

Capital Circulation, Digital Platform Expansion and the Solow Paradox—An Analysis of Total Factor Productivity Based on Marxist Political Economy

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Abstract. Starting from Marxist political economy, this paper reconstructs the Solow growth model with the theory of capital circulation, and redefines total factor productivity as capital appreciation efficiency jointly determined by production efficiency, value realization efficiency and the technical composition of capital. As a new form of commercial capital, digital platforms exert two opposite effects on industrial capital: "accelerating turnover" and "value extraction". In the early stage of development, platforms can shorten circulation time and improve capital turnover efficiency, thus promoting productivity growth. However, with the expansion of platform scale, the rise of market concentration and the enhancement of bargaining power, the effect of value extraction gradually surpasses that of turnover acceleration. Unproductive circulation costs continuously squeeze the profit margin and reproduction capacity of production sectors, turning total factor productivity from growth to decline, thus giving rise to the Solow paradox.

Keywords: Solow paradox, total factor productivity, capital circulation, digital economy

1. Introduction

Economist Robert Solow found that productivity did not improve significantly as expected amid the rapid penetration of information technology. "You can see the computer age everywhere but in the productivity statistics." [1] This phenomenon was later named the "Solow paradox" or "productivity paradox". It was first confirmed in empirical studies in the United States: despite a substantial increase in computing power and IT investment, productivity growth at the overall economic level slowed down. Scholars have put forward various explanations for the causes of the Solow paradox, which this paper summarizes into four main viewpoints. First, the statistical omission hypothesis. This hypothesis holds that the Solow paradox stems from the failure of traditional economic statistical methods to capture part of the added value, including consumer surplus [2], efficiency improvement in traditional industries [3], and the enhancement of product quality, diversity and innovation [4]. Second, the heterogeneity hypothesis. It argues that although the digital economy continues to expand at the macro level, there are significant differences in the absorption and transformation capacity of digital technologies across industries [5], upstream and downstream industrial chains [6], different types of enterprises [7] and regions [8]. When a large number of

agents with weak absorption capacity fail to effectively convert technology inputs into efficiency outputs, their negative effects drag down overall productivity, leading to the emergence of the Solow paradox at the macro level. Third, the time-lag effect hypothesis. This hypothesis suggests that technology is not useless for improving productivity but involves a time lag. The causes of the time lag mainly include the learning and adaptation of micro agents to new technologies [9], the penetration of technologies within and between industries [10], and the diffusion of technologies across regions [11]. Fourth, the factor misallocation hypothesis. It states that the misallocation between R&D investment in digital technologies and complementary R&D human capital is the core reason hindering productivity growth [12], thereby suppressing total factor productivity [13]. Among the above four explanatory approaches, the statistical omission hypothesis incorporates all utility brought by digital technologies to consumers into total social output and naturalizes the value of commodities, thus ignoring that the value of commodities is realized only under capitalist production relations, which is a traditional fallacy of mainstream economics. The remaining three viewpoints do explain and demonstrate several causes of the Solow paradox from different levels and perspectives, and are entirely reasonable.

Returning to the investigation of the Solow paradox itself, Marxist political economy holds that the essence of the Solow paradox is the contradiction between two types of productivity: the "labor-based efficiency view" and the "capital-based efficiency view". Total factor productivity originates from the Solow growth model, but is generally depicted by the Cobb-Douglas production function under the assumption of constant returns to scale. On the whole, total factor productivity is essentially the appreciation efficiency of capital and the production capacity of surplus value [14]. However, it has long been regarded as a measure of technological progress, ignoring that total factor productivity is an abstract description of "technology" and cannot be confused with specific technologies. Just like technologies including ICT and digital technologies are specific technologies, which are usually described by scholars as general-purpose technologies. They are basic technologies with wide applicability, continuous improvement and innovative complementarity, which can trigger structural changes in multiple industries and serve as the sole driving force for long-term technological progress and economic growth [15]. This theory confuses the relationship between abstract technology at the theoretical level and specific technology, assuming that specific technological progress can be directly transformed into the improvement of production efficiency and capital appreciation efficiency. Therefore, most digital platforms are essentially modern forms of commercial capital functioning in the circulation sphere, creating no new value and being unproductive.

