Proceedings of ICEMGD 2026 Symposium: The Role of Blue Economy in Promoting Human Sustainable Development
DOI: 10.54254/2754-1169/2026.GT34574

Pricing and Risk Analysis of CSI 300 Index-Linked Structured Products Based on Geometric Brownian Motion and Monte Carlo Simulation

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Abstract. Against the backdrop of the net asset value-based transformation of wealth management, index-linked structured products have gradually become important wealth management tools for banks. This paper takes the CSI 300 Index as the research object and obtains daily closing price data for the past three years. This paper uses geometric Brownian motion to describe the price dynamics of the CSI 300 Index, estimating drift and volatility parameters; it generates 10,000 Monte Carlo simulation independent price paths and calculates the product return for each path. Then, sensitivity analysis is performed, calculating the Greeks Delta and Gamma values to analyze the sensitivity of the product price to changes in the benchmark index. To verify the model assumptions, this paper conducts normality tests and ADF unit root tests on the logarithmic returns of the index. The study results indicate that although the logarithmic return series of the CSI 300 Index is relatively stable, it still exhibits leptokurtosis and heavy tails. The theoretical price of the product is 0.018213, with an annualized return of 3.69%. The product shows low sensitivity to index fluctuations. This study provides a basis for financial institutions' product design and risk management.

Keywords: Monte Carlo simulation, geometric Brownian motion, risk analysis, structured products, CSI 300 Index

1. Introduction

With the deep integration of the global financial market and the transformation of domestic asset management business, structured products with the characteristics of "fixed income + derivatives" have become an important tool for financial institutions to meet the diversified risk and return needs of investors [1]. Xie Shuhang pointed out that the number of newly issued structured wealth management products by wealth management companies increased by 109.26% year-on-year, which is a huge increase [2]. As the core broad-based index of the A-share market, the CSI 300 Index is often used as the benchmark for such products. The rationality of its pricing and the controllability of its risks are not only related to the product competitiveness of financial institutions, but also directly affect the asset security of investors [3]. Therefore, finding a pricing and risk analysis

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method for structured products that is suitable for the characteristics of the A-share market has great theoretical and practical significance.

Currently, international scholars have made significant progress in the pricing and risk analysis of structured products. The Black-Scholes option pricing model provides a theoretical basis for the valuation of financial derivatives. Geometric Brownian motion is often used to describe changes in the price of the underlying asset [4], providing a classic theoretical framework for simulating index price paths. Monte Carlo simulation can be widely used in the pricing of financial products. Yang Gang and Cheng Yuanlei used this method to verify its effectiveness in volatility estimation and insurance pricing [5]. Meanwhile, research shows that Monte Carlo simulation helps price complex path-dependent products [6]and is one of the mainstream numerical methods for pricing structured products. In derivative pricing, an important tool for measuring the sensitivity of product prices to changes in the underlying asset is sensitivity analysis, namely, the Greek value. Xiang Jiangming used Monte Carlo simulation to estimate the sensitivity parameters of American options and verified the feasibility of this method in calculating the Greek value [7]. In domestic research, scholars have focused on the characteristics of the local market and studied the impact of factors such as credit risk premium and A-share index volatility on the pricing of structured products, providing local experience support for the optimization of pricing models adapted to the A-share market.

This paper selects structured products linked to the CSI 300 index as the research object. It adopts the classic approach of using geometric Brownian motion to simulate the index price path and the Monte Carlo method for product pricing, and establishes a pricing model suitable for the A-share market. It quantifies the return and risk characteristics of the product, provides technical support for financial institutions to improve product design and strengthen risk control, and also provides a theoretical foundation for investors to understand product value and rationally allocate assets.

