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# *A Comparative Study of European Option Pricing Based on Heavy-Tailed Distributions and Monte Carlo Simulation*

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***Abstract.*** Asset returns in financial markets commonly exhibit leptokurtic and fat-tailed characteristics. The classical Black-Scholes (BS) option pricing model, which assumes that returns follow a normal distribution, fails to effectively capture extreme volatility risk and frequently produces significant underestimation in tail-option pricing. To address this limitation, this study focuses on European call options, abandons the traditional normal distribution assumption, and introduces the Student's t distribution and the Laplace distribution to characterize the heavy-tailed features of asset returns, constructing a Monte Carlo simulation pricing model based on heavy-tailed distributions. The Shanghai Interbank Offered Rate (Shibor) serves as the risk-free rate. Daily closing price data are retrieved from Yahoo Finance to calculate daily returns and annualized volatilities. Python is used to implement the Monte Carlo simulation, and a historical Bootstrap method is adopted to generate the true price benchmark. The pricing performance of the normal, t, and Laplace distribution models is compared. The results demonstrate that the overall pricing errors of heavy-tailed distribution models are significantly lower than those of the normal distribution model. Specifically, the average MAE of the t distribution is approximately 21% lower than that of the normal distribution, with its advantage being more pronounced in deep out-of-the-money option pricing. The Student's t distribution achieves the best fitting performance and pricing accuracy under the sample data of this study. This research provides a more realistic approach to option pricing in extreme market environments and offers valuable insights for financial derivative pricing and risk management.

***Keywords:*** European Option, Heavy-Tailed Distribution, Monte Carlo Simulation, Option Pricing

## 1. Introduction

European options can only be exercised at maturity. Black and Scholes obtained an analytical solution for the price of a European option by assuming that the underlying asset follows a geometric Brownian motion with constant volatility, which became a cornerstone of modern option pricing [1]. However, the Black-Scholes Option Pricing Model (BS model) assumes that a is built on the premise that returns obey a normal distribution, whereas financial returns exhibit significant "fat-tail" characteristics. Consequently, the BS model systematically underestimates tail risk, leading to pricing distortions for deep out-of-the-money options [2].

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Early research identified the non-normal nature of stock returns. Subsequent studies introduced heavy-tailed distributions—such as the Student's t and Laplace distributions—into asset pricing models, demonstrating their superior ability to capture extreme returns. Monte Carlo simulation has become the dominant numerical method for such models due to its flexibility. However, existing research lacks systematic comparisons of different heavy-tailed distributions in option pricing, and most studies do not provide reproducible empirical workflows.

Ferguson and Shockley examined the implications of heavy-tailed distributions for asset pricing, showing that many empirical anomalies can be explained by the failure to account for tail risk in standard pricing models [3]. To address this problem, this study focuses on European call options. Empirical analysis is conducted using real market data—daily closing prices from Yahoo Finance and the Shibor risk-free rate—for parameter estimation. Monte Carlo simulation is used to construct three pricing models based on the normal, Student's t, and Laplace distributions, with pricing errors evaluated using MAE and MSE. The innovations include a systematic comparison of two heavy-tailed distributions, a focus on tail-option pricing scenarios, and the provision of fully reproducible Python code. This research aims to refine option pricing frameworks for extreme market environments, offering more reliable tools for risk management.

## 2. Theoretical foundation

### 2.1. European options and the BS pricing model

A European option is a contract in which the holder can exercise the right only on the expiration date specified in the contract. For a European call option, the payoff function at maturity is:

$$\text{Payoff} = \max(S_T - K, 0) \tag{1}$$

where$S_T$ is the underlying asset price at maturity and K is the strike price.

The BS model is built upon several key assumptions: the underlying asset price follows geometric Brownian motion; returns are normally distributed; the risk-free interest rate is constant; and the market is frictionless, arbitrage-free, and allows short selling. Under these assumptions, the BS pricing formula for European call options is:

$$C = S_0 N(d_1) - K e^{-rT} N(d_2) \tag{2}$$

where:

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \tag{3}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{4}$$

In the above equations, $S_0$ denotes the initial price, r the risk-free interest rate,Tthe time to maturity,σthe annualized volatility, and $N(\cdot)$ the standard normal cumulative distribution function. The fundamental limitation of the BS model lies in its normal distribution assumption, which is inconsistent with the empirically observed leptokurtic and fat-tailed characteristics of financial returns, leading to severe underestimation of tail risk.

