

The Evolution of Option Pricing Models: From Black-Scholes and Heston Models to Differentiable Deep Neural Networks

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Abstract. Option pricing has moved away from the previous analytical and stochastic models to machine-learning-based methods. Therefore, a good model of the market and an efficient one for computation is required. This paper reviews the development of the Black-Scholes and Heston models to DNN- and DDN-based surrogate models, as well as their applications in quantitative finance. Give some attention to computational efficiency in the repeated pricing and calibration. Another issue in the paper is the trade-off between the accuracy of prices and the cost of computation; high-speed calculation may not be entirely precise in finance. Differentiability in the context of risk management and hedging, as well as Delta and Vega sensitivity measures, will be introduced. The problems with the current ones are: dependence on training data, lack of interpretability, and issues of robustness and generalization. In the future, one will aim to build a more stable and flexible surrogate model, enhance its interpretability, address financial constraints, and integrate traditional financial models with machine learning in a hybrid manner.

Keywords: Option Pricing, Heston Model, Volatility

1. Introduction

Option Pricing Supports the Operation of the Financial Market. Support derivative pricing and risk management for market making. Therefore, as a result of the work by Black and Scholes [1], option pricing theory has been widely used in modern quantitative finance. Their work established the first closed-form pricing formula for European options. However, it assumes that volatility is constant and does not fully reflect the market phenomenon of volatility smiles and volatility clustering. Later, Heston presented a stochastic volatility model that assumes that asset prices and variances are correlated stochastic processes to improve the accuracy of option price fluctuations [2].

Stochastic Volatility models are more appropriate for showing changes in the market but are also more difficult to apply. Many times, Heston model calibration is performed using numerical methods to find option prices. Therefore, the calibration process is relatively slow, and if the market conditions change frequently, updating the model parameters must be done often [2, 3].

Recently, with the development of machine learning, a neural-network surrogate has been constructed to replace a computationally intensive pricing engine for this problem. Previous studies

have shown that neural networks can enhance computational efficiency while keeping the high pricing accuracy [4]. High pricing accuracy is not sufficient for real-world application in quantitative finance alone. As Itkin [4] suggests, neural-network-based option pricing models should also consider the necessary financial properties of differentiability for accurate Greek calculation and consistency with the no-arbitrage condition. The above demands require to consider both computational efficiency and financial feasibility in the application of deep learning to option pricing.

Reviewing the development of option pricing models from traditional stochastic-volatility frameworks to machine-learning-based surrogate models in this paper. Investigate the applications of machine learning in supporting the implementation of traditional pricing models. Section 2 introduces the development of option pricing methods, such as the Black-Scholes and Heston models, and also presents neural network-based and differentiable deep neural network (DDN) approaches for studying them in finance through differentiability. Section 3 will introduce the actual applications and shortcomings of differentiable surrogate models in improving pricing, calibration, and risk management.

2. Evolution of option pricing models

Option Pricing Models have developed significantly over the past decades. This development has been driven by the continuous demand for more accurate representations of financial markets and more efficient computational methods. Early analytical models established the theoretical foundation for derivative valuation. Later stochastic models introduced more realistic assumptions regarding market dynamics. These models also became more complex and created new computational challenges. This led to the development of machine learning-based approaches and differentiable surrogate models.

2.1. Traditional option pricing models

Traditional option pricing models are based on assumptions about asset price dynamics, market efficiency, and stochastic processes. They are still used as benchmarks for evaluating modern computational methods. The Black-Scholes model represents the starting point of modern option pricing theory. Stochastic volatility models such as the Heston model provided a more flexible framework for modeling changes in volatility.

2.1.1. Black-Scholes model

The Black-Scholes model was introduced by Black and Scholes in 1973 [1]. It was a milestone in modern financial mathematics since it established a closed-form solution for European option pricing. The model also provided a theoretical framework for derivatives pricing based on the no-arbitrage principle.

The main assumption of the Black-Scholes model is that the underlying asset price follows a geometric Brownian motion with constant volatility:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad (1)$$

Where S_t is the underlying asset price, μ is the expected return, σ is the constant volatility, and W_t is a standard Brownian motion.

