

Model Comparison and Research Analysis of European Call Option Pricing Issues

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Abstract. European options, as a crucial tool in the financial derivatives market, possess unique design principles and operational mechanisms, holding an irreplaceable position in global financial markets. This paper will focus on European call options as the core research subject. Collecting six pricing models for European call options and explore their pricing principles. The aim is to understand the processes of these six models and compare the differences and similarities among the models. Thus, accumulate more necessary knowledge about pricing in the field of options. At the same time, the closing price of EUR/USD from 2025 to 2026 and the interest rates of EUR and USD in the past three months were collected from Dongfang Wealth Network for supplementation. The conclusion is that the pricing strategy of European call options is mainly based on the Black-Scholes-Merton Model model. The other five models are innovative based on it to avoid limitations. The pricing model organically combines time value and investment functional value, providing theoretical support for the foresight of the financial industry.

Keywords: European Option, European Call Option, Black-Scholes-Merton Model, Pricing Model

1. Introduction

European options typically appear in the international market in the form of stock index options and foreign exchange options. At present, all exchange-traded fund (ETF) options traded on domestic exchanges in China are European options. With the continuous deepening of economic globalization and the continuous expansion of the global foreign exchange market. The determinacy of the exercise mechanism of European options, the flexibility of pricing models, and their widespread application in specific fields have made European options an important derivative in China and even the global foreign exchange market. European options are typically defined as a type of option where the buyer only has the right to exercise on the expiration date. There are two types of European options: call and put. This article will focus on the pricing issue of European call options and analyze in depth how market participants calculate and quote based on theoretical foundations and existing models in actual trading.

The pricing of European options is mainly determined through the Black-Scholes-Merton Model (BSM). The BSM model applies to European stock options that do not pay dividends. Secondly, by reading relevant scholars' literature and based on their data and models, a framework for

determining the pricing of European call options was systematically constructed. Further innovated the calculation methods and data acquisition and summarization. Summarize the other five classic pricing models for European call options. They are respectively Monte Carlo Simulation, Scott's stochastic volatility model (Scott), Time-Fractional (2+1)-D European Call Option Model, Finite Difference Scheme, Garman-Kohlhagen Model.

The reason for studying the pricing of European call options is that they have significant significance and value in risk control, leverage operations, and financial investment. In the following section, this article will gradually elaborate on the relevant issues of pricing European call options from the perspectives of data analysis, model methods, and results and discussion. The discussion will mainly focus on six classic models. Recording present data, these models will be compared and analyzed to explore their practical significance in solving the pricing problem of European call options. This will lead to the core objective of this study: firstly, to introduce these models and understand their application principles, compare and analyze these models to understand their characteristics and their application environments. Secondly, it is necessary to master critical thinking and anticipate the future development trends of European options. Finally, systematically construct a theoretical system for option pricing and accumulate methods for option pricing, laying a solid foundation for future learning.

2. Methods

2.1. The historical process and research status of European options

The historical process of options can be divided into its embryonic stage, development stage, and modern standardization stage. Options can be traced back to about 500 years ago, when philosopher Thales obtained the right to use a press by predicting olive harvests. This is the first time that options have appeared in a practical form.

Later, during the Middle Ages, European traders used similar option contracts to lock in future prices for goods. From the 17th century to the 19th century, the famous tulip mania and the early OTC trading in the United States opened up the market for options. Since the 20th century until now, in 1973, the Chicago Board Options Exchange (COBE) was established and launched the first standardized stock option contract, marking the birth of the modern options market and an important milestone in the history of options development. In the same year, two scholars, Fisher Black and Myron Scholes, calculated the price of European options using the Black-Scholes-Merton model. Since then, the model has been specifically used for pricing European options. Nowadays, the pricing models for European options are becoming increasingly widespread. Different scholars have different theories and research methods. This has brought a new era to European options.