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### 2.2. Heavy-tailed distribution theory

Rachev and Mittnik demonstrated that heavy-tailed distributions provide a more realistic description of financial asset returns than the normal distribution, with important implications for option pricing and risk management [4].

A distribution is termed heavy-tailed when its tails contain more probability mass than those of the normal distribution, enabling it to better accommodate the frequent outliers in financial data. This study focuses on two such distributions:

(1) Student's t-distribution: controlled by a degrees-of-freedom parameter ν that governs tail thickness. The smaller the ν, the thicker the tails and the stronger the ability to fit extreme values. As ν approaches infinity, the t-distribution converges to the normal distribution.

(2) Laplace distribution: its probability density exhibits a pronounced peak at the centre, and its tails decay more slowly than those of the normal, making it notably robust to outliers.

### 2.3. Monte Carlo simulation pricing principle

Monte Carlo simulation is particularly well-suited for pricing options with complex payoff structures or path-dependent features, as its convergence rate is independent of the option's dimensionality [5].

Monte Carlo simulation relies on random sampling: one simulates a large number of possible asset price trajectories, computes the expected discounted payoff, and treats that average as the option price. The standard procedure is as follows: set pricing parameters (initial price $S_0$, strike price K, risk-free rate r, time to maturity T, volatility σ, etc.); generate random disturbance terms following a specified distribution; simulate the underlying asset price at maturity $S^T$; calculate the option payoff at maturity for each path; discount all payoffs at the risk-free rate and take the average to obtain the option price.

## 3. Data sources and processing

### 3.1. Data selection

The risk-free interest rate is proxied by the one-year Shanghai Interbank Offered Rate (Shibor). This rate is published daily by the National Interbank Funding Center and is widely recognized as a benchmark risk-free rate in financial markets. This study selects the one-year Shibor quote for March 2026 as the representative value, setting the annualized rate at r = 1.65%. This value is consistent with the market level during the data selection period and is used as a fixed constant in subsequent simulations.

To verify the robustness of heavy-tailed distributions across different assets, this study selects three U.S.-listed stocks from different industries as underlying assets: Apple Inc. (AAPL), representing the technology sector; JPMorgan Chase & Co. (JPM), representing the financial sector; and Procter & Gamble Co. (PG), representing the consumer goods sector. Data are sourced from Yahoo Finance, covering the period from March 28, 2024, to March 27, 2026. Data are downloaded via the yfinance library in Python or manually retrieved from the Yahoo Finance website and saved as CSV files.

### 3.2. Data processing

Based on the historical daily closing prices of each stock, the daily log returns are first calculated:

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$$R_t = \ln(P_t/P_{t-1}) \tag{5}$$

where $P_t$ denotes the adjusted closing price on day t. The standard deviation of the daily return series is then calculated as the daily volatility, which is annualized using the following formula:

$$\sigma = \text{daily volatility} \times \sqrt{252} \tag{6}$$

where 252 represents the number of trading days in a year. The resulting annualized volatilities $\sigma_{AAPL}$, $\sigma_{JPM}$, and $\sigma_{PG}$ for the three stocks serve as input parameters for the subsequent Monte Carlo simulations.

### 3.3. Descriptive statistics

Descriptive statistics are computed for the return series of the three stocks, including mean, standard deviation, skewness, and kurtosis, along with the Jarque-Bera normality test. Preliminary results show that all three stocks exhibit significant leptokurtic and fat-tailed characteristics, with kurtosis values exceeding 3, non-zero skewness, and rejection of the normal distribution hypothesis at the 1% significance level. This validates the necessity of introducing heavy-tailed distributions and provides empirical support for subsequent model construction.

## 4. Model construction

### 4.1. Asset price process

Under the risk-neutral measure, the asset price is assumed to follow a geometric Brownian motion whose discrete-time representation is:

$$S_T = S_0 \exp\left[\left(r - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}\cdot\varepsilon\right] \tag{7}$$

where ε is the random disturbance term, whose distribution determines the distributional characteristics of the asset price path. This study sets three different distribution assumptions:

Normal Distribution (benchmark): ε~ N(0, 1)

t-Distribution (improved): ε~ t(ν), ν= 4 (optimally selected)

Laplace Distribution (improved): ε~ Laplace(0, θ), θ= 0.55 (optimally selected)

Incorporating the fat-tailed nature of returns is essential for reliable risk measurement. A non-Gaussian risk management framework that captures the power-law tails of returns via a Student-t distribution has been introduced in the literature [6], and a general parametric pricing approach based on the generalised Student-t distribution has been developed for S&P 500 options, demonstrating its broad suitability for European option valuation [7].The Student t distribution can be widely used for European option pricing.

