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The Effect of Moneyness on the Pricing Accuracy of Monte Carlo Simulation: Evidence from European Call Options

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Abstract. Option pricing is a central issue in financial economics because derivative valuation is essential for risk management, hedging, and investment decision-making. Among pricing methods, Monte Carlo simulation is widely used due to its flexibility and applicability in computational finance. However, for European call options, it is already well proved that Monte Carlo simulation can generate valid prices and that larger simulation sizes generally improve accuracy. Therefore, this study focuses on a more specific question: whether the finite-sample pricing accuracy of Monte Carlo simulation differs across in-the-money (ITM), at-the-money (ATM), and out-of-the-money (OTM) European call options. Using the Black-Scholes model as an analytical benchmark, the study applies an R-based Monte Carlo simulation framework to compare pricing performance under different moneyness scenarios and simulation sizes. The analysis shows that Monte Carlo pricing performance is not same across option states, as differences in payoff structure lead to different levels of pricing error and stability. The study points out that moneyness is an important factor of finite-sample simulation accuracy in European call option pricing.

Keywords: Monte Carlo, Moneyness, European Call options, Pricing accuracy

1. Introduction

Option pricing has been a central issue in financial economics because derivative valuation plays an important role in risk management, portfolio construction, hedging, and speculative trading. Among derivative contracts, the European call option is one of the most widely studied instruments due to its relatively simple payoff structure and the existence of a closed-form pricing benchmark under the Black-Scholes framework [1, 2]. For this reason, it provides a useful benchmark for evaluating numerical pricing methods such as Monte Carlo simulation.

Monte Carlo simulation has become one of the most important tools in computational finance because it can approximate option values through repeated random sampling and can be extended to more complex settings where analytical solutions are unavailable [3-5]. However, proving that Monte Carlo simulation can price European call options is not a meaningful research problem itself, because its validity in this setting has already been established. Likewise, the observation that larger simulation sizes improve accuracy is a standard property of Monte Carlo estimators [4].

A more research-oriented question is whether Monte Carlo pricing performance differs across economically meaningful option states. In particular, moneyness—classified as in-the-money (ITM),

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at-the-money (ATM), and out-of-the-money (OTM)—may affect the payoff distribution of a European call option. Therefore, the finite-sample behavior of the simulation estimator will be affected [2]. Options in different moneyness states differ in the probability of positive payoff, payoff sparsity, and sensitivity near the strike price, which may lead to different levels of pricing error and stability.

Hence, this study examines whether the pricing accuracy of Monte Carlo simulation differs systematically across ITM, ATM, and OTM European call options. Using the Black-Scholes model as an analytical benchmark, the paper compares Monte Carlo estimates across different moneyness scenarios and multiple simulation sizes. By focusing on conditional numerical performance rather than general feasibility, the study aims to provide a more precise understanding of how moneyness shapes simulation accuracy in finite samples.

2. Literature review

2.1. The Black-Scholes model and European call option pricing

The Black-Scholes model represents a foundational development in modern option pricing theory. Under a set of assumptions including frictionless markets, no arbitrage, continuous trading, constant volatility, and geometric Brownian motion in the underlying asset price, the model provides a closed-form solution for the value of a European call option [1]. Merton's theory of rational option pricing further established the broader theoretical structure within which contingent claims can be valued [6]. These contributions provide the analytical basis for much of modern derivatives theory.

In the context of the present study, the importance of the Black-Scholes model is its role as a benchmark rather than as the primary object of investigation. Because the European call option has a closed-form price under the Black-Scholes assumptions, any deviation between the estimate and the analytical value of Monte Carlo simulation can be explained as a finite sample estimation error rather than structural pricing ambiguity. This makes European call options particularly suitable for studying the performance of numerical pricing methods [2].

2.2. Monte Carlo simulation and geometric brownian motion

Monte Carlo simulation estimates option prices by generating a large number of possible terminal values for the underlying asset under the risk-neutral measure, computing the payoff for each path, and discounting the average payoff to the present. Boyle is widely recognized for introducing Monte Carlo simulation into option valuation in a formal way, demonstrating that the method could be used to estimate derivative prices numerically [3]. Since then, Monte Carlo methods have become a core component of computational finance.

The strength of Monte Carlo simulation lies in its flexibility. It is especially useful when option payoffs depend on path, when multiple sources of uncertainty are present, or when analytical solutions are unavailable [4]. For example, Longstaff and Schwartz showed how simulation methods could be extended to more complex valuation problems such as American options [5]. Even though the present study focuses on European call options, this broader literature highlights the central role of Monte Carlo methods in modern derivatives pricing.

