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Stochastic Volatility Modeling in Emerging Markets: A Comparative Review of the Heston and SABR Models

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Abstract. Implied volatility surfaces are crucial to option pricing and risk management, as they encapsulate market expectations of future uncertainty across various strikes and maturities. In developed markets, well-documented volatility smiles and skews represent systematic deviations from the constant-volatility assumption underlying the Black–Scholes model, motivating the development of more advanced frameworks. Stochastic volatility models such as Heston's and the SABR model have become standard tools for capturing these dynamics. The Heston model introduces a mean-reverting stochastic variance process, yielding semi-analytical solutions for option prices. In contrast, the SABR model provides an approximate analytical expression for implied volatility that is computationally efficient and flexible in reproducing smile shapes. However, most comparative analyses of these models focus on developed markets, leaving less evidence on their performance in emerging, less efficient markets characterized by lower liquidity, higher volatility, and structural frictions. Features such as sparser strike grids, irregular maturities, and noisier quotes, create calibration challenges that differ materially from those encountered in mature markets. This review examines how the Heston and SABR models capture implied volatility surfaces in such environments, emphasizing calibration behavior, ability to replicate smile and skew patterns, term-structure consistency, and robustness under sparse or noisy data. Empirical evidence suggest that Heston offers stronger theoretical consistency and cross-maturity coherence, while SABR excels in calibration stability and local smile fitting under thin liquidity. The paper concludes with implications for practitioners and future research directions, including extensions incorporating jumps or regime-switching dynamics.

Keywords: stochastic volatility, Heston model, SABR model, implied volatility surface, emerging markets

1. Introduction

Accurate modeling of the implied volatility (IV) surface is essential for option pricing, hedging, and risk management. The IV surface, a three-dimensional representation of implied volatilities across strike price and time-to-maturity, encapsulates market expectations of future uncertainty. Since the introduction of the Black–Scholes model, which assumes constant volatility and log-normal returns, researchers and practitioners have recognized systematic deviations from this framework. These deviations appear as volatility smiles and skews, reflecting jumps, fat tails, and stochastic volatility.

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Emerging markets introduce further complexity. These markets tend to be less liquid and potentially less efficient, leading to noisier and more erratic implied volatility data. Market frictions, heterogeneous investor behavior, and structural asymmetries can result in volatility surfaces that differ qualitatively from those in developed markets. Examples include right-skewed volatility curves in China's SSE 50 ETF options [1], contrasting with the left-skew typical of U.S. equity options. Such distinctions underscore the need to assess whether models calibrated for developed markets can effectively capture the dynamics of emerging ones.

Early option-pricing frameworks, such as the Black–Scholes model, fail to account for these patterns, motivating the development of more sophisticated approaches. The Heston model introduces mean-reverting stochastic variance and provides a semi-analytical solution for European options [2]. The SABR model functions as a parametric implied-volatility formula [3] and is widely used for its fast, stable calibration.

Most comparative studies of these models, however, focus on efficient and liquid markets, such as those in the U.S. or Europe [4,5]. Emerging markets, such as India, China, Brazil, and South Africa, remain underexplored, despite growing derivatives activity. These markets exhibit lower liquidity, sparse strike coverage, and heightened sensitivity to macroeconomic shocks [6,7]. Understanding how the Heston and SABR models perform under such conditions is crucial for both practitioners and regulators. This paper addresses this gap by systematically reviewing their mechanisms, calibration behavior, empirical performance, and relative strengths in capturing implied volatility surfaces in emerging markets.

The central research question guiding this paper is: How do the Heston and SABR stochastic volatility models capture implied volatility surfaces in emerging, less efficient option markets? This paper develops a comprehensive literature review organized around five themes: (1) mechanisms and theoretical foundations of Heston and SABR, (2) calibration and empirical performance, (3) ability to reproduce smiles and skews, (4) term-structure and long-maturity behavior, and (5) evidence from emerging markets. The discussion section synthesizes these strands into a comparative analysis, focusing on calibration stability, robustness under sparse data, and the trade-offs between theoretical coherence and empirical fit. The paper concludes by summarizing findings, emphasizing implications for practice in emerging markets, and suggesting future research directions.

