

Market Risk Measurement and Backtesting of Stock Portfolios (China A-Shares) Based on VaR and ES

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Abstract. In the context of normalized volatility in the A-share market, traditional VaR models are unable to effectively capture the tail risk of extreme market conditions, while Expected Shortfall (ES) can make up for this deficiency. This paper selects 12 A-shares from two equally weighted stock portfolios within and outside the CSI 300 index, with a sample period from May 2022 to April 2026. It uses three methods—historical simulation, parametric method (normal), and GARCH (1,1)-t—to estimate VaR and ES, and conducts backtesting using the Kupiec POFF test, the Christoffersen conditional coverage test, and the Acerbi-Székely Z2 test. The results show that GARCH (1,1)-t can more accurately characterize tail risk at a 99% confidence level. The historical simulation method is generally robust, while the normal parameter method tends to underestimate systematic risk. ES is greater than VaR in all scenarios, which verifies its stronger ability to capture extreme losses. The conclusion remains unchanged after replacing the confidence level and portfolio weights. This paper recommends that A-share investors and regulatory agencies use ES as an important supplementary indicator to VaR, and prioritize the use of GARCH-t or historical simulation methods for risk measurement.

Keywords: VaR, Expected Shortfall, Backtesting, Market Risk, Stock Investment Portfolio

1. Introduction

With the A-share market (Chinese) now experiencing normalized volatility, the market risk of stock portfolios needs to be accurately measured. While the traditional VaR model is the mainstream risk measurement tool in the industry, it cannot effectively capture tail risk under extreme market conditions, whereas the ES model can fill this gap [1, 2].

VaR (Value at Risk) and ES (Expected Shortfall) are the two most mainstream risk measurement tools in current financial regulation and practice. Scholars such as Artzner proposed a consistent risk measurement axiomatic system, pointing out that VaR does not satisfy subadditivity and cannot capture extreme tail risks [1]. ES addresses this deficiency. Fissler and Ziegel proved the joint extractability of the two models, overcoming the theoretical bottleneck that makes it difficult for ES to perform effective backtesting independently [2]. Subsequently, Bayer and Dimitriadis constructed a regression-based independent backtesting framework for ES, and Barendse et al. corrected the scale distortion caused by parameter estimation errors in joint backtesting [3, 4]. While current research has established a mature theoretical framework, there is still ample room for empirical

research on comparing the effectiveness of multi-model risk measurement and backtesting optimization for Chinese domestic stock portfolios. Existing studies, such as those by Feng et al., have analyzed the numerical characteristics of A-share volatility [5]. Yue Haomin and Sun Yingjun compared the performance of parametric and nonparametric VaR methods on the CSI 300 Index; Xu and Huang compared the accuracy differences of VaR measurement under different distributions using GARCH family models; and Zhang Jian performed VaR estimation and Kupiec backtesting based on an improved GARCH-t model [6-8]. This paper first reviews the core theories of VaR and ES, constructs a dual-combination sample, and completes the calculation and backtesting of VaR and ES.

2. Research design & data

2.1. Sample selection and data sources

The benchmark portfolio selects stocks from the CSI 300 index, representing a large-cap blue-chip style; the control portfolio selects stocks outside the CSI 300, representing a small- and mid-cap style. The two groups are roughly corresponding in industry to control for the impact of industry differences on risk measurement. Each group consists of six equally weighted stocks to avoid a single stock dominating portfolio risk.

The sample period is from May 6, 2022, to April 30, 2026 (daily frequency), and the data comes from the Wind financial terminal. The core indicators extracted include adjusted closing price (daily frequency), daily turnover, annualized volatility, Beta coefficient, and adjusted closing price of the CSI 300 Index. The model uses the daily logarithmic return of individual stocks and the equal-weighted daily return of the portfolio. The rolling window extrapolation uses an initial window of $W = 500$ trading days, rolling forward daily to dynamically update the model parameters.

2.2. Variable definition

Individual stock log return

$$R_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right) \quad (1)$$

Where $P_{i,t}$ is the ex-dividend adjusted closing price of stock i on day t .