The main merit of the Black-Scholes model is its analytical simplicity and computational efficiency. Whereas its assumption of constant volatility is a limitation. Empirical evidence suggests that the constant volatility assumption of the Black-Scholes model cannot fully explain observed market behavior. Recent research showed that when the model is applied to market option data, implied volatilities across options differ, indicating deviations from the constant-volatility assumption [5]. These findings led to the development of more flexible models that allow volatility to change over time.

2.1.2. Stochastic volatility models: Heston model

Stochastic Volatility Models have gradually been employed to model fluctuating levels of volatility over time. In 1993, Heston presented a stochastic volatility model [2]. The above models are more flexible than the constant-volatility model in describing the empirical features of the option market. Recently, some scholars have extended the Heston model to enhance the explanatory power for complex market phenomena. Ahn and others are examples; they used a multi-factor Heston model to find that adding more volatility factors significantly improved the model's fit to the data [6].

The key contribution of the Heston model is the introduction of a separate stochastic process of variance. Instead of assuming σ is constant, the model assumes that variance evolves according to a mean-reverting process:

$$dv_t = \kappa(\theta - v_t)dt + \xi\sqrt{v_t}dW_t^v \quad (2)$$

κ is the speed of mean reversion, θ is the long-term variance level, ξ is the volatility of variance, commonly referred to as volatility of volatility, and W_t^v is another Brownian motion process.

The asset price dynamics follow:

$$dS_t = \mu S_t dt + \sqrt{v_t} S_t dW_t^S \quad (3)$$

where S_t is the underlying asset price, μ is the expected return, V_t is the instantaneous variance, and W_t^S represents the Brownian motion process associated with asset price movements.

The model further incorporates the correlation between asset returns and variance changes:

$$dW_t^S dW_t^v = \rho dt \quad (4)$$

where ρ captures the relationship between changes in asset prices and volatility. This correlation structure allows the Heston model to capture asymmetric volatility effects and implied volatility patterns that cannot be fully described by the Black-Scholes model.

Stochastic variance and correlation are added to the Heston model to reflect the real-world shape of the implied volatility surface and market fluctuations that the Black-Scholes model cannot explain. Thus, in recent years, derivative pricing, risk management and volatility modelling have been broadly applied to it.

Although the Heston model has the above advantages, it is also relatively computationally intensive. The Black-Scholes model has a closed-form solution, but the Heston model needs to repeatedly compute the characteristic function and option prices numerically. Many times during calibration, the model needs to calculate option prices based on market data by adjusting its parameters. The above calculations will increase the cost of calibration. Therefore, some research

has been carried out to reduce the repeated pricing cost and maintain the financial structure of stochastic volatility models.

2.2. Machine learning-based option pricing models

The demand for faster derivative valuation has led to the use of machine learning as a data-driven approach to option pricing. Machine learning models aim to approximate the relationship between model inputs and option prices. This approach reflects a shift from traditional model-based pricing to surrogate modeling. Machine learning-based surrogate models do not need to replace the financial assumptions of classical models. Instead, they can approximate the outputs of existing pricing engines and reduce the time required for valuation and calibration.

2.2.1. Deep neural network-based pricing models

One of the first uses of machine learning techniques in derivative pricing was introduced by Hutchinson, Lo and Poggio [7]. Their study put forward a nonparametric learning framework for pricing and hedging derivative securities using learning networks. The former saw how the derivative price changed because of the market change firsthand. This was not in line with the assumptions of the standard option pricing model for a particular asset price process. Hutchinson and others have applied learning networks to the problems of derivative pricing and hedging. They have pointed out that the financial pricing model should be reasonably accurate and incorporate risk control [7].

Following these early developments, the interest in neural-network-based option pricing models also increased. Deep neural networks (DNNs) became particularly useful because of their ability to approximate highly nonlinear relationships and process high-dimensional financial data. Ruf and Wang reviewed the use of neural networks in option pricing and hedging. Their review showed that neural networks have been applied to derivative valuation, implied volatility modeling, and hedging tasks that require models capable of providing reliable risk-related outputs beyond price prediction [8].

In option pricing applications, DNNs are often used as surrogate models for computationally expensive pricing functions. DNN-based surrogate models aim to reproduce the outputs of existing pricing engines. For example, a neural network can approximate the relationship between market variables, model parameters, and option prices:

$$\hat{C} = f_{\theta}(x) \quad (5)$$

Where f_{θ} is a learned approximation function with parameters θ , $\hat{C}$ is the predicted option price, x is the input variables.

