

An Empirical Comparison of Black-Scholes and Binomial Tree Models in Pricing US Technology Stock Options

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Abstract. Option pricing is an important problem in quantitative finance because an option's value depends on the underlying stock price, volatility, interest rates, and time to maturity. For European call options, the Black–Scholes (B–S) closed-form formula and binomial tree model represent two mainstream pricing approaches, yet their pricing accuracy diverges substantially when calibrated against real-world market data. This paper compares the two models by pricing at-the-money European call options with about six months to expiration on four US technology stocks: Apple (AAPL), Microsoft (MSFT), NVIDIA (NVDA), and Alphabet (GOOGL). Using daily stock prices from Yahoo Finance and risk-free rate data from FRED, this paper estimates historical volatility from daily log returns, calculates theoretical prices under both models, and compares those prices with observed market premiums. This paper also tests how quickly binomial tree prices approach B-S prices as the number of time steps increases. The results show that the binomial tree price converges to the B-S price with a convergence rate of $O(1/N)$. This research further finds that both models produce similar pricing errors for more liquid options, while pricing errors increase when historical volatility differs from market-implied volatility.

Keywords: Option Pricing, Black-Scholes Model, Binomial Tree, Empirical Analysis, US Stock Market

1. Introduction

Options are widely used for hedging and speculation [1]. Most option pricing models start from the no-arbitrage principle: in an efficient market, mispriced derivatives cannot generate risk-free arbitrage profits for investors. In a risk-neutral framework, the fair price of a derivative equals its expected payoff discounted at the risk-free rate [2].

This paper focuses on two standard models. The Black-Scholes model, introduced in 1973 [1], gives a closed-form price for European options under constant volatility, continuous trading, and frictionless markets. By contrast, the binomial tree model, established in 1979 [3], simulates underlying asset prices via discrete upward and downward movements across finite time steps, and calculates option values through backward induction from terminal payoffs. The binomial tree converges to the B-S formula as the number of steps approaches infinity, but the rate and practical significance of that convergence still matter when the model is applied to market data [4, 5].

This paper examines the convergence speed of the binomial tree model toward the B-S solution as the number of steps increases. It also compares the prices generated by each model with observed market prices for major US technology stocks.

The dataset comprises daily closing stock prices retrieved from Yahoo Finance and risk-free rate indicators obtained from FRED. Historical volatility is estimated from daily log returns. Both models are then used to price at-the-money European call options with roughly six months to expiry [6, 7].

2. Methodology

2.1. Theoretical foundation of option pricing

The fundamental theorem of asset pricing establishes that for any complete arbitrage-free market, an equivalent martingale measure exists such that discounted asset prices follow martingale processes [2]. Under this risk-neutral measure, the fair price of a European call option with strike price K and time to maturity T is:

$$C = e^{-rT} E^Q [\max (S_T - K, 0)] \quad (1)$$

where r is the risk-free rate and S_T is the asset price at maturity under the risk-neutral process.

2.2. Calculation of log returns and historical volatility

Historical volatility is estimated from daily closing prices using log returns:

$$R_t = \ln \left(\frac{S_t}{S_{t-1}} \right) \quad (2)$$

the annualized historical volatility is computed as $\sigma_{\text{hist}} = \sqrt{252} \times \text{std}(R_t)$, where 252 represents the typical number of trading days per year.

2.3. Construction of option pricing models

2.3.1. The multi step binomial tree model

The binomial tree discretizes time into N intervals of length $\Delta t = T/N$. At each node, the asset price can move up by factor $u = e^{\sigma\sqrt{\Delta t}}$ or down by factor $d = 1/u$, with risk-neutral probability $p = (e^{r\Delta t} - d) / (u - d)$ [3]. Option prices are computed by backward induction: starting from terminal payoffs $\max(S_T - K, 0)$, the option value at each preceding node is the discounted expected value under the risk-neutral measure.