2. Methods

2.1. Data source

This study selects the CSI 300 Index as the underlying asset, with data sourced from the Tonghuashun financial data terminal. The sample period is from March 3, 2023, to March 3, 2026, obtaining daily closing price data for the past three years at daily frequency. Missing values due to market closures are removed to form a continuous time series, and the logarithmic return series is calculated. In addition to index price data, this study will use Excel to perform Monte Carlo simulations and sensitivity analyses, and the SPSSAU online statistical analysis platform to validate the data and visualize the results.

2.2. Selection and explanation of indicators

The core variables in this study include: ① Index price series: The daily closing price of the CSI 300 Index is directly used as the price of the underlying asset S_t, they are used to estimate parameters of geometric Brownian motion; ② Logarithmic returns are used to test the normality of the return distribution and the stationarity of the time series. ③Product revenue structure: Setting the initial pricing date$(t = 0)$ and mid-term observation day $(t = \tau)$, two key time nodes, the formula for the product's final return is:

$$Y = f\left(S_0, S_\tau\right) \tag{1}$$

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2.3. Method introduction

Geometric Brownian Motion: This study uses geometric Brownian motion to describe the price dynamics of the CSI 300 Index, assuming that asset prices satisfy：

$$dS_t=\mu S_t dt+\sigma S_t dW_t \tag{2}$$

Where μ is the drift rate, σ is the volatility, and W_t is the standard Brownian motion. The parameters estimate volatility using historical data. $\sigma=\sqrt{Var(r)/\Delta t}$, Drift rate $\mu=\overline{r}/\Delta t+\sigma^2/2$.

Monte Carlo simulation: Generate N = 10,000 independent price paths in Excel. The price of each path on the initial fixing date and the intermediate observation date is obtained recursively by $S_{t+\Delta t}=S_t\exp[(\mu\text{-}\sigma^2/2)\Delta t+\sigma\sqrt{\Delta t}\epsilon]$, where ϵ is a standard normal random number. The formula for estimating the product price, $Y^{(i)}$ for each path, is as follows:

$$\hat{V}=\frac{1}{N}\sum_{i=1}^{N}Y^{(i)} \tag{3}$$

Sensitivity analysis: Calculate Delta ($\Delta=\partial V/\partial S$) and Gamma ($\Gamma=\partial^2/\partial S^2$). The sensitivity of product prices to changes in a benchmark index is approximated by using the numerical difference method.

3. Results

3.1. Descriptive statistics and model testing

This study selected daily closing price data of the CSI 300 Index from March 1, 2021, to February 29, 2024, obtaining a sample of 724 trading days. Table 1 reports the descriptive statistics and normality and stationarity tests of the logarithmic return series. As shown in Table 1, the mean of the logarithmic returns is close to zero, consistent with the efficient market hypothesis. The standard deviation is 0.011, corresponding to a daily volatility of approximately 1.1%. The skewness is -0.245, slightly left-skewed; the kurtosis is 11.914, significantly higher than the kurtosis value of 3 for a normal distribution, indicating that this series has a typical "leggy" or "fat-tailed" characteristic.

Table 1. Normality test results

name	Sample size	Average value	Standard deviation	Skewness	Kudo	K-S test		Shapiro-Wilk test	
						Statistic D-value	p	Statistic D-value	p
Logarithmic return	724	-0.000	0.011	-0.245	11.914	0.079	0.000^{*}	0.888	0.000^{**}

*p<0.05 **p<0.01

Regarding the normality test, as shown in Table 1 and Table 2, the p-values of both the K-S test and the Jarque-Bera test are less than 0.01, rejecting the null hypothesis of a normal distribution at

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the 1% significance level. This result is consistent with the typical characteristics of financial time series, namely, the probability of extreme values occurring is higher than predicted by a normal distribution. Referring to the SPSSAU analysis recommendations, data with an absolute kurtosis of less than 10 and an absolute skewness of less than 3 can be generally accepted as normally distributed. In this study, the kurtosis of 11.914 slightly exceeds the standard, but considering the subsequent stationarity test, geometric Brownian motion can still be used for modeling.