Equally-weighted portfolio daily return

$$R_{p,t} = \frac{1}{N} \sum_{i=1}^N R_{i,t} \quad (2)$$

Where N is the number of stocks in the portfolio.

The dependent variables in this paper are the VaR and ES values. The non-independent variable is the holding period (daily).

2.3. Research methods

Historical Simulation Method: This method does not rely on distributional assumptions; instead, it directly estimates VaR using the empirical α -quantile of the portfolio's logarithmic return series over the past W trading days, and takes the mean of the losses exceeding the VaR as the ES [9].

$$\text{VaR}_t^{\text{HS}}(\alpha) = -\text{Percentile}(\{R_{p,j}\}_{j=t-W}^{t-1}, 1-\alpha) \quad (3)$$

$$\text{ES}_t^{\text{HS}}(\alpha) = -\frac{1}{\alpha W} \sum_{j \in E_t} R_{p,j}, \quad E_t = \{j : R_{p,j} \leq -\text{VaR}_t^{\text{HS}}(\alpha)\} \quad (4)$$

GARCH (1, 1) Model [9]:

$$R_{p,t} = \mu + \varepsilon_t, \quad \varepsilon_t = \sigma_t z_t \quad (5)$$

$$\omega > 0, \quad \beta \geq 0, \quad \gamma \geq 0, \quad \beta + \gamma < 1$$

Parametric method (variance-covariance / GARCH model). Fitting the conditional volatility of portfolio returns using the GARCH(1,1)-t model proposed by Bollerslev [9]. After obtaining the conditional volatility, this paper assumed—separately—that the residuals follow a normal distribution (Parametric Method–Normal) and a t-distribution (GARCH-t), and calculate the corresponding VaR and ES at the specified confidence levels.

$$\text{VaR}_t^{\text{Normal}}(\alpha) = -(\mu + \sigma_t \cdot \Phi^{-1}(1-\alpha)) \quad (6)$$

$$\text{ES}_t^{\text{Normal}}(\alpha) = -\mu + \sigma_t \cdot \frac{\phi(\Phi^{-1}(1-\alpha))}{1-\alpha} \quad (7)$$

Conditional variance model of the GARCH (1,1)

$$\sigma_t^2 = \omega + \gamma \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (8)$$

GARCH-t

$$\text{VaR}_t^{\text{GARCH-t}}(\alpha) = -(\mu + \sigma_t \cdot t_v^{-1}(1-\alpha)) \quad (9)$$

$$\text{ES}_t^{\text{GARCH-t}}(\alpha) = -\mu - \sigma_t \cdot \text{ET}_\nu(\alpha) \quad (10)$$

2.4. Backtesting methodology

This paper employs the following standardized backtesting framework: the Kupiec POF unconditional coverage test (LR_uc), under the null hypothesis that the actual breach rate equals the nominal confidence level α [10]. The null hypothesis for Christoffersen's independence test (LR_ind) and joint test (LR_cc) is that the breach events are mutually independent over time [11]. Acerbi-Székely Z2 test; p-value obtained based on 1,000 bootstrap iterations [12].

3. Empirical results and analysis

3.1. Descriptive statistics and preliminary hypothesis tests

Table 1. Constituent stocks of the sample portfolios

Portfolio	Stock Name	Stock Code	SW Industry Classification
Benchmark Portfolio (within CSI 300)	Industrial and Commercial Bank of China	601398.SH	Banking
	China Shenhua	601088.SH	Coal
	China Petroleum & Chemical Corporation	600028.SH	Petrochemicals
	Dawning Information Industry	603019.SH	Computer
	Inspur Electronic Information Industry	000977.SZ	Computer
	China Yangtze Power	600900.SH	Utilities
Control Portfolio (outside CSI 300)	Suken Agricultural Development	601952.SH	Agriculture, Forestry, Animal Husbandry & Fishery
	Bank of Nanjing	601009.SH	Banking
	Bank of Nanjing	600170.SH	Building Decoration
	Chongqing Water Group	601158.SH	Utilities
	Hongcheng Environment	600461.SH	Environmental Protection
	Sun Paper	002078.SZ	Light Manufacturing

This paper selects two groups comprising a total of 12 A-share stocks, with a sample period spanning from May 6, 2022, to April 30, 2026—totaling 968 trading days—as shown in Table 1. The benchmark portfolio consists of six equally weighted CSI 300 constituent stocks, while the control portfolio consists of six equally weighted stocks outside the CSI 300 index.

