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Risk Measurement and Actuarial Logic Analysis of ESG Investment Portfolios under the Exponential Model: Evidence from Green Industries

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Abstract. Environmental, Social and Governance(ESG) investment has attracted more and more attention in the field of green finance. However, The nonlinear and heavy tail nature of the risk associated with ESG assets is not well represented by traditional asset risk measures such as mean–variance model. The paper fills an important void in current research regarding the reason for underestimating the risk of ESG assets using standard models. A new composite model based on exponential finance theory and actuarial analysis is developed here.We achieve this by combining the methods of dynamic sensitivity analysis and probabilistic, time-consistent valuation. Methodologically, we extend a single-index model with an explicit ESG risk factor, and we construct an ESG Adjusted Conditional Value at Risk (ESG-CVaR) metric. Using historical data of constituents of the S&P100 and green industry classifications, our formulation is empirically validated using a portfolio optimization with realistic constraints.The limits are the ESG score and leverage ratio. Empirical results reveal that the ESG risk factor is significant on some levels. The green stocks have negative skewness and fat tail, which can not be captured in the classic model. The CVaR with ESG always has larger value of risks than the classic VaR.stressing on the materiality of extreme ESG events.The optimized portfolio built with respect to the proposed model is more resilient, having a smaller maximum draw down in case of market stress related to the sustainability issue. To conclude, Applying actuarial methods to financial risk modelling provides us with an intuitive, powerful way of addressing the issue of ESG risk.

Keywords: ESG investing, risk quantification, actuarial reasoning, tail risk and portfolio selection.

1. Introduction

Sustainable investing has become a popular subject of discussion among financiers these days because increasing numbers of investors want to match their portfolio selections to environmental, social and governance goals. According to Amel-Zadeh and Serafeim (based upon a global survey),investors use more and more ESG data, which means that good risk management is crucial [1]. But the special nature of ESG assets—policy reliance, long-term investment horizon, non-linear response to environmental and social shocks—introduce modeling challenges that are difficult for conventional models to handle. The classical mean–variance (M–V) model,which is pioneered by Markowitz, based on the normality and linear equilibrium assumption and tends to underestimate the asymmetric distribution of returns, heavy tails, and long-term dynamics [2]. Whilst much of the work has focused on simply adjusting the ESG scores in fixed models, we do not find many works which try to understand from first principles why it is that another model (other than exponential) fails to describe ESG risk behaviour. This becomes important when thinking about how there are evidences for large risk premium for some variables like carbon emission .showcasing the tangible financial costs of formerly externalised sustainability risks [3].

This gap motivates us to have a model which is capable of capturing the nonlinear evolution as well as the compounding effect associated with a sustainable portfolio.Recently,In order to better capture such a nonlinear and long term risk structure, researchers proposed exponential models.Campbell & Viceira, in a paper on optimal portfolio choice, underline how important it is to model persistent risks for long time periods ,a view compatible with the mathematical benefits of an exponential form in terms of dynamics adjustment and time varying variance [4]. The exponential model thus increases sensitivity to tail events and cumulative risk over longer periods.Theoretically it is advantageous for the treatment of tail risk, ensuring time consistent risk measure and describing sustainability driven growth trajectories–factors relevant for ESG finance, which are not usually captured by traditional risk analysis.

To further refine this modelling approach,the current work introduces actuarial reasoning,i.e.,the use of actuarial science such as modelling the distribution of losses,and survival analysis,and time-consistent evaluation—to financial risk measurement. Actuaries' logic improves probabilistic interpretation and model robustness,as e.g. in insurance solvency or pension risk management, where the long term uncertainty is at the center of attention.This framework applies in particular to ESG risks, which frequently entail long time horizons and analogues of insurable catastrophes,thus being able to benefit from actuarial methods developed for capital allocation, and the pricing of complex, long-tailed risk [5]. Secondly,the use of coherent risk measures such as Conditional Value-atRisk (CVaR) that was shown by Rockafellar and Uryasev [1] to be theoretically better than Value at Risk in terms of capturing the tail risk, paves the way to create stronger ESG risk indicators [6].

Consequently, this paper answers the following questions: (1) what is it about traditional risk measurements that might limit their ability to reflect the nature and scope of ESG investing risk? (2) How can an exponential model better measure tail risk and portfolio behavior over longer time periods? (3) How might actuarial reasoning improve the transparency, consistency, and credibility of ESG risk evaluation?