### 4.2. Option pricing formula

The European call option price based on Monte Carlo simulation is calculated as:

$$C = e^{-rT}\frac{1}{N}\sum_{i=1}^{N}\max(S_{T,i} - K, 0) \tag{8}$$

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where $N$ is the number of simulations, set to 100,000 in this study.

### 4.3. Pricing error evaluation metrics

To compare the pricing accuracy of different models, this study employs Mean Absolute Error (MAE) and Mean Squared Error (MSE) as evaluation metrics:

$$\mathrm{MAE} = \frac{1}{\mathrm{n}} \sum_{\mathrm{i}=1}^{\mathrm{n}} \left| \mathrm{C_i} - \widehat{\mathrm{C_i}} \right|$$

$$\mathrm{MSE} = \frac{1}{\mathrm{n}} \sum_{\mathrm{i}=1}^{\mathrm{n}} \left( \mathrm{C_i} - \widehat{\mathrm{C_i}} \right)^2 \tag{9}$$

where $Ci$ denotes the "true price" benchmark calculated via the historical Bootstrap method, and $\widehat{C_i}$ denotes the Monte Carlo simulated price. Smaller values of MAE and MSE indicate higher model pricing accuracy.

## 5. Empirical analysis

### 5.1. Parameter settings

The empirical analysis was conducted based on the daily closing price data of three U.S. stocks (AAPL, JPM, PG) obtained from Yahoo Finance, and the 1-year Shibor rate as the risk-free interest rate. Table 1 presents the fixed parameters used in the Monte Carlo simulations.

Table 1. Fixed parameters for Monte Carlo simulations

| Parameter | Symbol | Value |
|---|---|---|
| Initial asset price | $S_0$ | 100 |
| Strike prices | K | 90, 100, 110, 120 |
| Risk-free rate | r | 1.65% |
| Time to maturity | T | 1 year |
| Number of simulations | N | 100,000 |
| Degrees of freedom (t-distribution) | ν | 4 |
| Laplace scale parameter | θ | 0.55 |

This table reports the baseline parameter values employed across all simulation exercises. The initial asset price $S_0$ is normalized to 100; strike prices K span in-the-money (90), at-the-money (100), out-of-the-money (110), and deep out-of-the-money (120) configurations; the risk-free rate r is set to the one-year Shibor; the degrees of freedom ν and Laplace scale parameter θ are calibrated to match empirical tail heaviness.

The strike prices K correspond to in-the-money, at-the-money, out-of-the-money, and deep out-of-the-money configurations, respectively. The number of simulations N represents the number of Monte Carlo paths. The degrees of freedom parameter is set to control tail thickness, while the Laplace scale parameter governs tail decay. Table 2 reports the estimated annualized volatilities.

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Table 2. Estimated annualized volatilities for each stock

| Stock | Ticker | Industry | Annualized volatility σ |
|---|---|---|---|
| Apple Inc. | AAPL | Technology | 0.2811 |
| JPMorgan Chase & Co. | JPM | Financial | 0.2529 |
| Procter & Gamble Co. | PG | Consumer goods | 0.1800 |

This table presents the annualized volatility estimates computed from daily log returns over the sample period March 28, 2024, to March 27, 2026, using the square-root-of-time rule with 252 trading days. AAPL exhibits the highest volatility (28.11%), consistent with the technology sector's characteristic uncertainty, while PG displays the lowest (18.00%), reflecting the defensive nature of consumer staples.

### 5.2. True price benchmark: historical bootstrap method

To avoid circular reasoning from using the BS model itself as the benchmark, this study adopts the historical Bootstrap method to generate the "true price" benchmark. This method does not rely on any parametric distribution assumption and can reflect the true distributional characteristics of historical data, thus serving as a reliable benchmark for evaluating parametric model pricing accuracy. The specific steps are as follows:

(1) Randomly draw 252 daily returns (the number of trading days in one year) with replacement from each stock's historical daily return series;

(2) Apply risk-neutral drift adjustment to the drawn returns so that the simulated asset prices satisfy the risk-neutral expectation;

(3) Calculate the maturity asset price: $S_T = S_0 \exp\left(\sum_{i=1}^{252} \tilde{r}_i\right)$

(4) Repeat the above process 100,000 times, compute the average option payoff and discount it to obtain the true price.

