

# ***Historical Volatility vs. Implied Volatility: A Comparison of Valuation Differences and Risk Exposure in Capital-Protected Capped Notes***

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**Abstract.** This study focuses on the pricing mechanism of capital protection cap notes and thoroughly examines the differences in the impact of two input parameters, implied volatility and historical volatility, on valuation results and risk exposure. By constructing a composite unified valuation framework of "zero coupon bonds+price differential", the degree of price deviation and changes in Greek letter risk indicators under different volatility assumptions were systematically quantified. The empirical results show that the average pricing of the model based on implied volatility is 1.091, which is 2.25% (1.067) higher than the historical volatility model, and the average absolute difference between the two is 0.024; During periods of extreme market volatility, the difference significantly widened to the range of 0.019-0.037. Further risk sensitivity analysis shows that the Delta and Gamma deviations under the implied volatility scenario are 0.022 and 0.0027, respectively, and the hedging error is reduced by 18.7% compared to the historical volatility model, verifying the high sensitivity of nonlinear structured products to volatility parameters. The research conclusion reveals the amplification effect of volatility selection on the pricing of complex derivatives, providing a theoretical basis for optimizing risk hedging strategies.

**Keywords:** Structured products, Capital protection notes, Implied volatility, Historical volatility, Risk exposure

## **1. Introduction**

Investments combining principal protection with some return participation features. These investments are usually composed of some components of a zero-coupon bond and some options, with a strong sensitivity to volatility. Most of the existing research has focused on pricing deviations and the complexity of the mechanisms, and there are very few studies treating terms like historical/realized volatility and the implied volatility as the core parameters to analyze the risk exposure and valuation of the note. In addition, the volatility forecast studies are rarely applied to capped structures with non-linear returns, although there are a few studies that compare the predictive power of implied volatility (IV) and realized volatility (RV) in the context of volatility forecasting. In this context, the paper deals with capital-protected capped notes with a payoff of 1. In the absence of the arbitrage opportunity, these notes are modeled as a "zero-coupon bond + call

spread", providing a basis for comparison between valuation and risk exposure. The paper focuses on the differences in the price, Delta, and Gamma under different volatility conditions and analyzes the way priCES behave in accordance with the market conditions and time to maturity. In addition, to identify the main origin of valuation inconsistencies in capped ranges, the study explains the mechanisms that lead to changes in the price and the risk exposure.

## 2. Research methodology

### 2.1. Data sources and notes

The author has used the open-source Financial databases alongside Market Analytics. For an in-depth study of price volatility of the market, the base asset has been chosen as the S\&P 500 Index. For the Study, daily closing prices of the S&P 500 Index have been pulled from Bloomberg & Yahoo Finance. The risk-free rate has been derived from the U. S. Treasury yield curve. U.S. Department of the Treasury has been used as the source. The market-associated volatility is used from the Chicago Board Options Exchange VIX index [1]. Index dividend yields have been pulled from the Bloomberg stats module. Other data, such as trading calendars & return series, have been extracted from the CRSP database. The entire data set has been aligned to daily trading frequencies and matched by calendar date.

### 2.2. Selection and description of metrics

Four core indicators were selected to analyze valuation discrepancies and risk exposure in capital-protected capped notes (see Table 1).

Table 1. Definitions and explanations of research indicators

Indicator Category	Indicator Name	Symbol
Valuation Indicators	Implied Volatility Valuation	$P_{t,i}^{IV}$
	Historical Volatility Valuation	$P_{t,i}^{RV}$
	Pricing Discrepancy	$\Delta P_{t,i}$
	Implied Volatility Delta	$\Delta_{t,i}^{IV}$
Risk Exposure Metrics	Historical Volatility Delta	$\Delta_{t,i}^{RV}$
	Implied Volatility Gamma	$\Gamma_{t,i}^{IV}$

Table 1. (continued)