2. Literature review

2.1. Mechanisms and theoretical foundations

The limitations of the Black–Scholes model in reproducing volatility smiles and skews prompted the development of stochastic volatility models such as Heston's and SABR's. The Heston model introduces stochastic variance through a mean-reverting Cox–Ingersoll–Ross (CIR) process. Under the risk-neutral measure, the asset price S_t and its variance ν_t evolve as

$$dS_t = rS_t dt + \sqrt{\nu_t} S_t dW_t^S, d_t = \kappa(\theta - {}_t)dt + \xi\sqrt{{}_t}dW_t$$

where r is the continuously compounded risk-free interest rate, and W_t^S and W_t are Wiener processes with correlation . The five key parameters in Heston's model are: initial variance ${}_0$, long-run mean variance θ, mean reversion rate κ, volatility-of-volatility ξ, and the correlation between the asset and variance shocks. Negative correlation ($\rho < 0$) generates the downward-sloping

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implied volatility skew typical of equity markets. The model's semi-analytical solution, achieved through characteristic functions, established it as a benchmark for pricing and calibration.

By contrast, the SABR model (Stochastic Alpha-Beta-Rho) was introduced as a practical model for implied volatility smiles, especially for interest rate options. SABR directly models the forward price or rate F_t and its volatility σ_t

$$dF_t = \sigma_t (F_t)^{\beta} dW_t, d\sigma_t = \nu \sigma_t dZ_t$$

with W_t and Z_t as two Wiener processes correlating . The parameters of SABR are ν , β , , and $\sigma_0 = \alpha$ is the initial volatility level. Here β $\in$ [0,1] controls elasticity, ν determines smile curvature, and governs skew. The model's asymptotic implied-volatility expansion provides a flexible and tractable method for fitting smiles, with arbitrage largely mitigated through appropriate constraints.

While Heston emphasizes stochastic variance dynamics grounded in theory, SABR prioritizes calibration flexibility, a contrast the underpins much of the empirical debate.

2.2. Calibration and empirical performance

Calibration is a key test of a volatility model's empirical reliability, and the Heston and SABR frameworks exhibit distinct trade-offs. Early studies using S&P 500 index options show that introducing stochastic volatility substantially reduces pricing errors relative to Black–Scholes, while jump-diffusion extensions improved extreme-strike fit at the cost of weaker hedging performance [4]. Christoffersen et al. further demonstrates that the standard one-factor Heston model struggles to jointly match returns, realized volatility, and option prices, especially at longer maturities, motivating richer volatility specifications [8].

The SABR model's literature emphasizes numerical stability, particularly under illiquid or incomplete markets. Empirical evidence documents SABR's stable parameter behavior and reliable maturity-by-maturity calibration under sparse or noisy data [9,10], with refinements reducing arbitrage violations and extending accuracy across strikes and maturities [11,12]. Overall, Heston offers global coherence but can be computationally sensitive, whereas SABR provides faster, more stable calibration without intrinsic cross-maturity consistency, differences that are especially pronounced in emerging markets.

2.3. Ability to capture smile and skew

Stochastic volatility models are motivated by their ability to reproduce implied volatility smiles and skews, a feature captured by both Heston and SABR models, though through distinct mechanisms.

In the Heston model, smile and skew arise endogenously from the joint dynamics of asset prices and stochastic variance. A negative correlation ($\rho < 0$) between returns and variance shocks generates the downward sloping skews typical of equity markets via the leverage effect [5]. While effective for short maturities, the square root variance process limits tail behavior, reducing accuracy at extreme strikes and longer maturities [13]. Calibration may compensate through high volatility-of-volatility (ξ) or slow mean reversion (low κ), potentially destabilizing parameters and motivating jump-based extensions, such as the Bates and Heston-Nandi models.

SABR, by contrast, captures smiles through direct parameterization: β controls elasticity, ρ governs skew, and ν determines curvature. This structure enables accurate per-maturity smile fitting,

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with refinements improving long maturity performance [12] and ensuring arbitrage-free wings [11]. However, independent calibration across maturities limits intrinsic term-structure coherence.

2.4. Term-structure and long-maturity behavior

Modeling not only the cross-sectional shape of implied volatility but also its evolution across maturities is essential for pricing, hedging, and risk management. Conceptually, this distinction corresponds to fitting individual volatility smiles at fixed maturities versus constructing a smooth volatility surface that evolves continuously along the maturity dimension. Figure 1 provides a conceptual illustration of such a surface, highlighting how implied volatility varies jointly across strike and maturity. Models such as Heston generate volatility surfaces endogenously through stochastic variance dynamics, whereas SABR is typically calibrated to isolated maturity slices rather than a unified surface.

