

CAPM, Fama-French, and APT in Portfolio Optimization: A Comparative Empirical Study

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Abstract. This paper empirically compares the application of the Capital Asset Pricing Model, Fama-French Five-Factor Model, and Arbitrage Pricing Theory in portfolio optimization under the mean-variance framework. Based on the selected representative stock samples, this study estimates the expected returns of each model through time series regression and accordingly constructs the minimum variance combination and tangent combination. Results show a significant divergence between models' in-sample fitting ability and out-of-sample investment performance. The FF5 model has the highest explanatory power, but its scattered expected returns lead to unstable, highly concentrated portfolio weights. In contrast, the expected returns generated by the APT model are more stable, and the investment portfolio constructed performs best outside the sample, with higher risk-adjusted returns and better downside risk control capabilities. CAPM models, due to their single-factor structure, although the results are relatively stable, have limited information content. Overall, the actual value of an asset pricing model does not lie in its statistical goodness of fit, but in whether it can provide a stable and reliable estimate of expected returns. Among the compared models, APT achieved a better balance between flexibility and robustness, making it more suitable for actual portfolio optimization.

Keywords: CAPM, Fama-French Five-Factor Model, Arbitrage Pricing Theory, Portfolio Optimization.

1. Introduction

Portfolio allocation represents one of the core issues in finance. Investors need to distribute wealth among risky assets and conduct trade-offs between expected returns and risks. The modern research foundation of this problem mainly comes from the mean variance framework, which formalizes portfolio selection as a trade-off between expected returns and variance [1]. However, further research suggests that the actual effectiveness of portfolio optimization largely depends on the quality of expected return estimates. The weight of the mean variance effective combination is usually sensitive to small changes in expected asset returns [2]. Therefore, the estimation of expected returns is the key issue in determining whether the results are optimized.

In this context, the significance of asset pricing models lies not only in explaining differences in asset returns, but also in their ability to provide estimated expected returns for portfolio optimization. CAPM provides a clear single factor benchmark model that suggests that the expected

return of an asset primarily depends on its market systemic risk [3]. APT further emphasizes that multiple systemic risk factors may jointly affect asset returns [4]. Subsequently, Fama French further expanded the factor pricing framework, pointing out that factors such as size, book to market ratio, profitability, and investment level are more effective in explaining the average return of stocks than individual market factors [5]. These findings indicate that traditional single factor models have limited explanatory power in real markets, while multi factor models have stronger applicability in displaying asset return characteristics. In recent years, related research has further expanded from traditional linear models to higher dimensional and more realistic investment environments to improve expected return estimates. This development emphasizes the continued importance of improving the estimation of expected returns in financial theory and practice.

Although these models are theoretically attractive, there are still critical gaps in understanding the correlation between asset pricing model selection and actual investment portfolio decisions. Specifically, this study addresses the following research question: How do different asset pricing models, including CAPM, Fama French factor model, and APT, affect expected return estimation and portfolio optimization results? To answer this question, the study will compare the expected return estimates generated by these three models and investigate how different model structures affect portfolio weights, risk return trade-offs, and the stability of the optimal investment portfolio. The main contribution of this study is to directly link asset pricing theory with portfolio construction, systematically evaluating the applicability and reliability of different return estimation frameworks in actual asset allocation, thereby providing more practical theoretical support for quantitative investment and modern asset management.

2. Theoretical framework and literature review

Modern portfolio optimization theory formalizes the intuitive principle of diversification into quantifiable investment standards [6]. Its core contribution is to shift asset valuation from an individual perspective to a portfolio perspective: the total risk of an investment portfolio is determined by the covariance of asset returns, rather than the simple sum of individual risks [7]. This makes portfolio selection a systematic task of balancing overall expected returns and total risk, and the derived effective boundary serves as a fundamental reference for optimal allocation.

Subsequent research has shifted from effective cutting-edge mathematical derivation to key issues in fundamental parameter estimation [8]. The optimal portfolio weights, especially in unconstrained environments, are highly sensitive to small expected return estimation errors. To address this issue, scholars have developed Bayesian estimation, shrinkage methods, and robust optimization techniques, all aimed at mitigating the impact of estimation errors on optimization results [9]. There is a consistent conclusion in portfolio theory literature that portfolio optimization is essentially a parameter estimation problem [10]. If there is no reliable return prediction, the optimal investment portfolio derived by the model lacks practical application value in actual investment scenarios. CAPM is based on market equilibrium and risk diversification, assuming that rational investors have homogeneous expectations in an efficient market. It decomposes risk into diversifiable unsystematic risk and non-diversifiable systematic risk, arguing that only systematic risk merits return compensation. CAPM Model is as Model (1):

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f] \quad (1)$$

Where $E(R_i)$ is asset i 's expected return, R_f is the risk-free rate, β_i is the market beta, $E(R_m)$ is the market portfolio's expected return, $(E(R_m) - R_f)$ is the market risk premium.