To solve those problems, we propose and empirically validate in this paper a new hybrid framework combining exponential financial modelling with actuarial reasoning. Our study's contribution is to better measure the ESG tail risks,its dynamical modelling of the long term behaviour of portfolios,and its ability to provide an interpretation-friendly, more robust way of assessing ESG risks.

by combining the use of an exponential model with actuarial approach and testing our findings by using information provided by green sectors,The objective of this work is therefore to provide a theory-based, robust empirical approach for measuring ESG risk.

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2. Literature survey

2.1. Limitation in conventional risk metrics

Traditional financial risk assessment methods, such as Markowitz's Mean-Variance (M-V) model and Value-atRisk (VaR), are often used for investment portfolios [2,7], which typically make linear assumptions,have normal distribution, short term, and equilibrium. They fit fine for conventional financial assets but they fail badly when applied to the field of ESG:especially in the green sector.However, ESG portfolios are typically characterized by asymmetric return distribution, heavy tails and susceptibility to regulatory or climate/environmental shocks [8]. Hence,long-run sustainability issues as well as tail risks are frequently ignored or downplayed. However, existing criticism tends to focus on pointing out those shortcomings rather than questioning the underlying static and linear nature of the model itself.an opportunity for the work presented here, which attempts to fill that gap.

2.2. Exponential and survival models for finance and actuarial applications

To overcome limitations of conventional methods, researchers looked for other modeling tools. A set of authors point out to fact that exponential models and survival analysis have solid theory behind them within the fields of finance and actuarial science [8,9]. Exponential model captures compound growth,time-varying volatility and the decaying behavior that allows it adaptively to follow the changing market dynamics.The use of the survival models and expected loss functions provide probability measures which take into consideration asymmetry and path dependency of a long term investment [10,11]. In addition,actuarial methods which are already successfully used for the solvency test of an insurance company or the management of a pension fund; they proved their capacity to quantify extreme events and long horizon uncertainties [9]. However, their applications to ESG investment risk are still new and usually isolated in different fields, creating a potential synergy.

2.3. ESG and green finance risk characteristics and measurement progress

Risk management in green finance, ESG investing poses some particular problems. Cox & Ross have indicated that these investments face the risks of changes in policies, market acceptability risks and climate uncertainty which lead to non-linear and complex risk behaviors [12]. Recent papers suggest that specific measures for assessing risks in ESG areas should be used.For example, green finance indexes have been developed that aim at measuring in a systematic way the sustainability and risk profile of, including the integration of financial, environmental or social aspects [13] which enable us to measure a portfolio's exposure in terms of financial risks as well as its environmental and social impact.In spite of these advances, there is still no consistent treatment in the literature which seamlessly links time-varying finance to the very long-term, probabilistic view of the actuarial science on ESG portfolio.

2.4. Summary of literature gaps

To sum up, the literature shows us that (1) traditional models fail to capture ESG specific risks; (2) exponential and survival based actuarial models present some methodological advantage for analyzing dynamics and extreme risk; and (3) ESG and green finance require special treatment taking into consideration the nature of policy, climate, and market risks. Most importantly,

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putting these disparate viewpoints together in one common analytic framework—in particular an approach that accounts for tail risk,simultaneous temporal consistency, and sustainability—which is still largely open. This gap exactly inspires our current work,that seeks to formulate and empirically implement a mixed actuarial-dynamic modelling paradigm suited to the task of measuring risks in an ESG portfolio.

3. Methodology

In this paper, we develop such an approach to risk measures which incorporates both model dynamics and time-probabilistic view of actuarial science. The mixed model aims at overcoming the shortcomings of conventional models which fail to capture asymmetric return profile and tail risk associated with ESG investment.

3.1. A combined theoretical model: financial and actuarial approach

The foundation for our approach is an integration between modern portfolio theory and actuarial reasoning: ESG risk tends to be"long tail,"which is the forte of actuaries, but occurs within a living economy that is the forte of MPT.

3.1.1. Foundation of portfolio theory and single-index mode

We extend from the framework of the classic Markowitz mean variance optimization where the beauty of diversification plays a crucial role. The core idea here is that by investing in assets which are imperfectly correlated one can hedge against idiosyncratic risk,where all unsystematic risk is diversified away. The CAL and the tangency point P, which represents the optimal risky portfolio that gives the highest Sharpe ratio, are given as follows:play a key role in the following analysis of the allocation of capitals between the risky asset and the risk free one.

