes improves portfolio efficiency, although the magnitude of the improvement remains relatively limited.

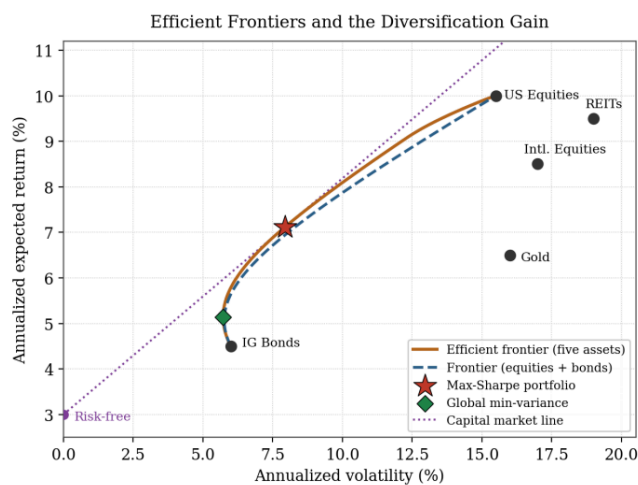


Figure 2. Efficient frontiers of the traditional and expanded opportunity sets with global minimum-variance, maximum-sharpe (tangency), and capital market line portfolios

4.3. Optimal portfolio allocation and diversification benefits

Table 2 summarizes the portfolio weights and risk-return characteristics of the optimized and benchmark portfolios. The traditional tangency portfolio allocates 44.6% to US equities and 55.4% to investment-grade bonds, achieving a Sharpe ratio of 0.496. In comparison, the expanded tangency portfolio allocates 42.2% to US equities, 43.0% to bonds, and 14.9% to gold, while assigning zero weights to international equities and REITs. This expanded portfolio achieves a Sharpe ratio of 0.519, indicating an improvement of approximately 4.5% over the traditional portfolio. At the fixed target return of 6.95%, the required volatility decreases from 7.97% to 7.63%, representing a reduction of approximately 4.2%.

Table 2. Portfolio weights and annualized risk-return characteristics of optimized and benchmark portfolios (risk-free rate = 3.0%)

Portfolio	US eq.	Intl eq.	Bonds	Gold	REITs	E(R) (%)	Volatility (%)	Sharpe ratio
Tangency, traditional	44.6	–	55.4	–	–	6.95	7.97	0.496
Tangency, expanded	42.2	0.0	43.0	14.9	0.0	7.12	7.94	0.519
Minimum variance, traditional	10.4	–	89.6	–	–	5.07	5.76	0.359
Minimum variance, expanded	10.1	0.0	85.6	4.3	0.0	5.14	5.73	0.374
Target return 6.95%, expanded	39.5	0.0	46.5	14.0	0.0	6.95	7.63	0.518
Equally weighted, five assets	20.0	20.0	20.0	20.0	20.0	7.80	10.34	0.464

The minimum-variance portfolios exhibit a similar pattern, with the expanded opportunity set slightly reducing volatility from 5.76% to 5.73% while maintaining a comparable expected return. The equally weighted five-asset portfolio achieves a Sharpe ratio of 0.464, which remains below the optimized traditional portfolio (0.496) and the expanded tangency portfolio (0.519). Notably, gold receives a meaningful allocation in all optimized portfolios despite having the lowest standalone Sharpe ratio among the five assets, whereas international equities and REITs receive zero weights. These results show that diversification benefits are primarily determined by cross-asset correlations rather than standalone risk-adjusted performance.

Given that the optimizer treats the calibrated inputs as fixed, the resulting allocations, particularly zero-weight decisions, may be sensitive to parameter assumptions. Reducing the correlation between US equities and REITs from 0.65 to 0.40 introduces an 11.4% allocation to real estate in the tangency portfolio, while lowering gold's expected return by one percentage point reduces its allocation from 14.9% to 8.6%. Nevertheless, the qualitative diversification pattern remains robust: assets with lower marginal correlation contribute more effectively to portfolio efficiency, whereas highly correlated assets provide limited incremental benefits.