A typical feed-forward neural network contains several hidden layers. Each layer performs a nonlinear transformation to the output of the previous layer:

$$h^{(l)} = \sigma(W^{(l)}h^{(l-1)} + b^{(l)}) \quad (6)$$

Where $h^{(l)}$ is the hidden representation at layer l , $W^{(l)}$ and $b^{(l)}$ are network parameters, and σ is the activation function. Through multiple nonlinear transformations, DNNs are capable of approximating complex relationships between financial variables and option prices.

Culkin and Das [9] suggested that deep learning models can approximate option pricing functions and reproduce results from traditional pricing models with high accuracy. Their study also showed that neural networks can be used as efficient alternatives for computationally intensive pricing problems, especially when option prices need to be calculated many times.

While computational efficiency is a key advantage of DNN-based pricing models, practical derivative applications require more than accurate price prediction. Option pricing models are used for valuation, calibration, risk management, hedging, and portfolio optimization. These applications require reliable sensitivity measures, commonly referred to as Greeks. Greeks are derivatives of option prices with respect to underlying market variables. Therefore, the pricing function must be sufficiently smooth to provide stable sensitivity information. Such practical requirements have led to the development of differentiable neural network architectures that include derivative information in surrogate pricing models.

2.2.2. Differentiable deep neural networks for option pricing

Accurate Pricing is not sufficient for actual application in the derivatives market. Itkin believes that the financial demands of a stable calibration performance, reliable Greek computation, and consistency with financial principles must be met by the neural network-based pricing model [4].

Differentiable Deep Neural Networks extend the previous DNN-based surrogate model by adding differentiability to the pricing mechanism. DDNs also aim to preserve the differentiable structure of the pricing function and thus can compute derivatives with respect to input variables directly. The above property can provide both the value of a neural network and the corresponding sensitivity analysis required by finance.

DNNs are employed as the surrogate model because they can approximate the complex nonlinear function effectively. Theoretical research has shown that deep neural networks have sufficient expressive power to approximate a large number of nonlinear functions, and thus they can be used to approximate computationally intensive pricing functions [10].

The main advantage of DDNs is their ability to preserve gradient information. The derivatives of the learned pricing function can be calculated as:

$$\nabla C = \frac{\partial C}{\partial X} \quad (7)$$

where X is the relevant market variables or model parameters. These derivatives correspond to those used in option risk management. For example, Delta measures the sensitivity of the option price to changes in the underlying asset price and is defined as:

$$\Delta = \frac{\partial C}{\partial S} \quad (8)$$

where S represents the underlying asset price. By maintaining differentiability, DDNs can calculate these sensitivities through automatic differentiation. This avoids the repeated pricing evaluations used in traditional bump-and-reprice methods.

Chi presented a potential application of differential neural networks in financial modeling and added derivative information to a neural-network-based pricing system. A differentiable neural network can be used to leverage gradient information for enhanced estimation of financial indicators that depend on derivatives of the pricing function [11]. Therefore, for financial applications that do not aim to achieve better-than-price prediction, differentiability is not required.

In the context of stochastic volatility models, DDNs can also provide a potential solution for reducing the computational burden associated with repeated pricing and calibration procedures. A typical Heston-based DDN surrogate can be formulated as:

$$C_{DDN} = F_{\theta}(S, K, T, r, v_0, \kappa, \theta, \xi, \rho) \quad (9)$$

where S , K , T and r are the market variables, v_0, κ, θ, ξ and correspond to Heston model parameters. By learning the mapping between these inputs and option prices while maintaining differentiability, DDNs can provide fast pricing evaluation together with parameter sensitivities required for calibration and risk analysis.

Recently, some research has used differentiable neural network methods for stochastic volatility models. Zhang and others put forward a deep differential network framework for Heston model calibration. They have found that differentiable neural networks can approximate the Heston pricing function and provide derivatives for gradient-based calibration [12]. Based on the above approaches, the DDN framework will probably combine the financial interpretability of traditional stochastic volatility models with the computational efficiency of artificial intelligence. Most differentiable deep neural networks are extensions of neural network-based pricing models that meet the high-end mathematical requirements for financial applications.

3. Discussion and future perspectives

3.1. Computational efficiency and practical implications

DDN-based surrogate models can make complex stochastic volatility models more practical for repeated valuation and calibration. This is useful when option prices need to be updated frequently or when a large number of contracts need to be evaluated. In these cases, even a small reduction in the cost of each pricing operation can improve the efficiency of the overall modeling process.