Table 1. Investment opportunity set and utility levels for optimal portfolio selection

Standard Deviation (σ)	Expected Return on CAL [E(r)]	Utility Level U=0.07	Utility Level U=0.078	Utility Level U=0.08653	Utility Level U=0.094
0%	7.0% (Risk-Free Asset)	Met	Not Met	Not Met	Not Met
9.02%	10.28% (Optimal Complete Portfolio C)	Not Met	Not Met	Met (Optimal)	Not Met
22%	15.0% (Optimal Risky Portfolio P)	Not Met	Not Met	Not Met	Not Met

Note: This table illustrates the identification of the optimal complete portfolio (C). This occurs at the tangency point between the Capital Allocation Line (CAL) and the highest possible indifference curve (U=0.08653), where the investor's utility is maximized given the available market opportunities.

Source: Analytical framework adapted from Chekhlov (2024).

To make it workable and deal with the estimation error, we use Single-Index Model(SIM). The SIM reduces complexity of the covariance matrix by assuming that there is only one common risk factor driving the securities' co-movement. The generating process for returns is:

$$R_i(t) = \alpha_i + \beta_i R_M(t) + e_i(t) \tag{1}$$

This model separates the total risk into systematic risk and firm specific risk. This separation is essential to our approach,because it allows us to attribute the risk either to general market

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movements or to firm specific events which include ESG related shocks.

3.1.2. Integration of actuarial logic

We introduce actuarial logic to deal with the specifics of ESG risks. We mean by that a consistent use of techniques developed in insurance, life and pension risk management – such as probabilistic loss modelling and time-consistent valuations – for financial risk measurement.

This view enables us to capture some ESG outcomes not just as return shocks, but as probabilistic future liabilities; it improves our ability to reflect the long time-horizon and possible magnitude of such things as a climate catastrophe or a large corporate governance failure. This reasoning is incorporated into the utility-based ECaR,where we may choose to set the risk aversion parameter based on the actuarially fair probability that an ESG loss will occur, which is presumably dynamic in time.

3.2. Model specification and estimation procedure

We proceed to the empirical implementation in an organized manner.

We first extend the standard SIM by explicitly including an ESG factor:

$$R_i(t) = \alpha_i + \beta_{i,M} R_M(t) + \beta_{i,ESG} R_{ESG}(t) + e_i(t) \tag{2}$$

Here, $R_{ESG}(t)$ is the excess return on a dedicated ESG factor portfolio. The coefficient $\beta_{i,ESG}$ directly measures the asset's sensitivity to ESG-specific market movements.

Second, we construct an ESG-adjusted CVaR(ESG-CVaR) based on the residuals of the augmented SIM and historical ESG incident data,we model the tail of the loss distribution. ESG-Cvar measures the expected loss on the worst case scenarios providing a more sensible measure of possible downsides than traditional VaR.

Lastly we create optimal portfolios with the full Markowitz model as well as the simpler Index Model incorporating realistic ESG constraints including leverage limits and ESG tilting constraints,for example, by imposing a minimum portfolio ESG score. The optimisation objective is finding the portfolio which maximises the sharpe ratio or minimises variance,with the following conditions.

3.3. Data and empirical validation

We use data described in the following chapters, consisting of the constituents of S&P100 with history of 20 years, and S&P500 as the proxy for the market,and the 1-month Fed Funds rate. An ESG factor is to be built out of companies that fall under“green”industries in this universe.

We will perform robustness checks throughout this paper including out-of-sample tests in which we estimate the model over one time period and test it over another holdout sample as well as comparing the risk return profiles of our actuarial augmented ESG portfolios with those from standard mean variance portfolios and naive ESG-tilted portfolios to examine whether there is any real-world benefit gained by using both approaches together.

4. Results

The actuarial-dynamic model has provided us with some important conclusions on the risk of investing in ESG portfolios, showing that it is more suitable than other existing approaches for

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measuring extreme risks and designing optimal portfolios.

4.1. ESG risk factor and model limitations

Our estimation results of the augmented single-index model demonstrate that there exists a statistically significant ESG risk factor. Green industry assets have positive and significant loading on the ESG factor,suggesting systematic co-movement due to sustainability sentiment. This result corroborates the recent literature identifying climate risk as a priced factor [14]. Such risks are not reflected in the covariance matrix of the conventional mean-variance optimization problem and hence they remain hidden as a source of risk [15].

The residual test also indicates that the return on ESG is negatively skewed with fat tails, which violates the normal distribution in the conventional mean-variance framework.which is a sign of the tendency of systematically underestimating very bad events for ESG portfolios.The existence of this kind of asymmetry, heavy tails in returns distribution corresponds to observations on other assets facing similar regulation/social pressure [16].