5. Discussion

Based on the optimal allocation results, diversification benefits depend primarily on an asset's marginal contribution to portfolio covariance rather than its standalone risk-adjusted performance. Gold receives a meaningful allocation despite having the lowest standalone Sharpe ratio because its low correlation enhances diversification. In contrast, real estate and international equities, although exhibiting stronger standalone performance than gold, provide limited incremental diversification benefits once domestic equity exposure is already included due to their higher correlations with existing assets. The limited improvement in the efficient frontier further indicates that the traditional

equity-bond combination has already captured a substantial portion of the available diversification benefit. Therefore, increasing the number of assets alone does not necessarily improve portfolio efficiency, as demonstrated by the lower Sharpe ratio of the equally weighted portfolio.

The observed allocation patterns support prior evidence on alternative assets' portfolio roles. The role of gold in the optimized portfolios agrees with evidence that gold can provide hedging and safe-haven benefits during periods of market stress [3, 4, 10]. The limited allocation to real estate is also consistent with evidence that REIT returns often exhibit equity-like characteristics over shorter horizons and that their diversification benefits vary across markets and periods [5, 13, 14]. However, gold's diversification benefits are state-dependent, as previous studies show that its hedging effectiveness varies across crises and market conditions [11, 12].

The portfolio results are subject to the estimation uncertainty inherent in mean-variance optimization. Portfolio weights are sensitive to estimated expected returns, volatilities, and correlations, which may contain substantial estimation errors [15, 18]. In addition, correlations can vary during market stress, potentially weakening diversification benefits when they are most valuable [2, 11, 12]. These parameter uncertainties may result in unstable allocations and overly optimistic in-sample performance, motivating alternative approaches like covariance shrinkage, Black-Litterman models, and other robust allocation methods [15, 16]. Besides, the mean-variance framework focuses only on the first two moments of return distributions and does not explicitly account for higher-order characteristics, such as skewness, kurtosis, and extreme downside risk [16, 17, 19]. Therefore, the portfolio improvements observed in this analysis should be attributed to the diversification contribution of alternative assets rather than their standalone returns. Though the specific allocations rely on the assumed return and correlation inputs, the underlying mechanism remains that assets with lower marginal covariance provide greater portfolio benefits.

6. Conclusion

This study evaluated whether the inclusion of gold and real estate improves the risk-return efficiency of a traditional equity-bond portfolio, using long-only mean-variance optimization over five asset classes with calibrated long-run parameters. Based on the current data and assumptions, three principal findings emerge. In particular, the expansion of the investment opportunity set improves measured efficiency, increasing the maximum attainable Sharpe ratio by around 4.5% and reducing the volatility required to reach the traditional target return by roughly 4.2%. Meanwhile, the improvement is concentrated in a single asset: gold enters the optimal risky portfolio at almost fifteen percent despite the lowest standalone Sharpe ratio, because its low assumed correlation with equities reduces aggregate risk. Moreover, under the same assumptions, real estate and international equities receive no allocation, as their high correlation with domestic equities leaves them little marginal role. The sensitivity checks indicate that the qualitative pattern is robust to moderate input changes while the precise allocations are not.

Under these assumptions, the results confirm a core portfolio theory principle: diversification value comes from low covariance with existing holdings rather than standalone performance. The practical implication is that potential diversifiers should be evaluated based on their relationship with the rest of the portfolio rather than solely on their individual performance. However, the analysis assumes fixed long-run estimates of returns, volatilities, and correlations, which may vary over time. Future research could incorporate time-varying correlations, regime-dependent parameters, and higher moments of return distributions, and compare out-of-sample performance with simple allocation rules and robust methods such as resampling and the Black-Litterman

approach. These extensions would clarify whether the diversification benefits identified under long-run assumptions persist across different market conditions.

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