Table 2. Jarque-Bera test

Name	Sample Size	χ^2	df	p -value
Logarithmic Return	724	4224.609	2	0.000

According to Table 3, the ADF test statistic is -11.374, which is less than the 1% critical value of -3.440, and p = 0.000. This indicates that the sequence is highly stationary and meets the requirements of geometric Brownian motion for incremental processes.

Table 3. Logarithmic return-ADF test table

Difference Order	t	p	Critical Value		
			1%	5%	10%
0	-11.374	0.000	-3.440	-2.866	-2.569

Figure 1 shows the Q-Q plot of the logarithmic returns. The horizontal axis represents the theoretical normal quantile, and the vertical axis represents the actual observed values. The scatter points are mostly located on the 45-degree diagonal, but slightly deviate at both ends, further confirming the leptokurtic and heavy-tailed characteristics of the series—the middle part follows a normal distribution, while the tails are thicker than normal.

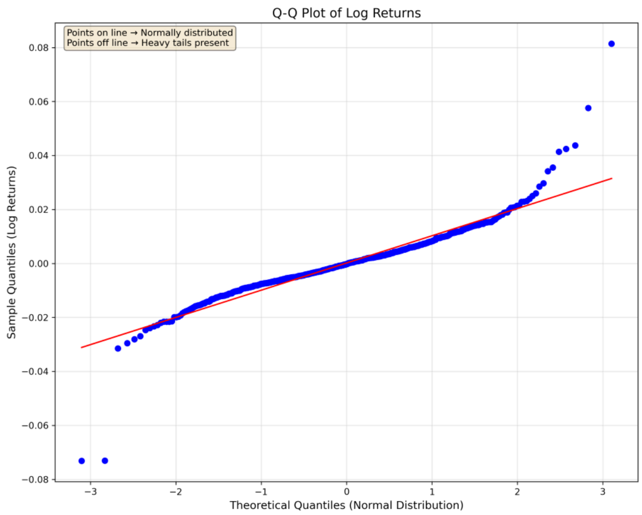

Figure 1. Q-Q plot of logarithmic returns

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3.2. Parameter estimation

Table 4. Core parameters of geometric Brownian motion

Parameter	Symbol	Estimated Value
Annualized volatility	σ	27.94%
Annualized drift rate	μ	-0.264%

Based on historical data, the maximum likelihood method was used to estimate the two core parameters of geometric Brownian motion, and the results are shown in Table 4.

The annualized volatility of 27.94% is within the reasonable range for A-share market indices, reflecting the average volatility level of the CSI 300 Index during the sample period. The annualized drift rate of -0.264% indicates that the overall market showed a slight downward trend during the sample period, consistent with the actual performance of the index.

3.3. Simulation results

Based on the above parameter estimation results, 10,000 independent index price paths were generated in Excel. The simulation period was one year, with the mid-term observation date set at day 180 ($\Delta_t = 0.493$ years). The product return structure was set as follows: if the index on the observation date is not lower than 95% of the initial level, investors will receive an annualized return of 5%; otherwise, they will receive a guaranteed return of 2%.

Table 5. Monte Carlo simulation results

Project	numerical values
Simulated path number	10,000
Number of high-yield scenarios (5% annualized)	5,574
Guaranteed minimum return (2% annualized)	4,426
High-yield path proportion	55.74%
Percentage of guaranteed return paths	44.26%
Product price estimate $\widehat{V}$	0.018213
Corresponding annualized rate of return	3.69%

As shown in Table 5, in 10,000 Monte Carlo simulations, 5,574 paths (55.74%) triggered the high-yield condition, i.e., the index on the observation date was not lower than 95% of the initial level; another 4,426 paths (44.26% of the total paths) would obtain a guaranteed return. The average return of all paths was 0.018213; thus, according to the theory, the price of the product should be 0.018213 for a principal of 1 unit. That is to say, the annual return will be 3.69%, which is between the guaranteed rate of 2% and the high-yield range, and fits the distribution path.