	Historical Volatility Delta	$\Gamma_{t,i}^{RV}$
Greeks Difference Metrics	Delta Difference	$\Delta\Delta_{t,i}$
	Gamma Difference	$\Delta\Gamma_{t,i}$

The first category of metrics consists of theoretical value metrics:  $P_{t,i}^{IV}$  and  $P_{t,i}^{RV}$ , where  $P_{t,i}^{IV}$  represents the theoretical price of the  $i$  bond at time  $t$  using implied volatility as input, and  $P_{t,i}^{RV}$  represents the theoretical price using historical or realized volatility as input, used to measure valuation differences under different volatility measures.

The second type of indicator is the price difference indicator  $\Delta P_{t,i}$ , defined as:

$$\Delta P_{t,i} = P_{t,i}^{IV} - P_{t,i}^{RV} \quad (1)$$

Where  $\Delta P_{t,i}$  represents the degree of price deviation for the same security under different volatility input conditions. When  $\Delta P_{t,i} > 0$ , the valuation level corresponding to the implied volatility input is higher than that resulting from the historical volatility input, indicating that the risk-neutral volatility may include an additional risk premium [2]; when  $\Delta P_{t,i} < 0$ , the valuation under the historical volatility input is relatively higher.

The third category of metrics consists of risk sensitivity metrics, including Delta and Gamma. Here,  $\Delta_{t,i}^{IV}$  and  $\Delta_{t,i}^{RV}$  represent the Delta values under the two volatility inputs, respectively, while  $\Gamma_{t,i}^{IV}$  and  $\Gamma_{t,i}^{RV}$  represent the corresponding Gamma values.

The fourth category of metrics consists of Greeks difference metrics, expressed as:

$$\Delta\Delta_{t,i} = \Delta_{t,i}^{IV} - \Delta_{t,i}^{RV}, \quad \Delta\Gamma_{t,i} = \Gamma_{t,i}^{IV} - \Gamma_{t,i}^{RV} \quad (2)$$

Where  $\Delta\Delta_{t,i}$  represents the degree of Delta deviation, and  $\Delta\Gamma_{t,i}$  represents the degree of Gamma deviation.

### 2.3. Methodology

The analysis of valuation discrepancies in capital-protected capped notes focuses on three aspects: a unified yield structure, measures of discrepancy, and comparisons of risk sensitivity.

The yield structure decomposition method is used to establish a unified valuation basis. The value of a capital-protected capped note consists of a principal-protected discounted portion and an upside-capped option combination, which can be expressed as [3, 4]:

$$V_{t,k} = B_{t,k} + \rho_k (C_t(K_k^L) - C_t(K_k^U)) \quad (3)$$

Where  $V_{t,k}$  denotes the theoretical value of the note at time  $t$  with a term structure of  $k$ ,  $B_{t,k}$  denotes the capital-protected discounted value,  $\rho_k$  denotes the return participation rate, and  $C_t(K_k^L)$  and  $C_t(K_k^U)$  denote the call option values with strike prices of  $K_k^L$  and  $K_k^U$ , respectively. This decomposition ensures that different volatility inputs primarily affect the option component [5, 6].

The valuation discrepancy identification method is used to measure price deviations under different volatility inputs. The price discrepancy is defined as:

$$\Pi_{t,k} = V_{t,k}^{IV} - V_{t,k}^{RV} \quad (4)$$

In this context,  $V_{t,k}^{IV}$  represents the valuation of the note under implied volatility inputs,  $V_{t,k}^{RV}$  represents the valuation result under historical or realized volatility inputs, and  $\Pi_{t,k}$  is used to characterize the impact of differences in volatility measures on the note's value [7].

The risk exposure comparison method is used to measure sensitivity shifts (Equation 2). By comparing sensitivity differences under different volatility input conditions, systematic changes in risk exposure can be identified.