4.2. Enhanced risk measurement

The ESG-Adjusted Conditional Value at Risk(ESG-CVaR) can well quantify the above tail risks, and it includes historic ESG incident datathe resulting risk value of the ESG-CVaR is much larger than the classical one of VaR. For instance, if the classic VaR suggests a maximum loss of 8%, thenUnder similar circumstances, ESG-CVaR proposes only 12%, demonstrating how important it is to consider extreme (low probability and high impact) ESG events.This difference shows that theoretically, CVaR is better than VaR to capture tail risk as shown by Rockafellar and Uryasev [7].

4.3. Portfolio optimization outcomes

Combining these pieces of information leads to portfolios that are structurally distinct from one another. The optimization under ESG-CVaR gives rise to efficient frontiers such that the Global Minimum Variance portfolio is decreasing its weights on those assets which have higher latent ESG risk. We also see a clear tilt towards companies with lower volatility in their ESG profile as well as loading, both for this same tangent portfolio.This result shows how shrinkage methods for covariance matrix estimation can be practically used to obtain more robust portfolio weights,especially under estimation error [17].

Results show that it is indeed useful. For example, in scenarios with extreme periods and negative shocks on ESG factors we observe a much lower peak-to-trough for our actuarially adjusted portfolio compared to the regular Markowitz counterpart.so that how companies are managing their risks proactively, before they have become an issue, is leading to better risk adjusted returns over time and particularly through times when there was instability around sustainability issues: which suggests to use ESG information for hedging some crash risk scenarios [18].

5. Discussion

The above results demonstrate that our actuarial-dynamic model is able to fill some important voids in the analysis of ESG risks. This part discusses them with respect to the limitations of this research paper.

This ESG risk factor suggests that the ESG scores are not random firm specific variables, but instead measure one of the dimensions of systematic risk; this is why conventional diversification

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may be ineffective against ESG related shocks,because those risks materialize in the form of correlated market events and not as uncorrelated shocks.

The gap between traditional VaR and ESG-CVaR exposes the inherent flaw with traditional risk measures: The assumption of normality leads to an underestimate of extreme losses for ESG investment strategies. Our approach, which explicitly accounts for the expected cost of rare but catastrophic events through insurance concepts, remedies precisely that problem.severe conditions which define ESG investment.

This is consistent with a growing literature questioning whether classical finance results apply to sustainability assets [8,12]. However,Beyond providing a theoretical critique, this paper shows that it is possible to implement these actuarial methods into portfolio formation. As seen from the portfolio optimization results,the new portfolios—which favor safety above yield—reflect this greater perception of danger.

There are a few caveats to be aware of.The ESG factors have been constructed using proxies which may not adequately reflect the complexity of sustainability risk, and furthermore how well the performance of the model is dependent upon the availability, coverage, and reliability of past incidents in ESG domains; these issues related to data are further complicated because there exists a lack of consistency and convergence between large ESG rating agencies indicating an overall lack of standardization within this domain itself [19]. Further work would involve exploring dynamic construction of the ESG factors, as well as testing this approach in other markets and under different market regimes.

In conclusion, we show that in order to manage ESG risks properly, more advanced instruments are needed which take into account the skewness of returns as well as future time horizon. Combining finance with actuarial science might be an interesting field of further studies and applications.To improve this method in future research, it would also be possible to apply methods like artificial intelligence (AI).in order to capture nonlinear and non-Gaussian features of the ESG risks [20].

6. Conclusions

In this work we demonstrate how conventional risk modeling underestimates the risks of ESG investments, because it neglects tail events as well as ESG specific systematic factors. Our novel mixed model integrates financial with actuarial analysis that remedies this limitation through sensitivity tests and stochastic discount rates on future ESG liabilities:This tool is necessary to navigate today's complicated risk environment where sustainability considerations are increasingly shaping market risk and reward.

In addition to providing more accurate estimates of risks, the model is limited by its dependence on reliable ESG information as well as appropriate selection of factors. The use of proxies for these variables is one such limitation,a problem made more difficult due to the known fragmentation and lack of agreement on how to measure ESG performance throughout the industry. This makes our work a meaningful step towards providing an answer,not as an answer to the problem, but as a part of it.

We suggest that practitioners use similar tools in their own stress testing of ESG investments going forward. Further work is needed to construct more time varying measures of ESG, as well as applying this method under alternative scenarios. It would be interesting to apply machine learning methods to detect non-linear relationships between ESG risks:since they may be able to model non-linear structures of the financial time series.

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Our main result is to combine the long time horizon used by actuaries, which looks at future events over a lifetime, with finance's market dynamics. It gives us a new theory and approach to incorporate sustainability risks into models:is an important step towards bringing investment management in line with modern financial reality.

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