2.3.2. The Black-Scholes formula

The Black-Scholes pricing formula delivers an analytical closed-form solution for European call options [1]:

$$C = S_0 \Phi(d_1) - K e^{-rT} \Phi(d_2) \quad (3)$$

where $d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$, $d_2 = d_1 - \sigma\sqrt{T}$, and $\Phi(\cdot)$ is the standard normal cumulative distribution function.

2.3.3. Theoretical connection: limit convergence

A well-established result in financial mathematics states that as the number of time steps N approaches infinity, the binomial tree price converges to the Black-Scholes price at a rate of $O(1/N)$ [2]. This convergence arises from the Central Limit Theorem, as the binomial approximation to the log-normal distribution improves with finer partitioning of the time interval.

2.4. Data acquisition and parameter setting

Daily closing equity prices of AAPL, MSFT, NVDA and GOOGL were retrieved from Yahoo Finance, spanning a complete three-year sample window comprising 752 trading days up to June 2026. The risk-free rate was estimated as the average 10-year US Treasury yield over the same period from FRED, yielding $r = 3.18\%$. For each stock, six-month European call options were selected with strike prices matching the prevailing spot price (at-the-money, ATM).

The annualized historical volatilities estimated from log returns are 25.87% for AAPL, 24.09% for MSFT, 46.81% for NVDA, and 29.71% for GOOGL. NVDA has the highest volatility in the sample, which is consistent with its larger price swings during the period.

3. Empirical analysis

This section reports the empirical comparison between the Black-Scholes formula and the binomial tree model. It first presents the convergence test, then compares model prices with market option premiums, and finally examines how option values respond to changes in volatility.

3.1. Convergence test with increasing time steps

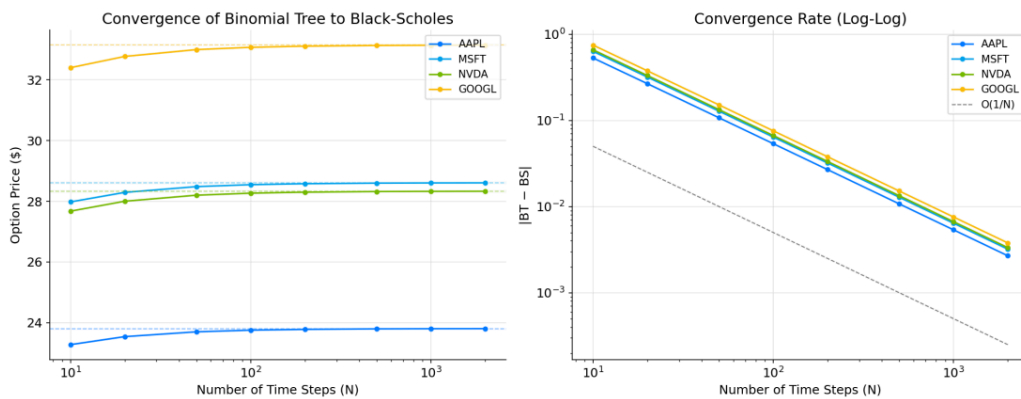


Figure 1. (a) Convergence of binomial tree to Black-Scholes; (b) Convergence rate (log-log)

To test convergence, binomial tree prices were computed for $N = 10, 20, 50, 100, 200, 500, 1000, 2000$ and compared against the B-S analytical price for each stock. Empirical outputs validate the theoretical $O(1/N)$ asymptotic convergence rate: absolute pricing deviations between binomial tree outputs and B-S analytical values shrink

proportionally as N increases. At $N = 500$, the error is already below 0.01 for all four stocks. At $N = 2000$, the error falls below 0.004 for all cases. Figure 1 illustrates this convergence behavior.