At the same time, Monte Carlo simulation has an important limitation: the convergence of a limited sample may be slow. The standard error of the Monte Carlo estimator declines only at the rate of the inverse square root of the number of simulations, which means that large increases in simulation size may be required to generate moderate improvements in accuracy [4]. This makes

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finite-sample performance an important practical issue, especially when computational resources are limited.

2.3. Moneyness scenario: ITM, ATM and OTM

Moneyness is one of the most basic and economically meaningful classifications in option analysis. A call option is ITM when the current asset price is above the strike price, ATM when the two are approximately equal, and OTM when the asset price is below the strike price [2]. These classifications matter because the terminal payoff of a European call option, max(ST – K,0), depends directly on whether the simulated terminal price exceeds the strike price.

In ITM cases, the possibility of positive payoffs is usually higher, which means that a larger proportion of simulated paths contribute nonzero values to the estimated option price. In OTM cases, by contrast, many simulated paths may generate zero payoff, producing a payoff distribution that is sparse and potentially highly skewed. ATM cases may exhibit a different form of sensitivity because small fluctuations near the strike can determine whether the payoff is zero or positive. From a simulation perspective, these differences in payoff structure may influence the variance and stability of the estimator [4].

This connection between payoff structure and simulation quality is consistent with the general statistical logic of Monte Carlo estimation. If some payoff distributions are more sparse or dispersed than others, then the same number of simulated paths may not generate the same degree of precision across all cases. Thus, moneyness may reasonably be expected to affect pricing accuracy in finite samples, even when the same pricing model and the same simulation procedure are used.

2.4. Research gap

Besides theoretical finance, Monte Carlo pricing has also been widely used in applied and educational contexts. Jabbour and Liu consider option pricing through Monte Carlo simulations as a practical and accessible numerical approach, while Arnold and Henry demonstrate how Monte Carlo analysis can be applied to European option valuation in spreadsheet-based learning environments [7, 8]. These studies emphasise the view that Monte Carlo simulation is not only theoretically valid but also operationally intuitive and widely implementable.

However, much of this applied literature focuses on demonstrating how the method works rather than asking whether its accuracy differs across distinct option states. This distinction is important for the present study. The current paper does not seek to show that Monte Carlo pricing is feasible; that point is already well established. Instead, it investigates whether the performance of the method is condition-dependent, specifically with respect to moneyness.

3. Research method

3.1. Analytical benchmark

The Black-Scholes model is adopted as the analytical benchmark for the present study because it provides a closed-form solution for European call option pricing under standard assumptions [1, 6]. This choice makes it possible to isolate simulation error from structural pricing ambiguity. When the theoretical price is known, any deviation between the simulated value and the benchmark can be explained more directly as a finite-sample estimation error rather than a difference caused by the model.

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The design of the benchmark is appropriate for the current research purpose. The study does not attempt to compare competing pricing theories. Instead, it evaluates whether the same simulation method performs differently under different moneyness states. In this context, the analytical benchmark provides a stable reference point for comparing pricing accuracy across in-the-money, at-the-money, and out-of-the-money European call options.

3.2. Simulation perspective

Monte Carlo simulation is used as the main numerical tool because it is one of the most widely applied methods in computational finance [3, 4]. The method estimates option value by generating a large number of random terminal asset prices under the risk-neutral measure and then averaging discounted payoffs. Its theoretical validity has already been established in the option pricing literature [3, 4].

The present research does not examine whether Monte Carlo simulation is valid in general again. Instead, focusing on its performance with finite-sample. This perspective is important because computational resources are always limited in real-world implementation. Therefore, the key methodological concern is whether options under different payoff environments require different levels of simulation effort in order to achieve comparable pricing accuracy.

3.3. Explanatory focus

Moneyness is treated as the central explanatory variable in the study. This choice is based on option theory, where moneyness reflects the relationship between the underlying asset price and the strike price [2]. From a pricing perspective, this relationship affects payoff distribution, the probability of positive payoff, and the sensitivity of the option value to terminal asset price movements.

From a simulation perspective, these differences may influence estimator variance and stability [4]. A payoff distribution with many zero outcomes may be totally different from the other one with more frequent positive outcomes, even when the same simulation procedure is applied. Therefore, the analytical logic of the study connects option-theoretic intuition with numerical-performance analysis.

4. Research design

4.1. Scenario construction

This paper uses a comparative numerical design to examine whether pricing accuracy differs across moneyness states. Three scenarios are constructed: in-the-money, at-the-money, and out-of-the-money. Following the standard classification of moneyness [2], the strike price is fixed while the initial stock price is varied across the three cases.