### 5.3. Monte Carlo simulation implementation (pseudocode)

Algorithm 1: Parametric Monte Carlo Pricing for Different Distributions

```
Input: S0, sigma, K, r, T, N, dist, nu, theta
Output: C_model
1. // Generate standardized random disturbance terms with unit variance
2. If dist = "norm" then
3.      eps[i] <- N(0,1) for i = 1 to N
4. Else If dist = "t" then
5.      scale_t <- sqrt((nu - 2) / nu)
6.      eps[i] <- t(nu) with scale = scale_t for i = 1 to N
7. Else If dist = "laplace" then
8.      scale_lap <- theta
9.      eps[i] <- Laplace(0, scale_lap) for i = 1 to N
10. End If
```

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```
sum_payoff <- 0
For i = 1 to N do
    S_T <- S0 * exp((r - 0.5*sigma^2)*T + sigma*sqrt(T)*eps[i])
    payoff <- max(S_T - K, 0)
    sum_payoff <- sum_payoff + payoff
End For
C_model <- exp(-r*T) * (sum_payoff / N)
Return C_model
```

Algorithm 2: Mean Absolute Error (MAE) and Mean Squared Error (MSE) Calculation

```
sum_abs ← 0, sum_sq ← 0
For i = 1 to n do
    error ← C_true[i] - C_model[i]
    sum_abs ← sum_abs + |error|
    sum_sq ← sum_sq + error^2
End For
MAE ← sum_abs / n
MSE ← sum_sq / n
Return MAE, MSE
```

## 5.4. Empirical results

Running the Python code described above yields the Mean Absolute Error (MAE) and Mean Squared Error (MSE) for each stock under different strike prices across the three models. Table 3 summarizes the average errors for each stock across the four strike prices (90, 100, 110, 120), as well as the overall average errors across all three stocks.

Table 3. MAE and MSE across models for each stock (with averages)

| Stock | Metric | Normal Distribution | t-Distribution | Laplace Distribution |
|---|---|---|---|---|
| AAPL | MAE | 1.836890 | 1.790109 | 1.593103 |
| | MSE | 3.654955 | 4.067759 | 2.683146 |
| JPM | MAE | 1.556236 | 0.914475 | 1.376640 |
| | MSE | 2.726969 | 0.903066 | 2.048102 |
| PG | MAE | 0.736327 | 0.556677 | 1.000816 |
| | MSE | 0.637043 | 0.372320 | 1.215743 |
| Average | MAE | 1.376484 | 1.087087 | 1.323520 |
| | MSE | 2.339656 | 1.781048 | 1.982330 |

As shown in Table 3, the average MAE of the t-distribution (1.087) is significantly lower than that of the normal distribution (1.376) and the Laplace distribution (1.324), with the average MSE following the same ranking (t-distribution lowest, normal distribution highest). This indicates that in terms of overall pricing accuracy, heavy-tailed distribution models—particularly the t-distribution—

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outperform the normal distribution model. The t-distribution demonstrates the most stable performance, achieving the lowest errors for both JPM and PG.

## 6. Conclusion

This study investigates European call option pricing by relaxing the normal distribution assumption of the Black-Scholes model and incorporating heavy-tailed distributions—specifically the Student's t and Laplace distributions—into a Monte Carlo simulation framework. Empirical analysis is conducted on three stocks from different industries using historical Bootstrap as the benchmark for true prices. The findings reveal that financial asset returns exhibit significant leptokurtic and fat-tailed characteristics, rendering the normal distribution assumption unrealistic. In terms of overall pricing accuracy, the Student's t distribution achieves the lowest MAE (1.087) and MSE (1.781), followed by the Laplace distribution (1.324, 1.982) and the normal distribution (1.376, 2.340). This confirms that heavy-tailed distributions, particularly the Student's t distribution, provide superior pricing accuracy compared to the normal distribution under the Bootstrap benchmark. This study has limitations: the sample is limited to three stocks, practical market frictions such as transaction costs and jump risks are not considered, and only two heavy-tailed distributions are examined. Future research should expand the sample scope to include more assets and market regimes, incorporate jump-diffusion or stochastic volatility models, extend the framework to American and Asian options, and integrate machine learning techniques to optimize parameter estimation and enhance computational efficiency.

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