Despite its role as a pricing benchmark, CAPM's empirical performance is flawed. The single market factor fails to explain cross-sectional stock return anomalies, where small-cap and high book-to-market stocks outperform CAPM's theoretical predictions. This single-factor framework cannot fully capture real-world return drivers, creating a need for a more comprehensive factor structure.

Rooted in empirical facts and risk compensation, Fama-French models add firm characteristic factors to CAPM's market factor. Each factor represents un-diversifiable systematic risk: small-cap firms face higher liquidity/financing risk, value stocks bear fundamental distress risk, high-profit and conservative-investment firms have distinct risk profiles. An asset's expected excess return is the sum of compensation for its exposure to all these systematic risks. The Fama-French Five-Factor Model is as Model (2):

$$E(R_i) - R_f = \alpha + \beta_i M [E(R_m) - R_f] + \beta_i SMBE(SMB) + \beta_i HMLE(HML) + \beta_i RMW E(RMW) + \beta_i CMAE(CMA) \quad (2)$$

Where SMB is a size factor, HML is a value factor, RMW is a profitability factor, CMA is an investment factor, $\beta_i M$, $\beta_i SMB$, $\beta_i HML$, $\beta_i RMW$, $\beta_i CMA$ are factor loadings, $E(R_m) - R_f$, SMB, HML, RMW, CMA are factor risk premiums.

The Fama French model extends from a single factor framework to a multi factor framework, explaining the return anomalies that CAPM cannot explain and capturing more cross-sectional return driving factors. It refines risk measurement by decomposing system risk into market risk and enterprise characteristic risk, improves the accuracy of expected return estimation by aligning with real market return patterns, and provides more stable parameters for portfolio optimization. However, this model has obvious limitations: factor selection relies on empirical induction, it has inherent factor redundancy, and factor effects exhibit significant market heterogeneity, highlighting the urgent need for a more universal multi factor pricing theoretical framework. In this context, arbitrage pricing theory elevates multi factor pricing from empirical induction to theoretical derivation, laying a rigorous and universal theoretical foundation for all multi factor models.

APT assumes asset returns are driven by limited orthogonal common systematic risk factors, with idiosyncratic risk being diversifiable. In an arbitrage-free market, an asset's expected return has a linear relationship with its factor loadings—higher exposure to a premium factor leads to higher expected return. APT does not predefine factor number/type, leaving flexibility for empirical factor identification. APT Model is as Model (3):

$$E(R_i) = R_f + \beta_{i1}\lambda_1 + \beta_{i2}\lambda_2 + \dots + \beta_{ik}\lambda_k \quad (3)$$

Where: k is the number of common systematic risk factors, β_{ij} is asset I's loading on factor j, λ_{jis} factor j's risk premium.

The estimation of expected returns remains a long-standing and unresolved bottleneck in portfolio optimization, seriously undermining the practical utility of the Markowitz mean variance framework. Each classic asset pricing model has its unique advantages and inherent limitations: CAPM provides a concise single factor benchmark, but has weak empirical explanatory power. The Fama French multifactor framework captures a wider range of return patterns, but it suffers from factor redundancy, cross market heterogeneity of factor effects, and weak theoretical foundations. Arbitrage pricing theory provides a flexible and universal theoretical structure for multi factor pricing, although it is only a theoretical foundation and not a direct empirical solution. Although existing literature has extensively explored individual asset pricing models and portfolio parameter

estimation, there is a lack of systematic comparative empirical analysis on how model selection affects the accuracy of expected return estimation and portfolio optimization results - this is the core research gap that this study attempts to address. For portfolio optimization applications, the value of pricing models lies in their ability to generate stable and reliable estimates of expected returns. Therefore, comparative empirical analysis of CAPM, Fama French models, and APT is not only necessary in theory, but also relevant to quantitative investment and modern asset management in practice.

3. Methodology

In order to improve the comparability of results from different asset pricing models, this study adopted an industry focused sampling method. The sample is not randomly selected from the entire market, but from three related industries: agricultural and sideline food processing industry, capital market services, and software and information technology services. Within each department, the sample was narrowed down to 30 representative companies with complete return history and factor data, resulting in an initial total sample of 90 companies. This method has two key advantages. Firstly, it mitigates errors driven by excessive cross industry effects, ensuring that differences in estimated expected returns are more likely to reflect the model structure. Secondly, it makes portfolio results more interpretable in practice, as investors typically allocate capital within relatively concentrated niche markets rather than across the entire market. Firms with missing observations, abnormal data gaps or incomplete factor exposures are excluded after initial filtering. This study is based on 90 samples. For the convenience of reporting, some graphs are of 10 samples. For each stock, the dataset includes the return sequence, daily and monthly market return information, risk-free rate, and the factor sequence required by CAPM, the Fama-French five-factor model, and the APT model.