2.2. Research and comparison of model methods

This article mainly studies six classic models for pricing European options. Understanding the principles in the model and comparing them. The BSM is based on the no-arbitrage assumption and risk-neutral pricing. By constructing a hedging portfolio, it is assumed that asset prices follow geometric Brownian motion. Using Ito's lemma, the BSM model is derived.

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2) \quad (1)$$

Among them:

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, d_2 = d_1 - \sigma\sqrt{T} \quad (2)$$

The core idea of Scott's Stochastic Volatility Model regards the volatility of the underlying asset price as a stochastic process.

$$C = S \cdot N(d_1) - K \cdot e^{-rT} \cdot N(d_2) \quad (3)$$

$$d_1 = \frac{\ln(\frac{S}{K}) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \quad (4)$$

$$d_2 = d_1 - \sigma\sqrt{T} \quad (5)$$

The Scott model and BSM model are very similar in their deduction process, but the difference between the two lies in their different assumptions about volatility. The BSM model assumes that the volatility of the underlying asset is a constant. The Scott model allows volatility to vary over time. Scholars who have applied the Scott model to study pricing issues have stated that this model is very practical for option valuation under stochastic volatility. Sánchez et al. constructed a finite difference scheme to ensure that the numerical analysis in the finite difference scheme satisfies consistency conditions such as monotonicity and positive definiteness. Secondly, multiple numerical experiments were conducted on the pricing of European call options, and the results were compared with those of Monte Carlo simulations. It was found that the two were consistent, thus verifying their feasibility [1].

The Time Practical (2+1)-D European Call Option Model is used to implement the optimal algebraic solution and new solution methods for call option models. By deriving infinitesimal transformations and classifying the optimal system, the model is further simplified into integer order ordinary differential equations under geometric Brownian motion.

The geometric Brownian motion (GBM) standard form of the stochastic differential equation for geometric Brownian motion:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad (6)$$

First order stochastic differential equations contain deterministic terms $\mu S_t dt$ and random items $\sigma S_t dW_t$

Integer order ordinary differential equation:

$$\frac{dS}{dt} = \mu S \quad (7)$$

First order linear ordinary differential equation:

$$S(t) = S_0 e^{\mu t} \quad (8)$$

Meanwhile, Bira et al. also obtained various ordinary differential equations and constructed power series solutions, proving their convergence. Subsequently, numerical simulations were conducted on these ordinary differential equations and conservation vectors were found [2]. European call options are observed through interval volatility and interest rates. A new method has been introduced to solve the option pricing problem with uncertain parameters, which models the

interval optimization problem as an interval optimization problem constrained by partial differential inequalities [3].

The Finite Difference Scheme is a method of converting continuous partial equations (PDEs) or ordinary differential equations (ODEs) into algebraic systems of equations by approximating the derivatives of differential equations with the difference quotient of functions at discrete grid points. In order to extend a more accurate pricing method based on the finite difference scheme. Scholars further solved a convergent fourth-order finite difference scheme for the BSM equation. Due to the discontinuous differential payment function of many options in the financial market, and the possibility of large errors and low convergence speed caused by the first and second derivatives in the BSM equation, a convergent fourth-order finite difference scheme is proposed to solve this problem.

$$\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0 \quad (9)$$

Finally, the effectiveness and ability of the proposed algorithm were verified through various calculation results of European call options, which is a research method that can be continuously studied in depth [4]. Because the extension of ODE and PDE is stochastic partial differential equations (SPDE). And the SPDE is also one of the methods to solve pricing problems, so Wijayanti et al. conducted research on SPDE and used the finite element method numerical solution. Simultaneously, a white noise segmented constant stochastic process was added to estimate the price of European call options [5]. Some scholars have also used the stochastic volatility Bachelier model to observe the implied volatility of call options and Asian options. Using Malliavin calculus to calculate the implied volatility level when the maturity date approaches zero. They applied the general results to the stochastic Alpha Beta smoothing algorithm rho, fractional Bergomi, and local volatility models, and conducted some numerical simulations. Therefore, they confirmed the accuracy of the skewness asymptotic formula [6].