Table 1. Pricing model comparison for ATM call options

Stock	Spot price	Strike price	Historical volatility	Implied volatility	B-S price	Binomial tree price	Market price
AAPL	\$295.95	\$295.00	25.87%	27.49%	\$24.27	\$24.27	\$25.60
MSFT	\$378.91	\$380.00	24.09%	35.88%	\$28.06	\$28.07	\$40.49
NVDA	\$204.65	\$205.00	46.81%	44.48%	\$28.17	\$28.16	\$26.85
GOOGL	\$363.79	\$365.00	29.71%	37.07%	\$32.55	\$32.56	\$40.00

3.2. Pricing error comparison against market premiums

Raw market option valuations were extracted from Yahoo Finance option chain datasets. Since the official API failed to supply valid bid–ask spread quotes, closing last-traded prices were adopted as proxies for equilibrium market option premiums [8]. This choice matters. For actively traded options, the last price is usually informative; for less liquid options, it may be stale.

For AAPL, the B-S price (\$24.27) is close to the market price (\$25.60), with a pricing error of 5.19%. The binomial tree gives a nearly identical error of 5.18%. NVDA shows a similar pattern: both models underestimate the market price by about 4.9%. One likely explanation is that market prices include a volatility risk premium that is not captured by historical-volatility inputs.

Substantially wider relative deviations are observed for MSFT (30.69%) and GOOGL (18.61%). Such pronounced mismatches stem primarily from outdated last-traded proxy prices instead of fundamental misspecification within the two pricing models. Comparing implied volatility from market prices, computed through inverse B-S, with historical volatility gives $\sigma_{\text{impl}}=27.49\%$ versus $\sigma_{\text{hist}}=25.87\%$ for AAPL and $\sigma_{\text{impl}}=44.48\%$ versus $\sigma_{\text{hist}}=46.81\%$ for NVDA. The closer match for these two stocks helps explain why both models price them more accurately [6, 9].