The effect is also relevant to the way complex pricing models are used in financial practice. Models that are computationally demanding may become more practical when combined with efficient surrogate methods. This can support applications such as real-time derivative pricing, dynamic risk management, and large-scale portfolio analysis. From this perspective, the role of DDNs is not simply to provide faster predictions. They can also make computationally intensive financial models more practical for applications that require frequent and large-scale calculations.

3.2. Accuracy-efficiency trade-off in DDN-based pricing

Surrogate Pricing Models require computational efficiency, but they must also have good pricing accuracy for financial applications. A good surrogate model should reduce computational costs without increasing the price error significantly. A surrogate model is a substitute for an existing pricing model. A good result under a particular set of market conditions cannot be expected in all other cases. Model validation is thus required for machine learning applications in derivative pricing. A low prediction error on a fixed dataset is not sufficient. The model should still perform well in the changed market.

Market Conditions, volatility, and Asset characteristics are all changing constantly. A neural network will work better if the market conditions are close to those in the training data. However, the performance will also be lower in an unstable market environment or with different asset

characteristics. Therefore, the following research will explore how to enhance the robustness and generalization capabilities of surrogate pricing models under various market conditions.

3.3. Differentiability, Greeks, and hedging

Differentiability is Required for Risk Management and Hedging. The price of an option is its value, and the risk of a decrease in the market is not included. Delta and Vega are also used to show these kinds of risks. A Pricing Model with reasonable sensitivity estimates can support a finer analysis of portfolio risk. This requirement is also a problem for machine-learning-based pricing models. A neural network can find the option price accurately but is prone to unstable sensitivity estimates. Therefore, only the accuracy of pricing is insufficient for evaluating a machine-learning-based pricing model. The quality of the risk information is also a problem. DDN-based methods can be used to extend the application of neural networks to risk analysis.

Greeks are also employed for hedging. Delta and Vega show how an option portfolio changes when the price or volatility of the underlying asset changes. Accurate Sensitivity Data Help Traders Adjust Hedging Strategies Based on Changes in Market Conditions. DDNs may be suitable for both pricing and risk analysis at the same time. Accurate Greeks do not guarantee the success of hedging. Hedging results can also be affected by transaction costs, rebalancing frequency, market liquidity, and model risk. Thus, one cannot know how effective the differentiable pricing model is in practice without a real-trade environment. More than testing the accuracy of the calculated Greeks is needed.

3.4. Limitations and future research directions

DDN-based Pricing Models have some deficiencies despite their prospective strengths. Model Interpretability is also one type. The Parameters of the older stochastic volatility model are economically feasible. The above indices can guide the public's economic understanding. Neural-network-based models are less transparent than other models because they have many learned parameters and nonlinear relationships. Therefore, these may not be suitable for use in financial applications requiring model interpretation.

Model Robustness and Generalization are also problems. Neural-network-based surrogate models are based on the data used for training. Changes in the market environment, a shift in volatility regime, and altered asset features. In the future, some of these models will be adapted to meet the demands of the market.

Financial Constraints are also objects of future study. A neural network can be used to build the conditions of no-arbitrage and other financial rules. The above restrictions will increase the stability of the machine learning model in finance. In the future, some research will also examine the combination of traditional finance and machine learning. Physics-informed neural networks and arbitrage-aware learning frameworks are examples of this direction.

In summary, DDN-based surrogate models offer a link between traditional stochastic volatility models and modern machine learning methods. They do not need to replace classical financial models. Instead, they can extend their practical use by adding computational and differentiable capabilities. This combination provides a useful direction for the continued development of machine learning-based methods in quantitative finance.

4. Conclusion

The development of option pricing models reflects the need to balance financial representation and computational efficiency. The review shows that DDN-based surrogate models combine neural-network methods with differentiability. They can support pricing, sensitivity analysis, calibration, and hedging. These models still have some limitations. Their performance depends on training data. Their internal structure is also less transparent than that of traditional stochastic volatility models. Practical factors such as market liquidity, transaction costs, and model risk also need further study. Future research can focus on model robustness, adaptability, and interpretability. Researchers can also study financial constraints and hybrid approaches that combine traditional financial models with machine learning.

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