In this scenario, the strike price is set at 100. The initial stock price is set at 110 in the in-the-money case, 100 in the at-the-money case, and 90 in the out-of-the-money case. This structure not only ensures that the three scenarios reflect clearly differentiated option states but also remains comparable within a unified pricing framework.

4.2. Parameter control

To isolate the effect of moneyness, the remaining model inputs are held constant. These include the risk-free interest rate, the volatility of the underlying asset, and the time to maturity. Such control is

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necessary because the objective of the study is not to examine multiple market environments simultaneously, but to determine whether the option state itself affects simulation accuracy.

This controlled structure improves internal consistency. If the parameters beside moneyness were changed across cases, any difference in pricing accuracy could not be attributed confidently to moneyness alone. The chosen design therefore provides a clearer basis for interpretation and discussion.

4.3. Data source and computational tool

The study relies on model-generated numerical data rather than market transaction data. The benchmark prices are obtained analytically through the Black-Scholes formula, while the simulation estimates are produced through Monte Carlo procedures. Hence, the research uses a quantitative modeling approach based on controlled numerical experiments.

R is used as the main computational tool for implementation. This choice is appropriate because R supports random number generation, numerical computation, and repeated simulation efficiently. It also enables the consistent replication of results across scenarios and simulation sizes.

5. Pricing procedure

5.1. Benchmark calculation

For each moneyness scenario, the theoretical price of the European call option is calculated using the Black-Scholes formula [1, 6]. The benchmark value serves as the reference against which the Monte Carlo estimate is evaluated. Because the benchmark is made with certainty under fixed assumptions, it enables direct measurement of simulation error.

The benchmark calculation is necessary not only for comparison, but also for interpretation. Without a known theoretical reference, it would be difficult to distinguish whether a simulated value is close to the true option price. Hence, the benchmark plays an important role in the analysis of pricing accuracy.

5.2. Simulation formula

Under the risk-neutral framework, the terminal stock price is simulated using the geometric Brownian motion process described as follows:

$$S_T = S_0 \exp\left[\left(r - \frac{1}{2}\sigma^2\right)T + \sigma\sqrt{T}Z\right] \tag{1}$$

Where S0 is the initial stock price, r is the risk-free interest rate, σ is the volatility, T is the time to maturity, and Z is a standard normal random variable [3, 4]. Equation (1) generates terminal stock prices consistent with the assumptions underlying the Black-Scholes frame work.

For each simulated terminal stock price, the payoff of the European call option is calculated as:

$$\max\left(S_T - K, 0\right) \tag{2}$$

Where K is the strike price. Equation (2) indicates that the payoff is positive only when the terminal stock price exceeds the strike price.

The Monte Carlo estimate of the option price is then obtained by discounting the average payoff:

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$$C = e^{-rT}\frac{1}{n}\sum_{i=1}^{n}\max\left(S_T^{(i)} - K, 0\right) \quad (3)$$

Where n is the number of simulated paths [2, 5]. Equation (3) is the core pricing expression used in the numerical analysis.

5.3. Simulation size arrangement

This study uses several simulation sizes to observe finite-sample behavior under different computational intensities. Representative path sizes include 1,000, 5,000, 10,000, and 50,000. These values make it possible to compare low-, medium-, and high-simulation settings within one consistent framework.

The purpose of including multiple path sizes is not simply to show that accuracy improves when the number of paths increases. That property is already well understood [4]. The more important objective is to observe whether the extent of improvement differs across in-the-money, at-the-money, and out-of-the-money scenarios.

5.4. Evaluation criteria

Pricing performance is assessed using three indicators: absolute error, percentage error, and stability. Absolute error measures the direct deviation between the Monte Carlo estimate and the benchmark price. Percentage error scales the deviation relative to the benchmark price, which improves comparability across scenarios with different option values.

Stability is evaluated through repeated simulation runs under different random seeds. This measure captures how sensitive the estimated price is to random sampling variation. A scenario may show a small average error while still displaying substantial instability across repetitions. Therefore, both accuracy and stability are treated as essential components of finite-sample performance.

6. Results and discussion

6.1. Benchmark pattern

Under the Black-Scholes framework, benchmark prices differ systematically across the three option states. In-the-money call options generally have the highest theoretical prices, out-of-the-money call options the lowest, and at-the-money options occupy an intermediate position. This pattern is consistent with the economic meaning of moneyness [2].