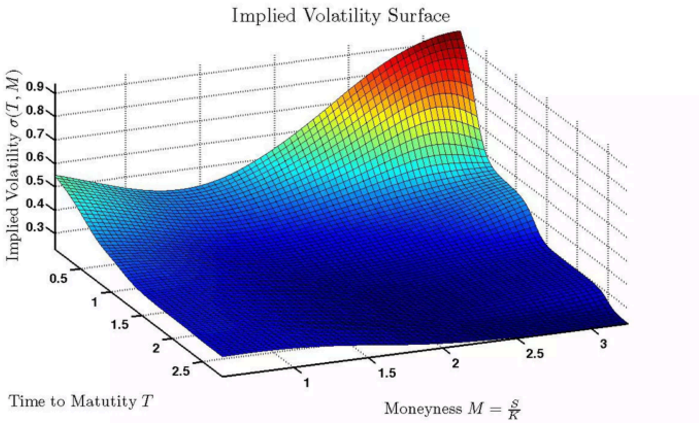

Figure 1. Conceptual implied volatility surface. Implied volatility is shown as a function of moneyness and time to maturity, illustrating the joint smile and term-structure effects and emphasizing cross-maturity coherence

Source: Adapted from Investopedia, Understanding the Volatility Surface in Options Trading [14]

The Heston model naturally generates a volatility term structure through its mean-reverting variance process, yielding smooth implied-volatility dynamics across maturities. Allowing parameters to vary across maturities has been shown to improve calibration quality in foreign exchange markets [15]. Although Heston may struggle with very short-dated smiles and extreme strikes, its joint calibration across maturities provides stability when market quotes are limited.

SABR, by contrast, does not inherently couple dynamics across maturities and relies on independent per-expiry calibration with exogenous smoothing [9]. While effective for short and medium maturities, its asymptotic approximation becomes less reliable at long horizons.

In emerging markets with sparse long-dated option data, these differences term-structure coherence is particularly important.

2.5. Emerging markets context

Emerging markets differ fundamentally from developed markets in their structural characteristics: lower liquidity, sparse strike coverage, episodic trading activity, and heightened sensitivity to macroeconomic or political shocks. These features produce implied volatility surfaces that are

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noisier, more irregular, and more prone to structural breaks, making them a stringent test for stochastic volatility models.

Evidence from India illustrates these dynamics. Analysis using Indian index options data showed that the Heston model significantly outperformed Black–Scholes in out-of-sample pricing accuracy across moneyness categories, volatility regimes, and maturities [7]. These results underscore the relevance of stochastic variance dynamics. Liquidity-weighted error measures further indicated that the Heston model did not disproportionately overfit illiquid strikes, highlighting its practical viability in thinly traded environments [7].

China provides a complementary case. Empirical evidence from the SSE 50 ETF options market shows that the Heston model's calibration becomes unstable under sparse strike grids and noisy option quotes [16], complicating the construction of smooth implied-volatility surfaces. SABR's numerical robustness is consistent with findings that fixing β enhances calibration stability in illiquid settings [10]. Episodes of right-skewed or highly curved volatility smirks during market stress have been documented [1]. Both Heston and SABR can represent such skews through positive ρ, but extreme curvature may necessitate very large volatility-of-volatility parameters or jump components.

Broader emerging-market evidence, such as the dynamics of realized volatility in South Africa [6], suggests roughness and long-memory properties not captured by classical one-factor models. These findings suggest that additional structural features may be required for full realism in emerging market volatility modeling.

3. Discussion

3.1. Calibration behavior and robustness

The calibration process reveals a fundamental contrast between the Heston and SABR models identified in the literature. Heston's calibration relies on a global optimization across strikes and maturities, producing internally coherent volatility surfaces when sufficient data are available. In emerging markets, however, sparse quotes and noise amplify sensitivity to initialization and optimization choices, particularly for the mean-reversion rate κ and the long-run variance θ. As a result, parameter estimates can be unstable across calibration windows, limiting robustness in less liquid conditions [7].

By contrast, the SABR model's calibration is localized by maturity and computationally tractable. Fixing the elasticity parameter β and calibrating the remaining parameters (α, ρ, ν) yields stable fits even in thinly traded markets [10]. In practice, SABR functions as a parametric smile model through its implied-volatility approximation, allowing rapid recalibration under changing market conditions. This stability makes SABR well suited for environments where data quality constrains global model estimation, though it comes at the cost of dynamic coherence across maturities

Overall, Heston's global structure favors consistency when data is rich, while SABR's localized calibration offers resilience under noisy or incomplete market conditions.

3.2. Ability to capture smiles and skews

Both models capture implied-volatility smiles and skews, but their empirical performance reflects the mechanisms emphasized in prior work. Heston generates skew endogenously through the correlation (ρ) between asset and variance shocks, enabling interpretation grounded in economic intuition. While effective for moderate skews, the model struggles to reproduce extreme short-term

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curvature or deep out-of-the-money smiles without auxiliary features such as jumps or stochastic correlation [13].