2. Reconstruction of the Solow growth model from the perspective of capital circulation: an analysis of total factor productivity

The standard form of the Cobb-Douglas production function, a special case of the widely adopted Solow growth model¹ is expressed as:

$$Y(t) = A(t)K(t)^\alpha L(t)^{1-\alpha}$$

The Solow growth model must satisfy two sets of assumptions. First, the assumption of constant returns to scale, meaning that output changes in the same proportion as the input factors of

¹ where $Y(t) = A(t)K(t)^\alpha L(t)^{1-\alpha}$ is equivalent to $Y(t) = K(t)^\alpha (A(t)L(t))^{1-\alpha}$

production. Second, the growth rate of a variable equals the rate of change of its natural logarithm, also referred to as the "balanced growth path" assumption. Therefore, it is necessary to investigate how fixed capital stock (K) and labor (L) are converted into output (Y), and to analyze the input-output ratio, namely total factor productivity (A). The theory of capital circulation put forward by Marx in Volume II of Capital argues that capital accumulation, and thereby capitalist economic growth, is realized through the constant movement of capital as it repeatedly shifts between different external forms, which is described by the general formula of capital as follows:

$$M-C(L+Mp)-P-C'-M'$$

Foley first formalized the theory of capital circulation in Volume II of Capital with a mathematical model using the container approach in his work Understanding Capital. Through integration, the quantitative relationship whereby any value flow grows along with capital circulation under expanded reproduction can be derived, taking the sales revenue flow as an example:

$$S(t) = (1 + pq)S(t - T_P - T_R - T_F)$$

To better depict the growth process of value flows and value containers over time in a continuous scenario, Foley assumes that all variables grow at a rate g under expanded reproduction in an exponential form. Thus, for any value flow or container variable:

$$\forall E \in \{Z, P, S, N, X, M\}, E(t+T) = E(t)e^{gT}$$

After rearrangement, the steady-state growth rate of each variable can be solved as $g = \frac{\ln(1+pq)}{T_P + T_R + T_F}$

Another modeling idea for the capital circulation model is the factor method. Instead of constructing abstract value containers based on the forms of capital movement, the factor method builds models directly on the basis of accounting accounts in corporate balance sheets in accordance with the laws of capital movement, and the unit of value is no longer abstract value but monetary value. For the modeling task to be completed in this paper, adopting the framework of the container method can simplify many factors beyond capital circulation.

The formal difference between fixed capital and circulating capital is that fixed capital is advanced in full at one time during its performance of productive functions, yet "enters successively into the realization of the value component of the commodity, and is withdrawn successively and gradually from circulation". We may assume fixed capital with the following characteristics: 1. All elements constituting fixed capital are continuous, homogeneous and identical; 2. Elements of fixed capital, whether subject to physical or moral depreciation, can be immediately compensated by the monetary reflux of products, meaning the reproduction of fixed capital is continuous; 3. The above two assumptions result in an infinite lifespan of fixed capital. We first conduct modeling analysis under discrete conditions. Starting from the initial advance of capital, assume the initial fixed capital is K_0 , the organic composition of capital is κ , the depreciation rate of fixed capital is δ , and the

proportion of constant circulating capital is σ . A comparison is made between simple reproduction and expanded reproduction:

Table 1. Capital circulation under simple reproduction in discrete conditions

	t_0	$+T_P$	$+T_R$	$+T_F$	t_1	$+T_P$	$+T_R$	$+T_F$
K	K_0	$(1-\delta)K_0$	$(1-\delta)K_0$	K_0	K_0	$(1-\delta)K_0$	$(1-\delta)K_0$	K_0
Z	$\frac{(\kappa+\sigma)}{K_0}$			$\frac{(\kappa+\sigma+\delta)}{K_0}$	$\frac{(\kappa+\sigma)}{K_0}$			$\frac{(\kappa+\sigma+\delta)}{K_0}$
P		$\frac{(\kappa+\sigma+\delta)}{K_0}$				$\frac{(\kappa+\sigma+\delta)}{K_0}$		
S			$(1+q)(\kappa+\sigma+\delta)K_0$				$(1+q)(\kappa+\sigma+\delta)K_0$	
$\frac{\Delta}{K}$				0				0