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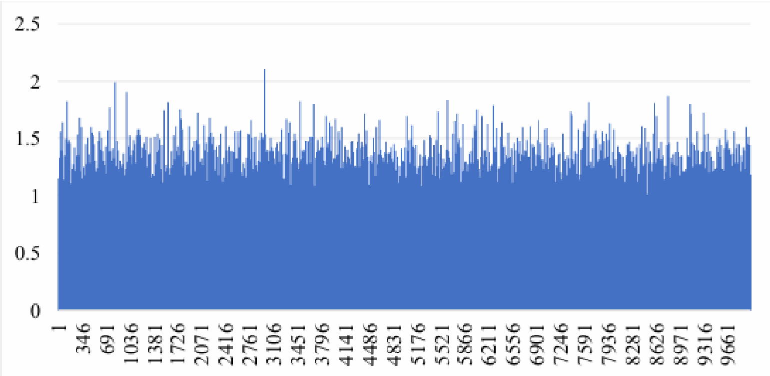

Figure 2. Histogram of simulated price distribution

Figure 2 shows the distribution of index prices over 10,000 simulated medium-term observation days. As can be seen from the figure, the simulated prices exhibit a right-skewed distribution, consistent with the log-normal distribution assumption. Prices mostly fall between 0.8 and 1.2, with extreme values occurring infrequently, consistent with the 30% volatility parameter setting.

3.4. Sensitivity analysis

To measure the sensitivity of product prices to changes in the benchmark index, two risk indicators, Delta and Gamma, were calculated. Using the numerical differencing method, with initial price changes set at ±1%, the Monte Carlo simulation was rerun, and the results are shown in Table 6.

Table 6. Sensitivity analysis results

circumstances	Initial Price S_0	Product Price V	Delta	Gamma
Down 1%	0.99	0.018200	-	-
Standard	1.00	0.018213	-0.00965	-3.65
Up 1%	1.01	0.018007	-	-

A delta of -0.00965 is close to 0, so the change in price will be small after a fluctuation in the index. As shown above, this is the expected structure of the product; that is to say, a guaranteed minimum return and a capped maximum return mechanism will reduce the price sensitivity to changes in the index. A 1% rise in the index will result in a decrease of 0.000206 in product prices; a 1% drop in the index will increase product prices by only 0.000013, and both are less than 0.02%. A Gamma of -3.65 shows that the price function is concave (convex downwards). When the index increases, Delta decreases (it will be a larger negative number), and when the index falls, Delta increases (it will be a smaller negative number). This non-linearity shows that the return structure of the product is convex; that is to say, it is very sensitive to declines in the index near the guaranteed minimum return and less sensitive to increases in the index at high-return levels.

4. Discussion

4.1. Reasons for and analysis of the research results

This study found that the kurtosis of the logarithmic return of the CSI 300 index reached 11.914, which is significantly higher than that of the normal distribution, showing a typical leptokurtic and heavy-tailed characteristic. This result is in line with the observation of the volatility characteristics

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of the same index by Li Bingqing and Zhang Tianqi [8]. The reason for this is that the price of financial assets is unstable after a sudden change in information. In terms of the degree, many factors have been occurring in the sample period 2023-2026, such as repeated outbreaks of the epidemic, geopolitical conflict, and restructuring of the economy. Due to the economy, there will be significant increases or decreases in the index on some trading days, thus changing the maximum value. This result is in line with the findings of domestic and foreign scholars on the return characteristics of emerging market indices. Due to the lower efficiency of information transmission and the different structure of investors, emerging markets have been relatively volatile. The product's Delta is -0.00965, and it is not very responsive to index changes. The reason for this result is that the structure of the product has set a guaranteed minimum return of 2% and a maximum return of 5%. When the index is fluctuating, the return for investors will be only between 2% and 5%; therefore, they cannot enjoy all the gains of an index rise or bear all the losses in an index fall. Thus, the effect of a product price change on index change is smaller. The result is in line with the Design goal of the product; that is to say, to provide low-risk investors with a relatively stable income that is loosely correlated with the index. Gamma = -3.65 shows that the price function is concave. Due to the convexity of the return structure, when the index is near the guaranteed minimum return threshold (around 95% of its initial level), the product price is more susceptible to a decline in the index; once it drops below this threshold, the return will drop from the maximum return to the guaranteed minimum return, whereas it is relatively less sensitive when far away from the threshold. Because the payoff function has an inflection point characteristic, Gamma is negative, which is in line with the Gamma characteristic of put options in option pricing theory.