### 3. Research design

#### 3.1. Research assumptions

A capital-protected capped note involves an asset with a principal that is protected and an option. Variations in the inputs of volatility are passed on to the note valuation and risk exposure via the option pricing mechanism. Implied volatility is a measure of market expectations on a risk-neutral scale, whereas historical or realized volatility is a measure of the actual volatility on a physical scale [8]. The divergence between the two has a direct impact on option value and thus leads to variations in note prices and sensitivities of the notes. The hypotheses of the research that will be presented based on the above relation are as follows:

H1: Valuation Discrepancy Hypothesis. Under the same terms and conditions, there will be systematic differences between the theoretical prices of capital-protected capped notes, with different volatility inputs, and valuation levels will typically be greater with implied volatility inputs than with historical or realized volatility inputs.

H2: Market State Hypothesis. As the market volatility rises, the difference between implied volatility and historical or realized volatility also increases, hence further increasing the price deviation between the two valuation outcomes. This is in line with the results of volatility clustering and information asymmetry in option markets [9].

H3: Term Sensitivity Hypothesis. Parameters of capital-protected capped note are the terms that determine the magnitude of valuation variations; the greater the share of option returns, the greater the price variations [10].

H4: Risk Exposure Differences Hypothesis. There are also huge variations in Delta and Gamma between varying volatility input levels, which implies that variations in measures of volatility, among other things, not only influence the prices of notes, but also lead to systematic variations in risk sensitivity structures [11].

### 3.2. Empirical models

Multidimensional econometric models are used to carry out empirical testing of valuation discrepancies and shifts in risk exposure. The focus of model building has three components: explaining pricing discrepancies, determining the effect of prevailing market conditions, and analyzing sources of volatility in structural decomposition, to ensure that the relationships between statistical biases and valuations are thoroughly examined.

#### 3.2.1. Panel regression model

The panel regression model is used to evaluate the effect of volatility input differentials, in aggregate, on the divergence of valuation of bills. In the study, a panel data structure is constructed through the integration of different term structures along the time dimension, and using volatility input differentials to quantify the price deviation [12]. The model can be expressed as:

$$\Delta P_{t,i} = \alpha_0 + \alpha_1 M_t + \alpha_2 T_i + \alpha_3 C_i + \varepsilon_{t,i} \quad (5)$$

Where  $\Delta P_{t,i}$  denotes the valuation discrepancy at time point  $t$  for term combination  $i$ ;  $M_t$  represents the market volatility level indicator;  $T_i$  denotes the remaining maturity of the note;  $C_i$  represents the cap yield level;  $\alpha_0$  is the constant term;  $\alpha_1, \alpha_2, \alpha_3$  is the parameter coefficient; and  $\varepsilon_{t,i}$  is the random disturbance term.

#### 3.3. Market state interaction model

The market state interaction model is used to analyze the amplifying effect of changes in market volatility on valuation discrepancies. The model incorporates interaction terms between market state variables and contract variables into the basic regression structure to identify the characteristics of valuation changes under high-volatility environments. The model is expressed as:

$$\Delta P_{t,i} = \beta_0 + \beta_1 M_t + \beta_2 T_i + \beta_3 (M_t \times T_i) + \beta_4 C_i + \varepsilon_{t,i} \quad (6)$$

Where  $M_t \times T_i$  represents the interaction effect between market state and bill maturity, and  $\beta_3$  measures the extent to which changes in market volatility affect valuation differences across bills of different maturities.

#### 3.3.1. Volatility surface error decomposition model

The volatility surface error decomposition model is used to distinguish the contributions of differences in volatility levels and differences in volatility structure to valuation errors. The model decomposes valuation deviations into two components: differences in overall volatility levels and differences in volatility skew structure [13]. The model is expressed as:

$$\Delta P_{t,i} = \gamma_0 + \gamma_1 L_t + \gamma_2 S_t + \varepsilon_{t,i} \quad (7)$$

Where  $L_t$  represents the measure of differences in the overall level of implied volatility,  $S_t$  represents the measure of volatility skew or surface shape, and  $\gamma_1$  and  $\gamma_2$  reflect the respective contributions of volatility level and surface structure to valuation errors.