Table 2. Capital circulation under expanded reproduction in discrete conditions

	t_0	$+T_P$	$+T_R$	$+T_F$	t_1	$+T_P$	$+T_R$	$+T_F$
K	K_0	$(1-\delta)K_0$	$(1-\delta)K_0$	$K_0+\Delta K$	K_1	$(1-\delta)K_1$	$(1-\delta)K_1$	$K_1+\Delta K$
Z	$\frac{(\kappa+\sigma)}{K_0}$			$\frac{(1+pq)(\kappa+\sigma+\delta)}{K_0}$	$\frac{(\kappa+\sigma)K_1}{K_0+\Delta K}$			$(\kappa+\sigma+\delta)K_1$
P		$\frac{(\kappa+\sigma+\delta)}{K_0}$				$\frac{(\kappa+\sigma+\delta)}{K_0}$		
S			$(1+q)(\kappa+\sigma+\delta)K_0$				$(1+q)(\kappa+\sigma+\delta)K_0$	

Table 2. (continued)

$\frac{\Delta}{K}$	$\frac{pq}{1+\kappa+\sigma} (\kappa+\sigma+\delta)K_0$	$\frac{pq}{1+\kappa+\sigma} (\kappa+\sigma+\delta)K_1$
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To facilitate modeling and ensure uniform velocity of value flows under the container method, all flows are here treated as proportions of the fixed capital container (or factor), thus yielding the system of equations:

$$Z(t) = \frac{pq}{1+q} S(t-T_F)$$

$$P(t) = (\kappa+\sigma+\delta)K(t-T_P)$$

$$S(t) = (1+q)(\kappa+\sigma+\delta)K(t-T_P-T_R)$$

$$Z(t) = (\kappa+\sigma)K(t) + \Delta K(t)$$

$$\Delta K(t) = \frac{pq}{1+\kappa+\sigma} (\kappa+\sigma+\delta)K(t-T_F)$$

Rearrangement gives the growth relationship of fixed capital along with the capital circulation cycle:

$$K(t) = 1 + \frac{pq}{1+\kappa+\sigma} (\kappa+\sigma+\delta)K(t-T_P-T_R-T_F)$$

Since fixed capital grows exponentially at a constant rate, assume $K(t-T) = K(t)e^{-gT}$

Solving yields the growth rate of fixed capital: $g = \frac{\ln(1+pq \frac{\kappa+\sigma+\delta}{1+\kappa+\sigma})}{T_P+T_R+T_F}$. The remaining container renewal conditions satisfy:

$$\frac{dN(t)}{dt} = Z(t) - P(t)$$

$$\frac{dX(t)}{dt} = P(t) - \frac{1}{1+q} S(t)$$

$$\frac{dM(t)}{dt} = \frac{1+pq}{1+q} S(t) - Z(t)$$

According to the Solow growth model, the growth rates of output, capital and labor satisfy:

$$g_A = g_Y - \alpha g_K - (1-\alpha)g_L$$

By abstracting the individual capital circulation model into the social total capital circulation model, total output represents the total value embodied in products, including newly created value, intermediate goods and the value transferred from fixed capital. Thus, in individual capital circulation, the sales revenue flow forms part of total output. Since total output is the cumulative flow over a period, and sales revenue is essentially the time derivative of total output, total output can be expressed as the integral of the sales revenue flow:

$$Y(t) = \int_t^{t+\tau} S(t) dt$$

As both flows and stocks in the model follow a balanced growth path,

$$S(t) = S(0)e^{gt}$$

Total output at time t over the preceding interval τ is:

$$Y(t) = \int_t^{t+\tau} S(0)e^{gt} dt = \frac{1}{g} S(t)(e^{g\tau} - 1)$$

Further considering total factor productivity, according to the Solow growth model, total factor productivity is defined as:

$$A(t) = \frac{Y(t)}{K^\alpha(t)L^{1-\alpha}(t)}$$

This expression for total factor productivity is derived from the production function $Y(t) = F(K(t), L(t))$, which carries an implicit assumption that production is instantaneous: current factor inputs can be immediately converted into current output. In practice, however, factor inputs require a production time lag to be transformed into output, meaning current factor inputs are meaningful in the production function only when matched with output realized after production is completed. To resolve this problem, assume the total time of capital circulation is $T = T_P + T_R + T_F$. Output $Y(t)$ in period t should be determined by factor stocks $K(t - T), L(t - T)$ from T periods earlier. The revised production function is thus written as:

$$Y(t) = F(K(t-T), L(t-T))$$

In the individual capital circulation model, fixed capital stock is $K(t)$ and labor stock is $\kappa K(t)$. For total output, if individual capital is regarded as the total social capital, total output represents the total value embodied in products, including newly created value, intermediate goods and value transferred from fixed capital. Thus, in individual capital circulation, the sales revenue flow represents total output. However, according to the above derivation, the sales revenue flow is $S(t) = (1 + q)(\kappa + \sigma + \delta)K(t - T_P - T_R)$. Part of the value in the sales revenue flow comes from the one-time transfer of intermediate goods value. The circulation time of intermediate goods is the purchasing time for buyers and the selling time for sellers. For total social capital output, the time horizon should include the financial time lag T_F . Final total output is therefore:

$$Y(t) = \frac{1}{g} (e^{g\tau} - 1)(1 + q)(\kappa + \sigma + \delta)K(t - T)$$

Correspondingly, total factor productivity is:

$$A = \frac{(e^{g\tau} - 1)(1 + q)(\kappa + \sigma + \delta)}{g\kappa^{1-\alpha}}$$

From the derivation, we can draw the following conclusion: under steady-state growth, consistent with the Solow growth model, the growth rate of total factor productivity, as the technological residual, is constant and exhibits no long-term growth trend. Technology and capital are inseparable, and their external manifestations are parameters related to capital accumulation (p, q), value input (κ, σ, δ) and capital turnover time ($T = T_P + T_R + T_F$). Even if the modern concept of technology has expanded from a mere mechanical system to systematic knowledge for manufacturing products, applying processes or providing services.

3. How digital technology acts on capital circulation: theoretical deduction and the occurrence conditions of the Solow paradox

Based on the relationship between digital technology and production in the digital economy era, this paper focuses on the impact of digital platforms—the new form of commercial capital—and the

substitution of digital technology for productive labor on total factor productivity, and further deduces the occurrence conditions of the Solow paradox.

3.1. Digital platforms

Digital platforms are general digital infrastructures that can incidentally collect, process and transmit information generated in economic activities such as production, distribution, exchange and consumption during operation. First, digital platforms are typical two-sided markets. An increase in the number of users on one side will raise the willingness of users on the other side to participate, forming cross-network externalities. For survival and expansion, platforms must continuously "bring both sides of the market on board", a mechanism that intensifies competition and fosters monopoly. Second, platforms feature significant network externalities. Existing scale translates into competitive advantages, forming a positive feedback loop of "scale expansion – transaction concentration – further expansion". Third, platforms lie at the intersection of production, circulation and consumption information, enabling continuous collection of massive data such as transaction records, user preferences and supply chain feedback. The continuous accumulation of data further strengthens the above positive feedback mechanism, resulting in increasing returns and market lock-in. As this centralization trend continues, dual consequences gradually emerge: the positive effect of accelerating commodity circulation and shortening capital turnover time, and the negative effect of extracting industrial profits through monopoly power. Based on the capital circulation model established in the previous chapter, the expressions of profit volume and profit rate are deduced. The sales revenue flow is:

$$S(t) = (1+q)(\kappa + \sigma + \delta)K(t - T_P - T_R)$$

The realized profit flow from sales is:

$$\Pi(t) = S(t) - \frac{1}{1+q} S(t)$$

Substituting $S(t)$ yields:

$$\Pi(t) = q(\kappa + \sigma + \delta)K(t - T_P - T_R)$$

Taking the capitalization rate p into account, the actually realized profit is:

$$\Pi_r(t) = pq(\kappa + \sigma + \delta)K(t - T_P - T_R - T_F)$$

The total advanced capital at time t is expressed as a proportion of fixed capital:

$$TK(t) = (1 + \kappa + \sigma)K(t - T_P - T_R - T_F)$$

The profit rate of a single circulation is expressed as:

$$r_c = \frac{\Pi_r(t)}{TK(t)} = \frac{pq(\kappa + \sigma + \delta)}{1 + \kappa + \sigma}$$

The profit rate in continuous time is expressed as:

$$r = \frac{\frac{pq(\kappa + \sigma + \delta)}{1 + \kappa + \sigma}}{T_P + T_R + T_F}$$

To simplify the analysis, assume that the owners of manufacturing firms do not engage in unproductive consumption and that all sales profits are invested in expanded reproduction, so that $p = 1$. The profit rate thus becomes:

$$r = \frac{\frac{q(\kappa + \sigma + \delta)}{1 + \kappa + \sigma}}{T_P + T_R + T_F}$$

According to the task model, assume that the sales of goods by manufacturing firms can be divided into several tasks s belonging to the task set A , which can be split into the task set A_h performed by labor and the task set A_k performed by digital platforms. Assume that the time required for each task can be expressed as follows:

$$T_R(s) = t(s)P(s) + l(s)1 - P(s)$$

where P is a binary variable: $P = 1$ means the task is completed by a digital platform, and $P = 0$ means it is completed purely by manual labor. $t(s)$, $l(s)$ are the time spent completing the task by digital platform and pure manual labor respectively. Theoretically, digital platforms are usually more efficient than pure manual labor in completing sales tasks, so it is assumed that $l(s) \gg t(s)$. In addition, assume that platform technology always advances in tasks where pure manual labor is extremely inefficient, so that $l(s) - t(s)$ is relatively large. Thus we can define $Z = \frac{1}{l(s) - t(s)} \in (0, +\infty)$ as the current frontier index of the platform. When $\frac{1}{l(s) - t(s)} < Z$, manufacturing firms adopt digital platform services; when $\frac{1}{l(s) - t(s)} \geq Z$, they continue to use pure manual labor. Since firms aim to minimize sales time, A_k and A_h are mutually exclusive sets, expressed respectively as:

$$A_k = \left\{ s : \frac{1}{l(s) - t(s)} < Z \right\}$$

$$A_h = \left\{ s : \frac{1}{l(s) - t(s)} \geq Z \right\}$$

Total circulation time is defined as:

$$T_R = \int_{A_k} T_R(s) ds + \int_{A_h} T_R(s) ds$$

As digital platform technology innovates and progresses continuously, it is reflected in an increase in the value of Z , which in turn causes the measure $M \equiv A_k$ to expand. Such innovation may lead manufacturing firms to platformize sales tasks that were previously unsuitable for platformization in terms of sales expenses, where the efficiency gap between platform technology and manual labor is small. Under Bertrand competition, all manufacturing firms must follow the technological frontier Z and purchase the corresponding volume M of platform services at equilibrium, abandoning manual task completion. When Z improves slightly, the measure of A_k expands to A_k' and that of A_h shrinks to A_h . The differential form of the change in firms' sales time is written as:

$$dT_R = \int_{\Delta} t(s) - l(s) ds, \quad \Delta \equiv A_k' - A_k = A_h - A_h'$$