3.2. Descriptive statistics of portfolio returns

Descriptive statistics for the daily log return of the two equally weighted portfolios are presented in Table 2. The benchmark portfolio posted an average daily return of 0.0782%, a daily volatility of 1.1535%, and an annualized volatility of 18.31%; the control portfolio recorded an average daily return of 0.0091%, a daily volatility of 0.9433%, and an annualized volatility of 14.97%. The benchmark portfolio exhibited a skewness of 0.144, while the control portfolio had a skewness of -0.408; their respective kurtosis values were 4.884 and 11.498—both significantly higher than the value of 3 associated with a normal distribution. The JB test strongly rejects the null hypothesis of normality at the 1% significance level, indicating that the distribution of returns for the A-share portfolio exhibits characteristic leptokurtosis and heavy tails. This directly serves as the theoretical basis for this paper's adoption of the GARCH-t model and the historical simulation method [5, 13]. The p-values of the ADF tests are all less than 0.01, indicating that the return series is stationary. Figure 1 shows the cumulative net value curves of the two equally weighted portfolios. Figure 2 lists the distribution of daily log returns.

Table 2. Descriptive statistics of daily log returns of the two portfolios

Statistic	Benchmark Portfolio (within CSI 300)	Control Portfolio (outside CSI 300)
Sample size	968	968
Average daily return (%)	0.0782	0.0091
Daily volatility (%)	1.1535	0.9433
Annualized return (%)	19.70	2.28
Annualized volatility (%)	18.31	14.97
Skewness	0.144	-0.408
Kurtosis	4.884	11.498
Maximum drawdown (%)	-19.63	-15.38
Jarque–Bera test p-value	<0.001	<0.001
ADF test p-value	0.010	0.010

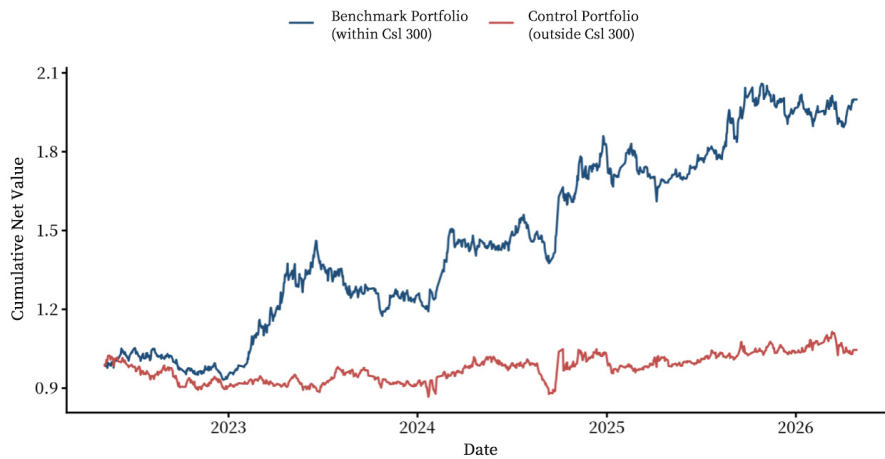


Figure 1. Cumulative net value curves of the two equally weighted portfolios (picture credit: original)