Monte Carlo Simulation is suitable for European option pricing problems. Using random sampling to simulate the future price path of the underlying asset, calculate the option yield at maturity, and discount the average yield to obtain the option price. In general, using frequency estimation to estimate expectations and using the law of large numbers to approximate real prices.

$$C_0 = e^{-rT} \cdot \frac{1}{N} \sum_{i=1}^N \max \left(S_T^{(i)} - K, 0 \right) \quad (10)$$

$$S_T = S_0 \cdot \exp \left(\left(r - \frac{1}{2} \sigma^2 \right) T + \sigma \sqrt{T} \cdot z \right) \quad (11)$$

Garman Kohlhagen Model (GK) is the foreign exchange version of BSM, which treats foreign currency as a stock with continuous dividends. The difference between it and BSM is that BSM has only one interest rate while GK has two interest rates, possessing the dual risk-free nature of foreign exchange.

$$C = S \cdot e^{-r_f T} N(d_1) - K \cdot e^{-r_d T} N(d_2) \quad (12)$$

$$d_1 = \frac{\ln \left(\frac{S}{K} \right) + \left(r_d - r_f + \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}} \quad (13)$$

$$d_2 = d_1 - \sigma\sqrt{T} \quad (14)$$

Some scholars have proposed a new numerical strategy, the color polynomial interpolation method, based on the algebraic graph theory background of European call options in the Black Scholes Merton Model. The birth of this method transforms the original model into an algebraic equation system with standard configuration points. Finally, calculations and charts demonstrate that the proposed numerical format is more reliable than modern numerical methods [7]. The pricing model of European call options is not limited to the six models involved in this study. By searching for relevant information, the binary tree model assumes that the stock price rises and falls, and under a risk neutral probability, the current option price is obtained by reverse discounting the European call option yield from the expiration date [8]. Viktor et al. followed the trajectory of GBM and found the moments returned by periodic logs, log moments, and expected general expressions. The corresponding function is obtained from the subordinate relationship. Combining different types of GBMs to generate generalized geometric Brownian motion. Thus, the empirical rules for pricing European call options can be summarized [9].

Table 1 presents five aspects: core ideas, scope of application, advantages, disadvantages, and characteristics. A summary of the six basic models studied in this article. Using a comparative table and summarizing their commonalities to gain a more intuitive understanding.

Table 1. Comparison table of six models

Model	Core concept	Range of application	Advantage	Disadvantage	Characteristic	Common ground
Black-Scholes-Merton Model	Constructing a risk-free investment portfolio consisting of the underlying asset and options, with the return equal to the risk-free interest rate. Derive the option price.	Pricing of European options in a market without dividends, without transaction costs, and with constant volatility.	Supporting risk management and strategy formulation, and promote market transparency.	Too idealistic	Closed-form solution formula, No arbitrage, no friction	All are based on no-arbitrage pricing as the core and the BSM model as the theoretical foundation. By depicting the dynamic changes of asset prices, the final goal is to achieve option pricing and risk analysis.
Monte Carlo Simulation	By randomly simulating the price path, the average return is discounted to obtain the current value.	Handling financial derivatives with path dependence, multiple factors, complex structures or no analytical solutions.	It can handle non-linear, significant fluctuations and "fat-tail" issues, and can respond quickly to market changes by setting a reduction factor.	The convergence speed is slow, there is an initial error, and the results are random.	The theoretical foundation is solid and highly flexible.	
Scott's Stochastic Volatility Model	Treating the volatility of the underlying asset as a random process, and assume that the volatility is different.	Mainly targeting European options and their derivative structures (such as compound, vulnerable options, etc.)	Allowing interest rates and volatility to fluctuate randomly, explain market anomalies such as "volatility smile".	The calculation is complex and parameter estimation is difficult.	Closely align with real market behavior	

Table 1. (continued)