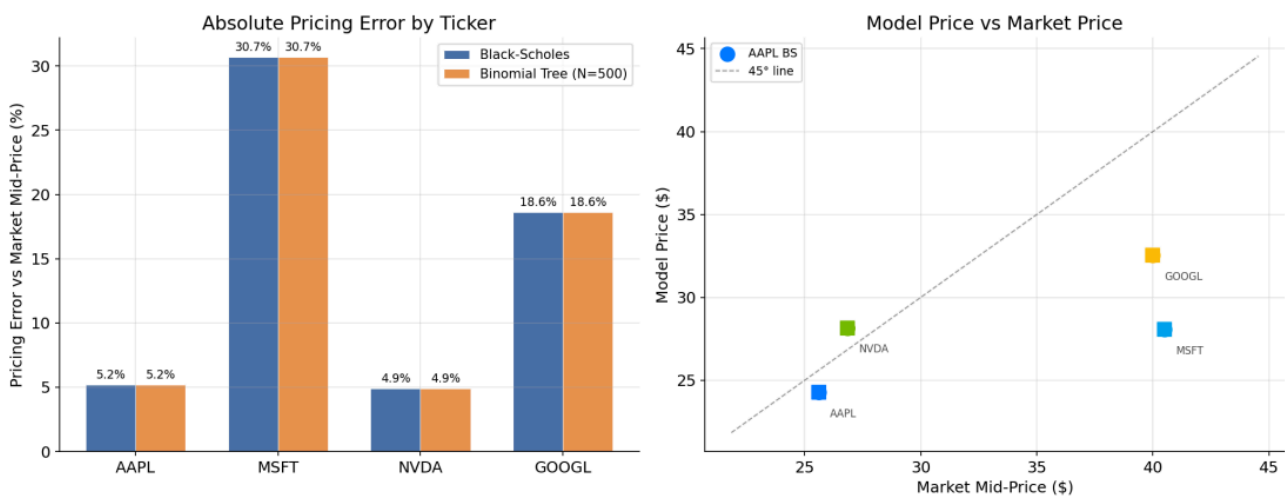


Figure 2. (a) Absolute pricing error by ticker; (b) model price vs market price

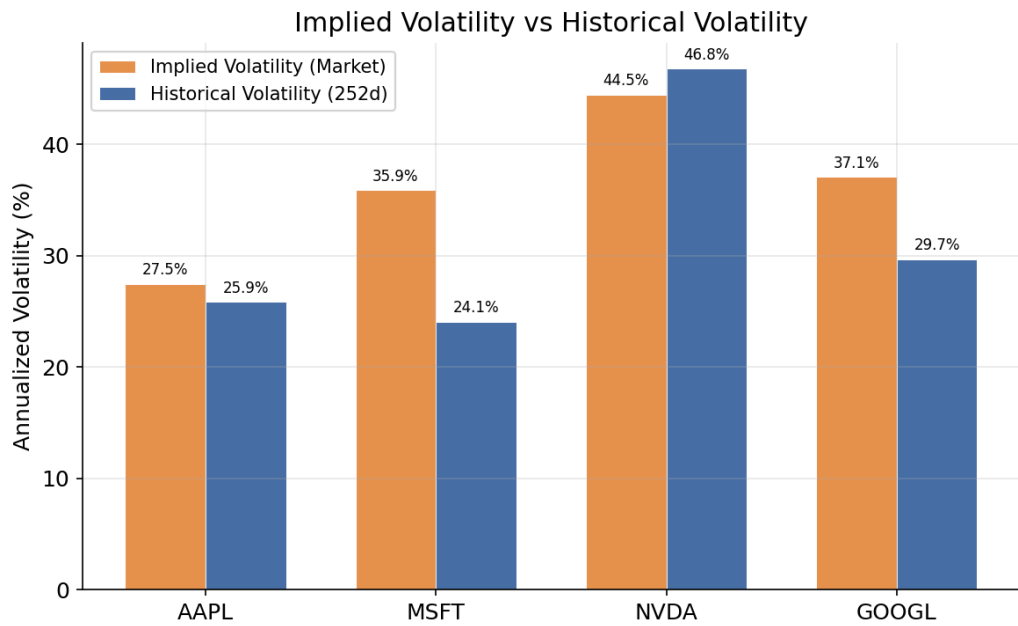


Figure 3. Implied volatility vs historical volatility

3.3. Volatility sensitivity analysis

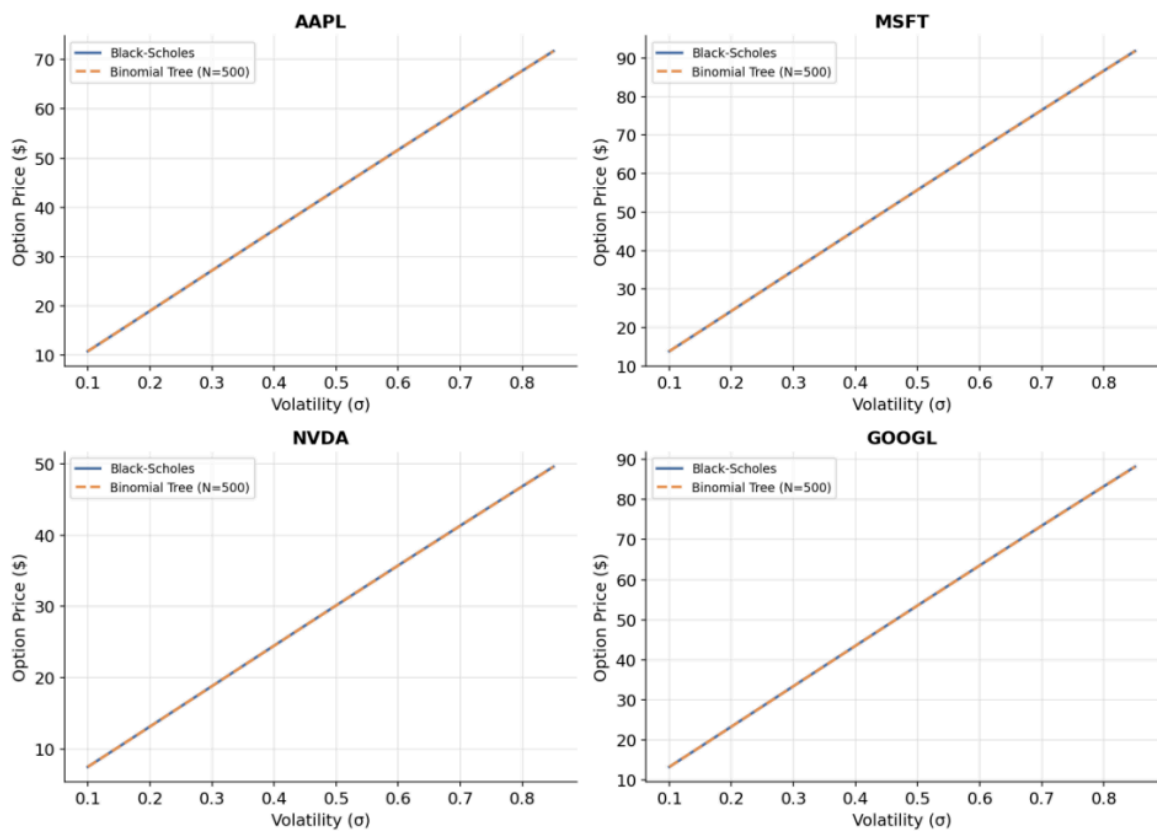


Figure 4. Sensitivity of option price to volatility

The volatility sensitivity analysis in Figure 4 shows that both B-S and binomial tree prices increase monotonically with volatility, consistent with the convexity of option payoffs. The two models produce almost identical price-volatility curves for all four stocks. With enough time steps, the binomial tree therefore works as a close numerical approximation to the B-S formula. NVDA's higher historical volatility (46.81%) also makes its option price more sensitive to volatility changes than a lower-volatility stock such as MSFT (24.09%).

4. Conclusion

This paper compared the Black-Scholes model and the binomial tree model for pricing European call options on US technology stocks. The empirical results are consistent with the theoretical convergence of the binomial tree to the B-S analytical solution at approximately $O(1/N)$. When calibrated against real-market option premiums, both frameworks yield comparable relative pricing deviations for highly liquid contracts within the sample dataset. The B-S model is more efficient for pricing European options in bulk, while the binomial tree is easier to adapt when an option has more complex features [7]. Both models, however, rely on a constant volatility assumption. In practice, asset returns often show volatility clustering and leverage effects, with volatility rising after price declines [9]. The gap between historical and implied volatility for MSFT and GOOGL suggests that a constant volatility input is too restrictive for parts of the sample. Future work could test whether time-varying volatility models, such as GARCH(1,1), provide a better fit to market prices [10].