The empirical analysis is carried out in three steps. First, time-series regression is conducted to estimate parameters for each model. CAPM links the excess returns of stocks with those of the market and generates a single market beta. The Fama-French Five-factor Model extends this framework by including market factor, SMB, HML, RMW, and CMA, thereby allowing expected return estimates to reflect size, value, profitability, and investment effectiveness. The APT model adopted in the study selects factors through a data-driven process. After extracting the common risk factors through the principal component analysis, this study incorporated market factors and two other factors of practical significance: random volatility and residual returns into the estimation process, based on their empirical explanatory power of stock returns. For each stock, the estimation coefficient and intercept are used to derive the implied expected return of the model. Secondly, these expected returns are incorporated into the mean-variance optimization framework. Two portfolios are constructed for each model: the minimum variance portfolio and the tangency portfolio. The minimum variance portfolio minimizes the risk of the portfolio with a given covariance structure, while the tangent portfolio maximizes the Sharpe ratio. Therefore, when optimizing to determine the hedging effect of certain assets, negative weights may occur. Thirdly, evaluate the result combinations based on the equal-weighted benchmarks in out-of-sample tests.

The performance evaluation combines model fitting and portfolio results. At the stock level, the main indicators are R-squared, estimated factor load, and the expected return implied by the model. At the portfolio level, the focus of the analysis is on the annual rate of return, annual volatility, Sharpe ratio, Sortino ratio, and maximum drawdown rate, as well as the cumulative return path. This study uses daily and monthly stock return data from the CSMAR database covering January 2022 to January 2026, and 1-year Chinese treasury bond yield as the risk-free rate, with an 80% in-sample

(2022-01 to 2024-12) and 20% out-of-sample (2025-01 to 2026-01) split for model estimation and portfolio backtesting. This combination is important because the model can well explain the returns within the sample, but it still generates unstable portfolio weights or weak risk-adjusted performance outside the sample. To deepen the comparison, the Sharpe ratio differences were also reported. In this way, this method not only judges the models through statistical fitting, but also by whether they can generate useful expected return estimates for the construction of actual investment portfolios.

4. Empirical results

The first group of results compares the statistical fit and implied expected returns of the three models. The FF5 model generates the strongest in-sample fit overall. As seen in Fig.1, R-squared values of the FF5 model are typically higher than those of the CAPM and slightly higher than those of the APT for most stocks. The value range of CAPM R^2 is approximately 0.73 to 0.96, that of FF5 is approximately 0.84 to 0.97, and that of APT is approximately 0.78 to 0.97. This improvement is particularly evident for stocks with relatively weak CAPM fit, such as S05 and S07. This indicates that a more diverse factor structure indeed captures a larger share of the changes in returns. Meanwhile, the market beta values estimated by CAPM and FF5 are roughly similar, which means that the additional FF5 factor does not replace market risk, only providing other sources of risk.

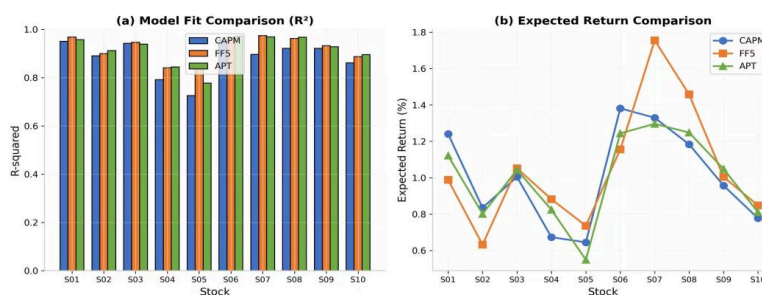


Figure 1. In-sample fitting performance and expected return estimates (Picture credit: Original)

Estimates of expected returns present meaningful differences. Average expected returns across models are close at approximately 1.00%, but dispersion levels differ significantly. The standard deviation of the expected return of FF5 is the highest, approximately 0.32, and the maximum expected return of FF5 is also much higher, approximately 1.76%. In other words, FF5 has produced a more positive cross-sectional differentiation, identifying some particularly attractive stocks. In contrast, APT has produced the most compressed return distribution, indicating that the ranking of opportunities is more moderate. For the minimum variance portfolio, the weights of CAPM and FF5 in this sample are the same, as the minimum variance portfolio mainly relies on the covariance structure rather than the differences in expected return predictions. Both models assign significant positive weights to stocks including S04, S05 and S10, and negative weights to S01, S06, S07 and S08. The general shape of the APT minimum variance portfolio is similar, but not the same. It places more weight on S05 and slightly reduces the positive position of S09.