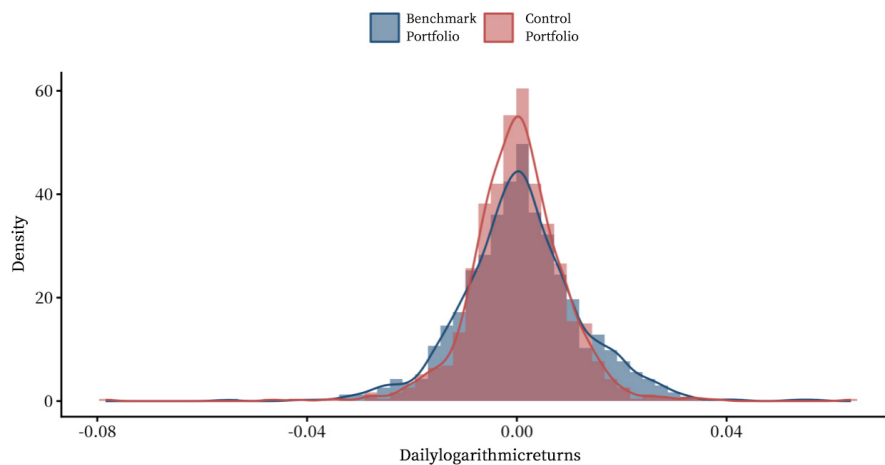


Figure 2. Distribution of daily log returns (picture credit: original)

3.3. Analysis of VaR and ES measurement results

To ensure the robustness of the window selection, this paper scans three rolling window lengths— $W \in \{125, 250, 500\}$ —under the historical simulation method (Table 3). The results indicate that $W=500$ demonstrates the overall best performance across both combinations and both confidence levels (yielding the least significant LR_uc test results); consequently, both the subsequent parametric method and the GARCH (1,1)-t model are extrapolated using a rolling window of $W=500$ [9, 10].

The main results for VaR and ES are presented in Table 4. While the VaR estimates derived from the three methods are comparable in magnitude at the 95% confidence level, at the 99% extreme tail, the VaR and ES values yielded by the GARCH (1,1)-t model are significantly higher than those of the other methods, thereby demonstrating its dual capability to capture both volatility clustering and heavy-tail characteristics. The average VaR/ES of the benchmark portfolio is higher than that of the control portfolio, which aligns with the benchmark portfolio's characteristics of having a higher weighting in the computer industry and individual stocks with greater volatility [9].

Table 3. Comparison of rolling window lengths for historical simulation

Portfolio	Confidence level	Window	Nominal violation rate (%)	Actual violation rate (%)	Average VaR (%)	LR_uc p-value	LR_cc p-value
Benchmark (within CSI 300)	95%	125	5.00	5.81	1.717	0.291	0.123
		250	5.00	4.87	1.765	0.877	0.276
		500	5.00	4.49	1.724	0.605	0.170
	99%	125	1.00	2.25	2.589	0.002	0.005
		250	1.00	1.39	2.763	0.318	0.527
		500	1.00	1.07	2.821	0.883	0.937
Control (outside CSI 300)	95%	125	5.00	5.10	1.338	0.893	0.981
		250	5.00	5.99	1.376	0.238	0.480
		500	5.00	5.77	1.451	0.456	0.666
	99%	125	1.00	1.66	2.111	0.078	0.167
		250	1.00	1.67	2.291	0.099	0.209
		500	1.00	1.71	2.372	0.161	0.326

Table 4. Summary of VaR / ES estimation results (rolling outofsample, $W = 500$)

Portfolio	Confidence Level	Method	Average VaR (%)	Average ES (%)	Violation rate (%)
Benchmark (within CSI 300)	95%	GARCH (1,1)-t	1.710	2.367	4.91
		Historical Simulation	1.724	2.488	4.49
		Parametric (Normal)	1.891	2.395	3.85
	99%	GARCH (1,1)-t	2.750	3.458	1.07
		Historical Simulation	2.821	3.541	1.07
		Parametric (Normal)	2.712	3.121	1.07

Table 4. (continued)

Control (outside CSI 300)	95%	GARCH (1,1)-t	1.460	2.034	4.27
		Historical Simulation	1.451	2.211	5.77
		Parametric (Normal)	1.614	2.029	4.70
	99%	GARCH (1,1)-t	2.365	3.002	1.71
		Historical Simulation	2.372	3.878	1.71
		Parametric (Normal)	2.292	2.628	1.71