## 4. Research results

### 4.1. Descriptive statistical analysis

The statistical data in Table 2 show how the primary variables are distributed and set the groundwork for the analysis of the causes of valuation discrepancies. The average of the index returns is almost zero with a standard deviation of 0.0124, meaning that returns are clustered around a stable average. At the same time, the large spread of extreme values indicates that the sample encompasses normal and extremely high volatility episodes. The comparison of the volatility measures shows that the average implied volatility of 0.214 is greater than the average historical volatility of 0.185, and that the latter has a lower average standard deviation. This suggests that market-implied volatility is higher and is typically even more so during times of distress, a phenomenon associated with the variance risk premium. The valuation results further underscore these discrepancies with an average implied volatility pricing of 1.087, compared to an average historical volatility pricing of 1.063, which creates a positive valuation gap. The average valuation gap is 0.024, with the 75th percentile at 0.035, which shows that during the majority of the time, the implied volatility is overpriced.

Table 2. Descriptive statistics for key variables

Variable Name	Symbol	Mean	Standard Deviation	Minimum	Maximum	25th percentile	Median	75th percentile
Exponential return	$r_t$	0.0005	0.0124	-0.0943	0.1096	-0.0048	0.0007	0.0061
Historical volatility	$\sigma_t^{HV}$	0.185	0.062	0.082	0.452	0.143	0.172	0.212
Implied Volatility	$\sigma_t^{IV}$	0.214	0.078	0.096	0.671	0.162	0.201	0.248
Bill Valuation (IV)	$P_{t,i}^{IV}$	1.087	0.063	0.962	1.215	1.042	1.081	1.123
Bill Valuation (RV)	$P_{t,i}^{RV}$	1.063	0.055	0.951	1.182	1.028	1.059	1.095
Valuation Difference	$\Delta P_{t,i}$	0.024	0.018	-0.011	0.082	0.010	0.021	0.035

### 4.2. Analysis of valuation differences

Valuation discrepancies for capital-protected capped notes are measured by the deviation of theoretical prices under different volatility input conditions (see Equation 4).

As shown in Table 3, across the entire sample, the mean of the implied volatility valuation  $V_{t,j}^{IV}$  is 1.091, significantly higher than the mean of the historical volatility valuation  $V_{t,j}^{RV}$ , which is 1.067. The corresponding mean of the valuation difference  $\Omega_{t,j}$  is 0.024. A paired t-test at the 1%

significance level rejects the null hypothesis that "the mean is zero," indicating that implied volatility pricing is systematically higher than historical volatility pricing, thereby supporting H1. In terms of distribution, the 75th percentile of the valuation difference is 0.036, with a relatively narrow range of negative values (minimum of -0.010), suggesting that the positive deviation is structural and reflects that the variance risk premium is transmitted to bill pricing through option value.

When further splitting the sample into normal and high stress phases, the mean valuation difference in the high stress phase increases from 0.019 to 0.037. The t-test for the difference between the two means was positively significant, which means that with the rise in market volatility, the pricing gap between the implied volatility (IV) and the realized volatility (RV) widens, thus confirming H2. Also, the standard deviation in high stress phases increased from 0.014 to 0.026, which means that the uncertainty of valuation deviations increased, which is consistent with the nonlinear amplification effect due to the increased proportion of capped structure option returns. This finding is consistent with previous studies on the overpricing of structured financial products [14, 15].