References

- [1] Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3), 637–654. JSTOR, <http://www.jstor.org/stable/1831029>
- [2] Hull, J. C. (2021). *Options, futures and other derivatives* (11th ed.). Pearson Press.
- [3] Cox, J. C., Ross, S. A., & Rubinstein, M. (1979). Option pricing: A simplified approach. *Journal of Financial Economics*, 7(3), 229–263.
- [4] Shvimer, Y., & Herbon, A. (2020). Comparative empirical study of binomial call-option pricing methods using S&P 500 index data. *The North American Journal of Economics and Finance*, 51, 101071.
- [5] Zhang, B. (2025). An empirical comparison of BSM and binomial tree models in delta hedging for S&P 500 ETF (SPY) forward options. *Advances in Economics, Management and Political Sciences*, 221, 122–132.
- [6] Salami, M. F. (2024). Empirical examination of the Black-Scholes model: Evidence from the United States stock market. *Frontiers in Applied Mathematics and Statistics*, 10, 1216386.
- [7] Wu, Y. (2022). Options pricing comparison between the Black-Scholes model and the binomial tree model: A case study of American equity option and European-style index option. *BCP Business & Management*, 32, 172-177.
- [8] Ding, H. (2023). Research on option pricing of Tesla based on Black-Scholes-Merton, binomial tree, and Cox-Ingersoll-Ross models. *Highlights in Business, Economics and Management*, 22, 35-41.
- [9] Tsay, R. S. (2012). *Analysis of financial time series* (3rd ed.). Wiley Press.
- [10] Yee, B. (2025). Comprehensive analysis of option pricing methodologies. *Research Square*. Word count: approximately 1,650 words (excluding abstract, tables, equations, and references).