Time-Fractional (2+1)-D European Call Option Model	In partial differential equations in 2D space and 1D time, fractional-order time derivatives are used instead of integer-order derivatives to describe non-local, memory, and anomalous diffusion.	Dynamic systems with memory effects, nonlocality and heterogeneous diffusion characteristics.	Describing market dynamic.	The calibration process is very challenging.	Introducing time fractional derivatives and possess a two-dimensional spatial structure.
Finite Difference Scheme	Using the difference quotient to approximate the derivative, the continuous differential equation is discretized into a system of algebraic equations for solution.	The initial-boundary value problems of PDEs and ODEs.	Rule domain, high precision requirements, fast computing efficiency	Poor stability, scattered values, and inflexibility in handling complex geometric boundaries.	Discretization is intuitive and has a wide range of applications.
Garman-Kohlhagen Model	Treating foreign exchange as a continuous income asset and modify the BS model to price foreign exchange options	Foreign currency options	It also incorporates the domestic risk-free interest rate and the foreign risk-free interest rate, providing a more accurate reflection of the discount structure of foreign exchange options.	During periods of high volatility or market instability, its pricing may deviate from the actual value.	Interest rate differences between the two currencies

This article heavily relies on scholarly literature for induction. So there may be time differences in data collection, leading to lag. To solve this problem, this article collected the closing price of EUR/USD from January 1, 2025 to January 1, 2026 (Figure 1), the interest rate of US treasury bond bonds from January 1, 2026 to March 1, 2026 (Figure 2) and the short-term risk-free interest rate of euro (Figure 3) in the Oriental Wealth Database, and made them into illustrations for display below. A clear overview of the current development trends of European options provides timeliness for research.



Figure 1. One year closing of European option (picture credit: original)

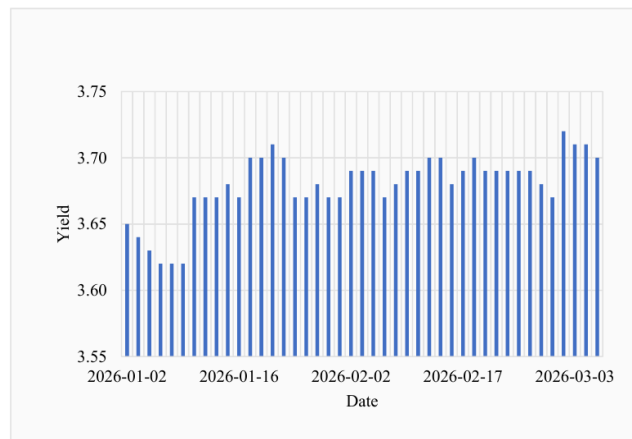


Figure 2. Three-month interest rate of Euro short-term (picture credit: original)

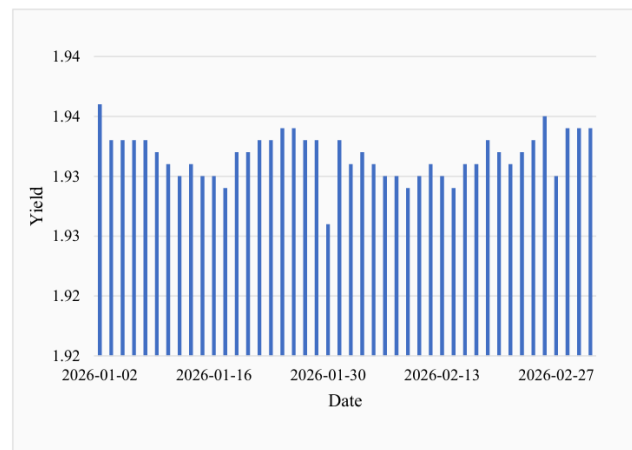


Figure 3. Three-month interest rate of Euro short-term (picture credit: original)