SABR, in contrast, achieves high-fidelity smile fitting by directly parameterizing curvature and skew. Empirical evidence from emerging markets, including China's SSE 50 ETF options, suggests that highly asymmetric or stress-induced smiles shapes can be accommodated more reliably by SABR under sparse strike coverage [16]. However, aggressive parameter values required to fit extreme curvature may introduce arbitrage concerns unless constrained appropriately.

Hence, while Heston offers interpretability and structural discipline, SABR prioritizes empirical accuracy at the individual maturity level.

3.3. Term-structure consistency and long-horizon implications

A key distinction between the models lies in their treatment of maturity dynamics. Heston's mean-reverting variance process ensures internally consistent evolution of implied volatility across maturities, producing smooth term structures that support long-horizon valuation and risk aggregation [15,17].

SABR lacks an intrinsic mechanism linking maturities, relying instead on independent calibration at each maturity, followed by parameter smoothing across expiries. While this limitation is less consequential in markets dominated by short-dated trading, it can distort valuation and hedging of longer-dated instruments in emerging markets, where maturity coverage is uneven.

Consequently, Heston's structural term-structure coherence complements SABR's flexibility, rather than directly competing with it.

3.4. Implications for emerging markets and model choice

Taken together, the findings suggest that no single model dominates across all dimensions in emerging markets. Heston provides a theoretically consistent framework capable of generating arbitrage-free surfaces and economically interpretable dynamics, but it requires sufficient data quality and careful calibration. SABR's simplicity and numerical robustness make it well suited for day-to-day pricing and interpolation in illiquid or rapidly changing environments, despite its lack of structural constraints.

In practice, emerging market participants may deploy SABR for short-term pricing and Heston for deeper valuation, stress testing, or regulatory analysis. This division of labor reflects a broader trade-off between theoretical coherence and empirical robustness. Hybrid approaches that embed stochastic variance dynamics within flexible calibration frameworks may offer a promising direction for future work, particularly in markets characterized by intermittent liquidity and structural instability.

4. Limitations of current research

Current research on stochastic volatility models in emerging markets faces several limitations. First, most empirical studies rely on small samples and limited time horizons, reflecting the relatively recent development of derivative markets in these economies. The scarcity of reliable high-frequency data impedes robust estimation of key parameters, particularly volatility-of-volatility and correlation that are central to stochastic volatility models. Second, direct comparative analyses between Heston and SABR remain relatively rare; most studies evaluate each model in isolation rather than conducting head-to-head performance tests under identical data and calibration settings.

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Moreover, as discussed earlier in the literature review, both models assume continuous sample paths, which may be restrictive in markets characterized by abrupt jumps and regime shifts. While extensions such as the Bates jump-diffusion model or Markov-switching Heston variants have been proposed to address these features, their empirical evaluation in emerging markets remains limited. As a result, the extent to which jump components improve pricing accuracy and stability relative to baseline stochastic volatility models is not yet well established.

Finally, the literature lacks standardized performance metrics. Studies employ different error measures, weighting schemes, and liquidity filters, complicating cross-market comparisons and limiting the generalizability of results.

5. Future directions

Future research can proceed along several promising lines. Empirically, broader datasets covering multiple emerging economies are needed to assess model robustness across diverse market conditions. Incorporating market microstructure effects may further improve calibration stability. Theoretically, hybrid models combining stochastic and local hold potential for reconciling the competing advantages of both frameworks.

From a policy perspective, stochastic volatility models may also support regulatory stress testing in emerging economies. Finally, more research into behavioral explanations for atypical skews, such as the right-skew phenomena observed in China, could refine model assumptions and parameter interpretations.

6. Conclusion

This review examined the question of how the Heston and SABR stochastic volatility models capture implied volatility surfaces in emerging, less efficient option markets. By integrating theoretical foundations with empirical evidence, the analysis evaluated each model's relative advantages across calibration behavior, smile and skew reproduction, term-structure dynamics, and robustness under sparse and noisy data.

The findings reveal a clear division in model suitability. The Heston model offers strong theoretical coherence through its stochastic variance dynamics, enabling internally consistent term-structure modeling and providing a principled framework for volatility evolution. However, its calibration can be unstable, particularly when data are limited or irregular, conditions that represent many emerging markets. These calibration challenges reduce its practical reliability, despite its structural strengths.

By contrast, the SABR model demonstrates empirical robustness and high calibration stability, supported by its closed-form asymptotic volatility formula and flexible parameterization. SABR reliably fits smiles and skews even when available strikes are sparse, and it performs well in the short- and medium-term maturities common in emerging markets. However, its lack of intrinsic maturity linkage limits its suitability for long horizon modeling.

Overall, no single model is universally superior. Model choice should reflect market liquidity, data availability, and analytical objectives. Future research should further explore hybrid specifications and conduct comparative studies to improve volatility surface modeling in emerging markets.

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