4.2. Extensions and recommendations of research findings

As the Delta of the product is close to zero, a static replication method will be employed by the financial institution. Therefore, in the issuance of such products, one does not need to build a complex dynamic hedging portfolio and can save on operating and trading expenses. A negative Gamma characteristic shows that in the event of a large fluctuation in the index, such as a rise of more than 3% in one day, the risk exposure of the product will be more volatile. A stress test system has been built to assess the impact of bad market conditions on the company. Conservative investors with a low-risk tolerance can choose to get bank deposits or an index fund from this product. The expected annual rate of return is 3.69 per cent; that is to say, the interest rate paid by banks is relatively low. It is suggested that the type of product be used as a "ballast" in the asset allocation, with an allocation range of 20%-40% of the total investment assets. They will earn a higher interest rate than in a bank deposit without reducing liquidity.

4.3. Research limitations and improvements

This study uses geometric Brownian motion modeling and assumes that the logarithmic return follows a normal distribution. However, the test results show that the K-S test $p < 0.01$, which strictly rejects the normality assumption. This limitation may lead to the model underestimating extreme events (such as market crashes). Future research can use a diffusion model with jumps (such as the Merton jump diffusion model) or a GARCH-type model to more accurately characterize the volatility characteristics of the index, especially its performance under extreme market conditions. Ma et al. used a sub-mixed fractional Brownian motion model to predict stock price behavior and proved that it is superior to the standard geometric Brownian motion model [9]. This provides a direction for improving the method of this study. The volatility and drift rate in the study

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are estimated based on historical data from the past 3 years, assuming that the future market characteristics are consistent with the past. However, in reality, volatility has a clustering effect and time-varying nature, and the drift rate is more difficult to predict. In the future, the volatility cone method can be introduced to compare the difference between historical volatility and implied volatility, or a rolling window estimation can be used to analyze the impact of parameter stability on pricing results. This study used Excel to perform 10,000 Monte Carlo simulations. If the number of simulations increases (e.g., 100,000) or the path becomes more complex (e.g., daily observation), the computational efficiency of Excel may become a bottleneck. Deviation from the normality assumption is also one of the limitations of this study. Chen Mingguang and Wang Yuanchang pointed out that the normal distribution is difficult to fit the data with sharp peaks and heavy tails effectively, and proposed the folded Gamma distribution as an improvement scheme [10], which provides a reference direction for subsequent research. In the future, professional tools such as Python and R can be used to improve computational efficiency and repeatability. At the same time, variance reduction techniques such as dual variables and control variables can be introduced to improve the estimation accuracy under the same number of simulations.

5. Conclusion

This study found that the leptokurtic and heavy-tailed characteristics of the logarithmic returns of the CSI 300 Index originate from the impact of macroeconomic events, and that the low Delta characteristic of the product aligns with the design intent of its guaranteed return structure. Based on the findings, targeted recommendations are made for financial institutions, investors, and regulatory agencies. Furthermore, limitations such as deviations from the normality assumption, time-varying parameter estimation, and product structure simplification are objectively analyzed, and corresponding improvement directions are proposed, providing a reference for future research.

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