To simplify the analysis, assume $t(s) - l(s) = k$ is a constant, so that $dT_R = k|\Delta|$, where $|\Delta|$ is a measure of the change in the task set. Here we analyze how technological progress, the expansion and rising concentration of the digital platform sector, and the increase in M affect the circulation time of manufacturing firms. From the above analysis, T_R can be regarded as a function of M , and there exists a critical value M^* that minimizes T_R . The relationship between M and T_R is as follows:

$$\frac{dT_R(M)}{dM} = \begin{cases} < 0, M < M^* \\ = 0, M = M^* \\ > 0, M > M^* \end{cases}$$

In the presence of the digital platform sector, platforms and manufacturing firms split profits based on the profit rate that would prevail without platforms. Due to the information advantage of digital platforms over manufacturing firms, their bargaining power is asymmetric. Using the

conclusion of the bargaining game model, the game can be regarded as maximizing the following Nash product:

$$p = \operatorname{argmax} \left[(1-p) \frac{\frac{q(\kappa+\sigma+\delta)}{1+\kappa+\sigma}}{T_P + T_R(M) + T_F} \right]^\theta \left[p \frac{\frac{q(\kappa+\sigma+\delta)}{1+\kappa+\sigma}}{T_P + T_R(M) + T_F} \right]^{1-\theta}$$

where p^2 is the profit-sharing ratio, and $\theta(M)$ is a monotonically increasing function mapping the measure M of A_k to $\theta \in (0,1)$, representing the relative bargaining power of digital platforms. Solving the above maximization problem yields:

$$p = 1 - \theta(M)$$

Accordingly, the profit rate of the manufacturing sector is:

$$r = \frac{q(1-\theta(M))}{T_P + T_R(M) + T_F} \frac{(\kappa+\sigma+\delta)}{1+\kappa+\sigma}$$

Assume that the owners of manufacturing firms do not engage in personal consumption and sales profits are only shared with the digital platform sector. Total factor productivity can then be written as:

$$A = \frac{(e^{g\tau}-1)(1+q)(\kappa+\sigma+\delta)}{g\kappa^{1-\alpha}}$$

$$g = \frac{\ln(1+1-\theta(M)) \frac{q(\kappa+\sigma+\delta)}{1+\kappa+\sigma}}{T_P + T_R(M) + T_F}$$

Next, take the derivative of A with respect to M . For convenience, first differentiate A with respect to g :

$$\frac{\partial A}{\partial g} = \frac{(1+q)(\kappa+\sigma+\delta)}{\kappa^{1-\alpha}} * \frac{e^{g\tau}(g\tau-1)+1}{g^2} > 0$$

By the chain rule, $\frac{\partial A}{\partial M} = \frac{\partial A}{\partial g} \frac{\partial g}{\partial M}$, so the sign of $\frac{dA}{dM}$ depends on $\frac{dg}{dM}$. We now differentiate g with respect to M . To simplify the expression, let the following letters denote constants and functions:

$$\begin{cases} B = \frac{q(\kappa+\sigma+\delta)}{1+\kappa+\sigma} \\ N(M) = \ln(1 + [1 - \theta(M)]B) \\ T(M) = T_P + T_R(M) + T_F \end{cases}$$

The derivative of g with respect to M is:

² note that p here has the same essential meaning as the profit capitalization ratio p above

$$\frac{\partial g}{\partial M} = g \left\{ -\frac{B\theta'(M)}{(1+[1-\theta(M)]B)N(M)} - \frac{T_R'(M)}{T(M)} \right\}$$

Under the assumption that $t(s) - l(s)$ is constant, it can be rewritten as:

$$\frac{\partial g}{\partial M} = g \left\{ -\frac{B\theta'(M)}{(1+[1-\theta(M)]B)N(M)} + \frac{(-k)|\Delta|}{T(M)} \right\}$$

Denote the two terms in $\frac{\partial g}{\partial M}$ respectively as:

$$P(M) = \frac{B\theta'(M)}{(1+[1-\theta(M)]B)N(M)} > 0$$

$$Q(M) = \frac{(-k)|\Delta|}{T(M)} > 0$$

That is:

$$\frac{\partial g}{\partial M} = g(Q(M) - P(M))$$

Now analyze the monotonicity of $Q(M)$ and $P(M)$. In $Q(M)$, according to the previous setup, $|k|$ first increases and then decreases with M , while $T(M)$ first decreases and then increases with M ; both reach their extreme values at $M = M^*$. It follows that $Q(M)$ first increases monotonically and then decreases monotonically. As shown in the figure, if the initial turnover efficiency of digital platforms is low, total factor productivity will keep declining, and the decline will accelerate as the digital platform sector expands. This suppresses the reproduction of the production sector and gradually gives rise to the Solow paradox.