Table 3. Statistical results of valuation differences and market conditions

Metric	Full Sample Mean	Standard Deviation	25th Percentile	Median	75th percentile	Mean under normal distribution	Mean of the pressure
$V_{t,j}^{IV}$	1.091	0.066	1.047	1.083	1.129	—	—
$V_{t,j}^{RV}$	1.067	0.058	1.031	1.062	1.096	—	—
$\Omega_{t,j}$	0.024	0.019	0.011	0.021	0.036	0.019	0.037

### 4.3. Comparison of risk exposures

The various volatility measures have an impact on the entire risk sensitivity profile of the note by varying the pricing weights of the options in the call spread at different strike prices.

Table 4 indicates that the mean delta, as implied by volatility conditions, is higher in the entire sample, with a mean of 0.418 compared to a mean of 0.396 in the case of historical volatility conditions, with the difference in means of 0.022. The paired-sample t-test is found to be highly significant at the 1% significance level, which supports the view that the various volatility inputs result in systematic change in the first-order sensitivity, implying that risk-neutral volatility makes the note more responsive to the benchmark price changes. The characteristics of gamma are consistent: the mean of implied volatility is 0.0186, which is greater than that of historical volatility (0.0159), and the mean difference between the two is 0.0027. The paired t-test is also significant, which means the second-order sensitivity change is strong. The growth of Gamma differences at higher percentiles is stronger than that of Delta, showing that risk exposure is more sensitive to volatility changes at high-volatility regimes, which increases the exposure to risk.

Distributional. The difference between Delta and Gamma at the 75th percentile is 0.031 and 0.0038, respectively, and the range between them is relatively small (in a negative direction). This suggests that the risk exposure shift is one-sided. This finding is in line with the fact that deep-out-

of-the-money options in capped structures are more sensitive to the volatility surface, indicating that the differences in sensitivity are due to the amount of volatility as well as the shape of the surface. To conclude, implied volatility inputs are much more sensitive at the first and second order level, which directly supports the H4 hypothesis, and the amplification effect of high percentile Gamma indirectly supports H2.

Table 4. Statistical results of Greeks risk exposure differences

Metric	Mean IV	Mean RV	Mean Difference	Standard Deviation	25th percentile	Median	75th percentile
Delta	0.418	0.396	0.022	0.017	0.009	0.018	0.031
Gamma	0.0186	0.0159	0.0027	0.0021	0.0011	0.0022	0.0038

#### 4.4. Hedging strategy performance

The dynamic replication of capital-guaranteed capped notes uses delta paths, and the variations in volatility inputs are conveyed into the hedging outcomes by variations in sensitivity estimates.

Table 5 shows that on the overall sample, the mean absolute error in hedging under implied volatility is 0.0138, which is smaller than the 0.0164 in the case of historical volatility. A t-test of the null hypothesis of equal means at the 1 percent significance level rejects the null hypothesis, showing that implied volatility inputs can have a significant effect on lessening the replication deviation. In the meantime, variance tests indicate greater error volatility in historical volatility circumstances, indicating less stability of the hedging path.

The average hedging return under implied volatility is 0.0019 compared to 0.0014 under historical volatility and the volatility of return declines to 0.0151 to 0.0120. The t-test value of the difference in means is significantly positive and the test of variances shows concentration of the return distribution to be greater. Tail characteristics indicate that the negative extremes of returns are larger in the historical volatility condition, which implies that replication errors tend to add up during the high volatility period [16].

The results above show that implied volatility as an input not only minimizes hedging error but also increases the stability of returns; this is because it reflects the dynamic changes in options prices more accurately. Implied volatility, together with sensitivity analysis, as previously mentioned, minimizes replication deviation by enhancing Delta and Gamma estimates as well as re-optimizing the rebalancing path. This finding also confirms H4 and completes a logical chain of transmission with the volatility amplification mechanism observed by H2 [17].

Table 5. Statistical results of Greeks risk exposure differences

Metric	IV	RV	Difference	Significance
Mean hedging error	0.0138	0.0164	-0.0026	***
Mean return	0.0019	0.0014	0.0005	**
Standard deviation of returns	0.0120	0.0151	-0.0031	**

#### 4.5. Analysis of regression results from the econometric model

The econometric model that is developed on Equations (5) to (7) econometrically tests valuation discrepancies across three dimensions: existence, amplification mechanism--source decomposition.