3. Discussion

The above content collects relevant literature on the pricing of European call options. By studying and analyzing the models used by scholars, it can be found that the core models are basically extended based on the BSM. More detailed and accurate models have been constructed around factors such as annualized historical volatility, implied volatility, closing price at maturity, and interest rates to ensure consistency and verifiability of the results. For nonlinear and dynamic markets, Xiao et al. combined the self-attention mechanism of Transformers with the temporal modeling of LSTMs. Scholars have captured global dependencies and local temporal patterns. Judging option pricing without restrictions is still challenging, with many strikes and underlying factors, requiring continuous enhancement of the impact of tools to meet pricing standards [10]. The reason for studying the pricing of European call options is that it has significant significance and value in risk control, leverage operations, and financial investment.

In the Chinese financial market, it has become the mainstream form and is widely used in the foreign exchange market, helping companies manage exchange rate risks and providing a favorable financial environment for investors. This article proposes several suggestions for the following entities based on the above content: Firstly, for investors, due to the current high market volatility and unstable market conditions, investing in long-term options carries greater risks. Therefore, it is

recommended to choose short-term or medium-term (2-3 months) European call options to avoid eroding profits due to time loss. Secondly, for relevant departments, regulatory agencies strictly control risks, strengthen regulatory systems, establish volatility linkage mechanisms, and dynamically adjust trading mechanisms to cope with high volatility risks. And for future technology, technicians are constantly expanding their technical fields, using artificial intelligence to optimize pricing models and improve pricing accuracy. Promoting intelligent chain services, automatically settle European options, and improve efficiency. Finally, for market entities, by establishing a more standardized and international financial market, it can promote the development of European-style options to serve the real economy and steadily expand opening up to the outside world.

4. Conclusion

The above is the core content of this study. Through in-depth analysis of the pricing models proposed by the aforementioned scholars, this article summarizes the following patterns: Most models are constructed through routine operations such as collecting past market data for descriptive statistics, stationarity testing, and correlation analysis. Most models are constructed by collecting past market data for routine operations such as descriptive statistics, stationarity tests, and correlation analysis. This will determine whether the data are studying is stable and avoid multicollinearity issues. Then, from these six basic models, new models can be designed to conduct more innovative research on the pricing of European call options. The main influencing factors on the pricing of European call options include the underlying asset, strike price, expiration time, risk-free rate, and volatility. The increase of these factors will affect the value of expired options. In analyzing models, it often encounter issues such as differences in option pricing between different models, the impact of volatility on option prices, the degree of influence of interest rates, exchange rates, and time on option value, and so on. These issues have been elaborated in detail in the main body of this article, which greatly helps to study the pricing problem of European call options.

Theoretical value is composed of the sum of intrinsic value and time value. The true value of options lies in whether, at a specific point in time, they can make correct and self-interested actions based on experience, to achieve maximum value. From another perspective, option trading can be mapped to the realm of philosophy. Small fluctuations in the underlying asset price do not generate value, while crossing key points can lead to several changes in option value, reflecting the accumulation process from quantity change to quality change.

In summary, this article only analyzes the pricing model behind European call options, not only to fully understand European options, but also to have a preliminary understanding of the entire foreign exchange field. The buying and selling behavior of European call options is not only a financial trading strategy, but also reflects the wisdom of investors. The buyer demonstrates acceptance of future uncertainty and pursues unlimited returns under limited risk. The seller, on the other hand, demonstrated rational restraint and a sense of social responsibility, and to some extent made compromises. They together constitute a closely interrelated and complementary relationship between risk and return, opportunity and inevitability, freedom and constraint, which is well-known to people. From the perspective of the connotation and pricing model of European call options, it reflects human cognition and value choices towards the world. Therefore, the discussion of these six classic models in this article not only enriches the theoretical system of derivative pricing, helps investors formulate trading strategies, and enables companies to invest in portfolios, but also deepens understanding of the operating laws of financial markets, the balance between risk and return, and the essence of time value. This has important theoretical value and practical significance for improving the efficiency of financial markets and perfecting the pricing system of derivatives.

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