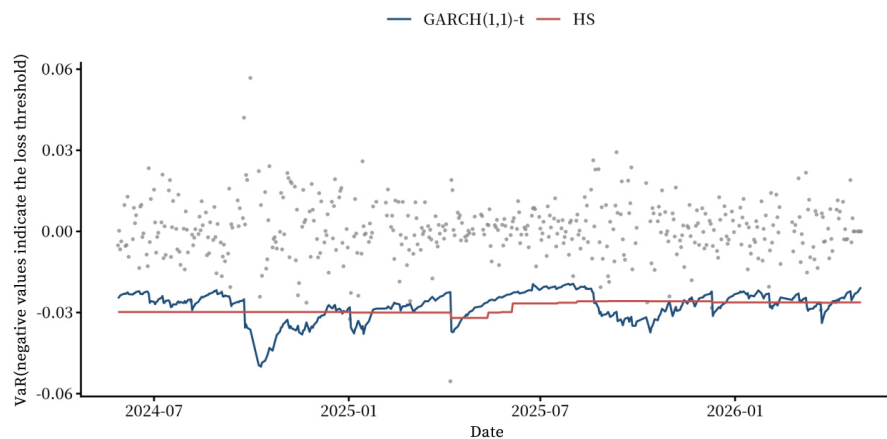


Figure 3. Rolling VaR at 99% for the benchmark portfolio (HS & GARCH) (picture credit: original)

As shown in Figure 3, both the volatility and the VaR/ES of the benchmark portfolio (within the CSI 300 index) are higher than those of the control portfolio—a finding that does not entirely align with the common intuition that "large-cap blue-chip stocks exhibit lower volatility." The underlying reason is that the benchmark portfolio was constructed using an equal-weighting scheme and includes two stocks from the computer sector (accounting for a combined weight of one-third); driven by the AI theme during the sample period, these holdings exhibited high volatility, thereby elevating the overall risk metrics of the portfolio. This result also indicates that, in the A-share market, portfolio risk characteristics depend more on industry composition and weighting methodology than on a simple large-cap/small-cap classification.

3.4. Backtesting results and validity analysis

This paper uses the Kupiec POF test (LR_uc), Christoffersen independence test (LR_ind), and joint test (LR_cc) to test the ES model using the Acerbi-Székely Z2 test (based on 1000 bootstraps). The results are summarized in Table 5 [10-12].

The results indicate that, at the 95% confidence level, neither the LR_uc nor the LR_cc statistics for the GARCH (1,1)-t model and the Historical Simulation method allow for the rejection of the null hypothesis at the 5% significance level, thereby demonstrating the validity of the models. The Normal Parameter Method likewise passed the unconditional coverage and independence tests; however, its actual violation rates (3.85% and 4.70%) both fell below the nominal level of 5%, exhibiting a tendency to underestimate risk. In light of the results in Section 4.2—which indicate that the return series significantly rejects the assumption of normality—the normal distribution

hypothesis fails to adequately capture the fat-tail characteristics of A-share portfolio returns, resulting in risk estimates that are biased toward optimism [13]. At a 99% confidence level, the GARCH (1,1)-t model consistently passes both the Kupiec test and the ES Z2 test; the Historical Simulation method also largely passes, whereas the Normal Parametric method systematically underestimates extreme tail risk. This result stands in contrast to the cross-country comparison conducted by Chawla (2025) across the U.S., U.K., and Indian markets, reflecting the unique characteristics of tail risk modeling in emerging markets [14].

Table 5. VaR / ES backtesting results

Portfolio	Confidence level	Method	Violations / Sample	LR_uc p-value	LR_ind p-value	LR_cc p-value	ES Z2 p-value
Benchmark (within CSI 300)	95%	GARCH (1,1)-t	23 / 468	0.932	0.123	0.303	0.992
		Historical Simulation	21 / 468	0.605	0.071	0.170	0.988
		Parametric (Normal)	18 / 468	0.233	0.719	0.461	0.982
	99%	GARCH (1,1)-t	5 / 468	0.883	0.742	0.937	0.929
		Historical Simulation	5 / 468	0.883	0.742	0.937	0.981
		Parametric (Normal)	5 / 468	0.883	0.742	0.937	0.978
Control (outside CSI 300)	95%	GARCH (1,1)-t	20 / 468	0.460	0.874	0.752	1.000
		Historical Simulation	27 / 468	0.456	0.612	0.666	0.982
		Parametric (Normal)	22 / 468	0.764	0.970	0.955	0.975
	99%	GARCH (1,1)-t	8 / 468	0.161	0.597	0.326	0.967
		Historical Simulation	8 / 468	0.161	0.597	0.326	0.982
		Parametric (Normal)	8 / 468	0.161	0.597	0.326	1.000

4. Robustness checks

This section follows the backtesting framework from 4.3, namely the Kupiec POF test, the Christoffersen conditional coverage and independence joint test, and the Acerbi-Székely Z2 test. All tests are performed based on out-of-sample predictions with rolling extrapolation.