To eliminate duplication, both basic regression and interaction models are tested jointly, and the error decomposition model is tested separately [18].

In the simple panel model (Equation 5), as seen in Table 6, the coefficient of market volatility is 0.083, and it is very positive at the 1% significance level, implying that an increase in implied volatility as compared to historical volatility increases the difference in valuation systematically. This finding directly confirms the H1 difference in valuation hypothesis. The maturity variable coefficient is significant (0.027) and the coefficient of the capital level is significant (0.041), implying that the greater the option proportion, the greater the sensitivity of the notes to volatility inputs and this gives empirical validation to the H3 clause sensitivity hypothesis. The volatility-times-maturity term coefficient in the extended model (Equation 6) is 0.052 and large and positive, indicating that the marginal effects of maturity on the valuation differences are exacerbated by market stress. This finding indicates that volatility deviations not only have a direct impact on option value but they are further enhanced by the time-to-maturity dimension, thus offering direct support of the market state hypothesis in H2. In the meantime, the R<sup>2</sup> of the model is rising to 0.41, meaning that the market state heterogeneity is a major parameter of valuation deviations.

Table 6. Regression results for the panel and interaction models

Variables	Coefficient	t-Statistic	Significance
Level of Market Volatility	0.083	5.43	***
Remaining Maturity	0.027	3.02	**
Maximum Return Level	0.041	3.74	***
Volatility × Maturity	0.052	3.06	**
R <sup>2</sup>	0.41	—	—

Error decomposition model (Equation 7) separates the components of valuation deviations caused by the differences in the h \textit{overall} volatility levels and the differences in volatility surface shape, which helps understand the components of price errors. In Table 7, the coefficient for the difference in volatility levels, which is 0.071 and is statistically significant, implies that the price upward movement is predominantly caused by higher IV compared to RV; the coefficient for skewness of volatility surface is 0.046 and is statistically significant, indicating that the capped structure tends to be more affected by the deep out-of-the-money volatility, therefore, the difference in the surface structure is also an important contributor.

Table 7. Results of the volatility error decomposition model

Variable	Coefficient	t-Statistic	Significance
Difference in Volatility Levels	0.071	5.06	***
Volatility Surface Skewness	0.046	3.87	***
R <sup>2</sup>	0.39	—	—

## 5. Conclusion

Overall, the empirical findings support the research hypotheses based on three dimensions, namely valuation differences, risk exposure, and hedging performance, with consistent findings. To begin with, the average of  $\Delta P_{t,i}$  has a significant positive value, which shows that implied volatility pricing tends to be higher in comparison with historical volatility, which directly supports H1.

Second, differences in valuation become significantly greater in high volatility times, indicating that market stresses increase the gap between IV and RV, which proves H2 to be true. The regression findings indicate that the coefficients on maturity and cap level are significantly positive, and thus, higher exposure to options increases the pricing deviations, which supports H3. In terms of risk exposure,  $\Delta\Delta_{t,i}$  and  $\Delta\Gamma_{t,i}$  will have a strongly positive skewness with growth at the high percentiles, which proves the systematic change in the sensitivity structure, so that H4 is proven.

There are still some limitations of the current analysis. Volatility models are usually founded on the VIX and rolling historical volatility, and not high-frequency realized volatility and stochastic volatility models, which can undermine the description of extreme market conditions; the yield structure is built on an arbitrage-free model and does not explicitly assume credit risk, liquidity spread and transaction costs; the hedging analysis uses a continuous rebalancing assumption and does not account for discrete trading constraints. In future studies, high-frequency data and stochastic volatility models can be used to describe the volatility structure, combine credit risk and transaction frictions into one model, and identify how discrete hedging strategies work in practice.

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