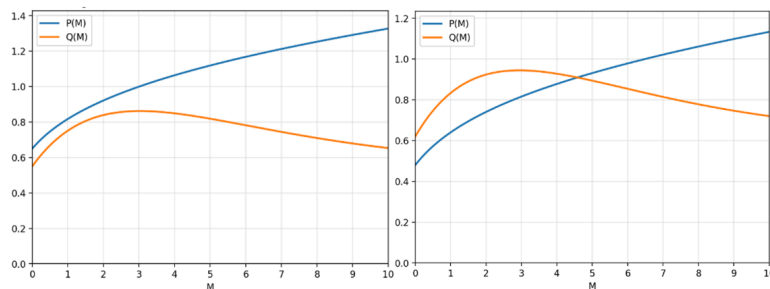


Figure 1. Analysis of changes in the differential of total factor productivity

3.2. Intermediate goods input

In the analytical expression of total factor productivity we have constructed, the parameter directly related to intermediate goods input is σ , which represents the ratio of intermediate goods input to the value of fixed capital. We first analyze the relationship between σ and A through mathematical deduction. Take the derivative of A with respect to σ :

For computational convenience, denote:

$$T = T_P + T_R + T_F$$

$$D = \kappa + \sigma + \delta$$

$$E = 1 + \kappa + \sigma$$

$$B = 1 + pq \frac{D}{E}$$

$$m = \frac{\tau}{T}$$

We obtain:

$$\frac{\partial A}{\partial \sigma} = (1+q)T\kappa^{1-\alpha} \left[\frac{B^m - 1}{\ln(B)} + D \frac{mB^m \ln(B) - (B^m - 1)}{B (\ln(B))^2} * pq \frac{1-\delta}{E^2} \right] > 0$$

This paper holds that idle production capacity is a key mechanism. Accordingly, idle capacity caused by accelerated technological iteration in the digital economy forms a specific transmission path for the emergence of the Solow paradox.

3.3. Depreciation and renewal of fixed capital

In the capital circulation model of this paper, δ stands for the depreciation of fixed capital, representing the proportion of value transferred into products in each round of capital circulation. The model assumes that every time the value of products is realized through market circulation, the portion belonging to fixed capital forms a depreciation fund, which is immediately compensated for physical and moral depreciation in the next round to maintain the stability of fixed capital value. Therefore, fixed capital has an infinite lifespan and is continuously renewed at every moment, so δ essentially represents the renewal speed of fixed capital.

$$\frac{\partial A}{\partial \delta} = (1+q)T\kappa^{1-\alpha} \left[\frac{B^m-1}{\ln(B)} + D \frac{mB^m \ln(B) - (B^m-1)}{B(\ln(B))^2} * \frac{pq}{E} \right] > 0$$

Thus, the relationship between the fixed capital renewal speed δ and total factor productivity is positive. If fixed capital becomes obsolete easily in general, leading to moral depreciation, with replacement cost even exceeding its own value and low or even unrecoverable scrap value after scrapping, the total factor productivity of capital tends to decline.

4. Conclusion

Digital platforms exert two opposite effects on industrial capital: "accelerating turnover" and "value extraction". In the early stage of development, the former dominates and helps improve productivity. However, with the expansion of platform scale, rising market concentration and enhanced bargaining power, the latter gradually takes the lead. Unproductive circulation costs continuously squeeze the reproduction space of industrial capital, turning total factor productivity from growth to decline.

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