4.1. Alternative confidence levels

After replacing 95%/99% with 90%/97.5%, the historical simulation method and GARCH (1,1)-t were rerun (Table 6). The conclusions are consistent with the main analysis: the GARCH model showed no significant p-values in the LR_uc and ES Z2 tests at both new confidence levels, and the breakthrough rate closely followed the nominal level; the historical simulation method performed well at the 90% level, and deviated slightly at the 97.5% level but still passed the joint test, indicating that the core conclusions of this paper are not sensitive to the choice of confidence level.

Table 6. Robustness checks (I): alternative confidence levels

Portfolio	Confidence level	Method	Violation rate (%)	Average VaR (%)	Average ES (%)	LR_uc p-value	LR_cc p-value	ES Z2 p-value
Benchmark (within CSI 300)	90.0%	GARCH (1,1)-t	8.97	1.264	1.918	0.452	0.748	0.984
		Historical Simulation	8.55	1.263	1.981	0.284	0.533	0.991
	97.5%	GARCH (1,1)-t	2.35	2.150	2.825	0.834	0.750	0.997
		Historical Simulation	2.14	2.348	2.937	0.606	0.703	0.978
Control (outside CSI 300)	90.0%	GARCH (1,1)-t	9.40	1.081	1.644	0.663	0.738	0.991
		Historical Simulation	10.26	0.996	1.700	0.854	0.858	0.960
	97.5%	GARCH (1,1)-t	2.99	1.840	2.437	0.509	0.521	0.987
		Historical Simulation	2.99	1.810	2.762	0.509	0.521	0.972

4.2. Alternative weighting schemes

Table 7. Robustness checks (II): Equal weighting vs. inverse volatility weighting (GARCH)

Portfolio	Confidence level	Weighting scheme	Violation rate (%)	Average VaR (%)	Average ES (%)	LR_uc p-value	LR_cc p-value	ES Z2 p-value
Benchmark (within CSI 300)	95%	Inverse volatility weighting	5.13	1.346	1.938	0.899	0.270	0.990
		Equal weighting	4.91	1.710	2.367	0.932	0.303	0.992

Table 7. (continued)

Benchmark (within CSI 300)	99%	Inverse volatility weighting	1.50	2.273	2.960	0.315	0.543	0.978
		Equal weighting	1.07	2.750	3.458	0.883	0.937	0.929
Control (outside CSI 300)	95%	Inverse volatility weighting	4.70	1.454	2.018	0.764	0.955	0.981
		Equal weighting	4.27	1.460	2.034	0.460	0.752	1.000
	99%	Inverse volatility weighting	1.71	2.345	2.962	0.161	0.326	0.977
		Equal weighting	1.71	2.365	3.002	0.161	0.326	0.967

Replace the equal-weighted scheme with a scheme weighted by inverse 60-month annualized volatility ($w_i \propto \frac{1}{\sigma_i}$, Normalization) , The VaR/ES of the two combinations and the backtest were recalculated using only GARCH(1,1)-t (as shown in Table 7).

After anti-volatility weighting, the average VaR/ES of both portfolios decreased, but the p-values of LR_uc, LR_cc, and ES Z2 tests did not change much compared with the equal-weighted scheme, and the model validity conclusion remained unchanged, indicating that the conclusions of this paper are robust to the portfolio weight design.