In a tangent portfolio, this contrast becomes even more intense, where the expected return valuation is very important. The CAPM tangent portfolio is relatively moderate, with positive weights distributed in most stocks and no particularly large risk exposure. However, the FF5 tangent portfolio is highly concentrated with a high leverage ratio. It has large positive positions in S04, S07, S08, and S09, and also has large negative positions in S01, S02, S05, and S06. This model reflects the wide dispersion of the implied expected returns of FF5. In contrast, the APT tangent

investment portfolio is more balanced. Although it also includes short positions, the leverage is much lower than that of FF5, and the final result is not dominated by a few factors like FF5. For the FF5 tangent portfolio, although it has the highest annual return rate, approximately 12.82%, it is accompanied by extremely high volatility, about 33.55%, with a maximum decline of approximately -43.12%. Its Sharpe ratio is only 0.307, the lowest among all tangent investment portfolios and lower than the benchmark of the same weight. Therefore, FF5 is clearly more in line with the cross-sectional area within the sample, but the large spread of the predicted returns has led the optimizer to generate excessive long and short positions. Once the samples are tested, these radical weights will become expensive. This result supports that better explanatory power does not necessarily mean better portfolio performance.

As seen in Fig.2, out-of-sample results constitute the most critical segment of the analysis. These results validate whether a stronger in-sample fit translates into improved portfolio performance. The APT tangent investment portfolio has the best overall risk adjustment effect, with an annual return rate of approximately 11.07%, an annual volatility of 12.58%, and a Sharpe ratio of 0.677, which is the highest among all strategies. Its maximum drawdown is only -6.88%, which is also one of the smallest values in the sample. The cumulative return chart shows a relatively stable upward path. This makes the APT tangent perform the strongest when both returns and downside controls are taken into account. The performance of the APT minimum variance combination is slightly better than that of the CAPM and FF5 minimum variance solutions. The Sharpe ratio is approximately 0.419, with a relatively small decrease of about -6.33%. This indicates that even if the expected return forecast is not used as actively as the tangential scenario, the APT framework still supports slightly better defensive configurations. FF5 has the best explanatory power and the highest average expected return, but its tangent portfolio is too extreme and performs poorly on a risk-adjusted basis. Empirical evidence strongly indicates that the APT tangent is the most attractive strategy in this sample.

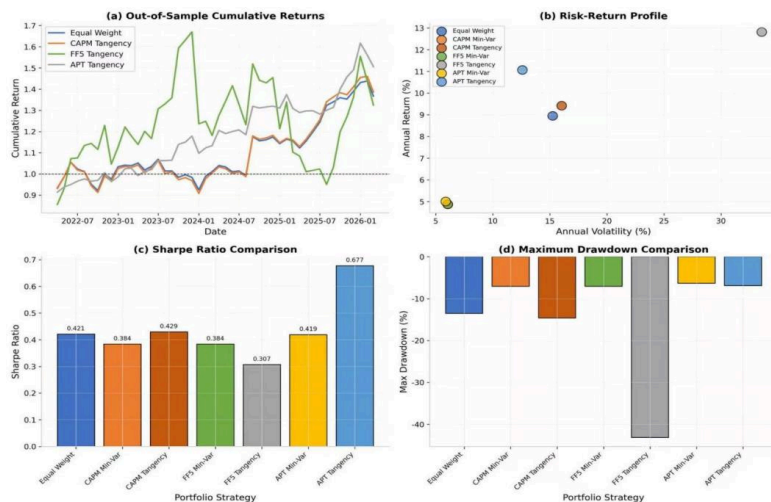


Figure 2. Out-of-sample performance of portfolios (Picture credit: Original)

5. Conclusion

This study compares the CAPM, the Fama-French five-factor model, and the APT not only as asset pricing models but also as tools for portfolio optimization. The results show that there is a significant difference between in-sample fitting and out-of-sample usefulness, as the actual value of an asset pricing model does not lie in its statistical goodness of fit, but in whether it can provide a

stable and reliable estimate of expected returns. APT offers the most balanced outcome. Its expected return estimate is less extreme. The tangential investment portfolio is more stable, with the highest Sharpe ratio and relatively limited decline.

The optimal model for practical portfolio optimization is not necessarily the model with the highest R-squared value. A more critical criterion is that the model generates expected return estimates with sufficient information and without excessive amplification of estimation errors in the optimization stage. In this study, APT performs best by achieving this balance. These conclusions must be interpreted with caution. The stock range is limited, the out-of-sample period is short, and transaction costs are excluded from the analysis. Therefore, future research can expand the sample range and add portfolio constraints.

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