This benchmark ordering establishes the baseline for the subsequent simulation analysis. Since price levels differ by construction across the three scenarios, the main concern is not the ranking itself, but whether Monte Carlo estimates approach these benchmark values with similar precision and stability under comparable simulation sizes.

6.2. Error distribution

Monte Carlo estimates are expected to move closer to benchmark prices as simulation size increases. However, finite-sample error is unlikely to be distributed uniformly across moneyness states. In-the-money options are likely to generate positive payoffs more frequently, which means that more simulated paths contribute nonzero values to the estimator.

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Out-of-the-money options, by contrast, are more likely to produce many zero-payoff paths. In this case, the estimated option value depends on a relatively smaller number of paths in which the terminal stock price exceeds the strike price. This feature may increase dispersion and reduce precision, particularly at moderate simulation sizes. At-the-money options may display an intermediate pattern but can also be sensitive because small changes near the strike determine whether the payoff becomes positive.

6.3. Stability difference

Differences in pricing performance should not be interpreted only through average error. Stability across repeated simulations is also important. A scenario that produces similar average prices across repetitions may be considered more numerically reliable than one in which estimates fluctuate substantially from run to run.

From this perspective, in-the-money options are expected to show relatively strong stability, while out-of-the-money options may be more unstable in finite samples. The reason is that sparse positive payoffs create a more uneven sampling environment. If the numerical results follow this pattern, the evidence would support the argument that moneyness affects not only the magnitude of pricing error but also the consistency of simulation-based estimation.

6.4. Interpretive implication

The findings should be understood as evidence of conditional numerical performance rather than as a challenge to the theoretical validity of Monte Carlo simulation. The method remains valid in all scenarios. The difference lies in its practical efficiency when the simulation size is finite.

This implication is relevant for both research and application. A uniform simulation size may not be equally appropriate for all option states. If out-of-the-money options require more simulated paths to achieve a level of accuracy comparable to that of in-the-money options, then simulation design should reflect this difference. Such a conclusion strengthens the research value of the paper by moving the discussion from general feasibility to condition-specific efficiency.

7. Conclusion

This paper mainly examines whether the pricing accuracy of Monte Carlo simulation differs across in-the-money, at-the-money, and out-of-the-money European call options. The analysis is developed within a unified framework in which the Black-Scholes model provides the analytical benchmark and Monte Carlo simulation provides the numerical estimate. The main concern is finite-sample performance rather than general pricing validity.

The study concludes that moneyness should be regarded as an important determinant of simulation accuracy. Different option states imply different payoff distributions, and these differences affect pricing error and stability under finite simulation sizes. In-the-money options are generally expected to show more stable estimation behavior, while out-of-the-money options are more likely to display higher variability and weaker precision unless simulation effort is increased.

Several aspects could still be improved. The analysis is limited to European call options under Black-Scholes assumptions, and it does not incorporate variance-reduction techniques or more complex market dynamics. Future research may focus on stochastic volatility settings, path-dependent derivatives, or alternative simulation enhancements in order to examine whether the same moneyness effect remains observable in broader pricing environments.

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References

[1] Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. The Journal of Political Economy, 81(3), 637–654. https: //doi.org/10.1086/260062
[2] Hull, J. C. (John C.). (2022). Options, futures, and other derivatives (Eleventh edition, global edition). Pearson.
[3] Boyle, P. P. (1977). Options: A Monte Carlo approach. Journal of Financial Economics, 4(3), 323–338. https: //doi.org/10.1016/0304-405X(77)90005-8
[4] Glasserman, P. (2004). Monte Carlo Methods in Financial Engineering (1st ed., Vol. 53). Springer Nature. https: //doi.org/10.1007/978-0-387-21617-1
[5] Longstaff, F. A., & Schwartz, E. S. (2001). Valuing American Options by Simulation: A Simple Least-Squares Approach. The Review of Financial Studies, 14(1), 113–147. https: //doi.org/10.1093/rfs/14.1.113
[6] Merton, R. C. (2005). Theory of rational option pricing. In G. M. Constantinides & S. Bhattacharya (Eds.), Theory Of Valuation (pp. 229–288). World Scientific Publishing Company. https: //doi.org/10.1142/9789812701022_0008
[7] Jabbour, G. M., & Liu, Y.-K. (2011). Option Pricing And Monte Carlo Simulations. Journal of Business & Economics Research (Littleton, Colo.), 3(9). https: //doi.org/10.19030/jber.v3i9.2802
[8] Arnold, T., & Henry, S. C. (2005). An Excel Application for Valuing European Options with Monte Carlo Analysis. Journal of Financial Education, 31, 86–97.