5. Conclusion

This paper uses two stock portfolios in the Chinese A-share market as samples to systematically compare the measurement effects of VaR and ES under three methods: historical simulation, parametric method (normal distribution), and GARCH (1,1)-t. The main conclusions are as follows::

First, the returns of A-share portfolios exhibit significant leptokurtic and heavy-tailed characteristics, rejecting the normal distribution hypothesis and providing empirical evidence for the use of GARCH-t and historical simulation methods. Second, at a 99% extreme tail confidence level, the VaR and ES values given by GARCH (1,1)-t are significantly higher than those of the normal parameter method, and the backtest shows the best performance; the historical simulation method is a robust nonparametric alternative. Third, although the normal parameter method passed the statistical test at a 95% confidence level, its breakthrough rate is systematically lower than the nominal level, indicating a tendency to underestimate risk. It is not recommended to use it as the sole basis for measuring the risk of A-share portfolios. Fourth, the estimates of ES are greater than VaR under all models and confidence levels, verifying ES's additional ability to capture extreme tail losses. Fifth, the conclusion remains unchanged after changing the confidence level and combined

weights, indicating good reliability. In summary, it is recommended that investors and risk management institutions in the A-share market introduce the ES indicator on the basis of VaR, and give priority to using GARCH (1,1)-t or historical simulation methods for risk measurement.

This study has the following limitations: First, the sample only includes 12 A-shares, resulting in limited industry coverage. Future studies could expand the sample to include more industries and market capitalization ranges.; Second, the GARCH model only uses (1,1) order and t distribution settings, does not involve asymmetric models such as EGARCH and GJR-GARCH, and does not introduce Monte Carlo simulation methods. In the future, it can be expanded to a combination of more industries and market capitalization ranges, introduce more effective weighted estimation methods to correct ES backtesting, and try to integrate prediction methods to improve model accuracy. Furthermore, extending the analytical framework of this study to industry rotation strategies and multi-asset portfolios, as well as exploring the feasibility of ES as a core indicator of risk management in A-shares from the perspective of Basel III regulation, are also directions worth exploring in depth.

References

- [1] Artzner, P., Delbaen, F., Eber, J. M., & Heath, D. (1999). Coherent measures of risk. *Mathematical Finance*, 9(3), 203–228.
- [2] Fissler, T., & Ziegel, J. F. (2016). Higher order elicibility and Osband's principle. *The Annals of Statistics*, 44(4), 1680–1707.
- [3] Bayer, S., & Dimitriadis, T. (2022). Regression-based expected shortfall backtesting. *Journal of Financial Econometrics*, 20(3), 437–471.
- [4] Barendse, S., Kole, E., & van Dijk, D. (2023). Backtesting value-at-risk and expected shortfall in the presence of estimation error. *Journal of Financial Econometrics*, 21(2), 528–568.
- [5] Feng, Y., Hua, X., & Gao, J. (2022). The numeric characteristics of Chinese A-share market index volatility, model simulation, and forecasting. *Social Sciences in China*, 43(2), 161–179.
- [6] Le, T. H. (2023). Forecasting VaR and ES in emerging markets: The role of time-varying higher moments. *Journal of Forecasting*, 42, 1–20.
- [7] Wang, Y., & Wang, Y. (2024). Forecasting VaRs via hybrid EVT with normal and non-normal filters: A comparative analysis from the Chinese stock market. *International Review of Economics & Finance*, 92, 92–110.
- [8] Tuysuz, S. (2023). Comparison of the accuracy of models in forecasting VAR and ES through time. *Journal of Economics, Finance and Accounting (JEFA)*, 10(3), 158–169.
- [9] Bollerslev, T. (1987). A conditionally heteroskedastic time series model for speculative prices and rates of return. *The Review of Economics and Statistics*, 69(3), 542–547.
- [10] Kupiec, P. H. (1995). Techniques for verifying the accuracy of risk measurement models. *The Journal of Derivatives*, 3(2), 73–84.
- [11] Christoffersen, P. F. (1998). Evaluating interval forecasts. *International Economic Review*, 39(4), 841–862.
- [12] Acerbi, C., & Székely, B. (2014). Back-testing expected shortfall. *Risk*, 27(11), 76–81.
- [13] Dlamini, S., & Shongwe, S. C. (2026). Comparative analysis of tail risk in emerging and developed equity markets: An extreme value theory perspective. *International Journal of Financial Studies*, 14(1), 11.
- [14] Chawla, A. (2025). Comparative backtesting of value-at-risk and expected shortfall models across the US, UK, and India (2005–2025). *International Journal of Innovative Science and Research Technology*, 10(8), 1649–1658.