The Application and Challenges of Monte Carlo Simulation in Financial Derivatives Pricing

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Abstract. The Monte Carlo simulation is a powerful numerical method based on random sampling. Renowned for its flexibility in handling high-dimensional problems, it serves as a cornerstone of modern finance. However, it faces a fundamental challenge—slow convergence and high computational cost limit its application in pricing financial derivatives. This paper explores the application of Monte Carlo simulation in option pricing, focusing on its benchmark role for European options and how the LSM algorithm addresses "backward pricing" in American options. Numerical experiments were conducted under the geometric Brownian motion model. Monte Carlo priced European options, while LSM priced American options. Results show Monte Carlo effectively prices European options, with error convergence matching theory ($1/\sqrt{N}$). For American options, LSM performs excellently, providing accurate estimates. However, its accuracy and stability depend heavily on the choice of basis functions and number of paths. Moreover, the computational complexity is higher than European pricing methods. It increases cost and amplifies pre-existing limitations. In summary, this study highlights the flexibility of Monte Carlo but also the persistence of its challenges in sophisticated applications like LSM. The findings offer insights regarding parameter selection and contribute to understanding trade-offs in accuracy, stability, efficiency and cost.

Keywords: Monte Carlo simulation, option pricing, LSM algorithm

1. Introduction

Accurately pricing financial derivatives is a central problem in modern quantitative finance, critical for risk management, trading, and investment strategies. Among these instruments, options, which confer the right but not the obligation to buy or sell an underlying asset at a predetermined price, are particularly important. The valuation challenge stems from their inherent dependency on the uncertain future path of the underlying asset's price. This challenge is compounded for American-style options, which allow the holder to exercise at any point up to expiration, introducing a complex optimal stopping problem that must be solved simultaneously with the valuation itself. The seminal work of Black, Scholes, and Merton in the 1970s provided a closed-form analytical solution for pricing European options. However, the practical applicability of such analytical models is often limited by their stringent assumptions, and the inability to price path-dependent or early-exercise

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features. Consequently, numerical methods have become indispensable. The Monte Carlo simulation method is widely used due to its flexibility and algorithmic strength. It generates a large number of possible future price paths for the underlying asset and calculates the average payoff across all these paths. Despite its usefulness, the Monte Carlo method faces two major challenges: (1) Slow Convergence Rate: The statistical error of the MC estimator decreases at a rate of ($1/\sqrt{N}$). This slow convergence implies that computational requirements increase substantially for high-precision pricing. (2) The American Option Dilemma: American options can be exercised at any time, requiring determination of an optimal exercise strategy, which traditional Monte Carlo simulations are ill-equipped to handle. Longstaff and Schwartz proposed the Least Squares Monte Carlo algorithm. This method embeds a dynamic programming solution within a forward-simulation framework, enabling efficient Monte Carlo pricing of American options. Although LSM addresses a major algorithmic obstacle, it does not resolve the inherent limitations of the Monte Carlo method. It still suffers from the slow ($1/\sqrt{N}$) convergence rate, necessitating a large number of simulated paths. The choice of basis functions becomes a critical risk factor, as an inappropriate selection may lead to significant pricing errors. Additionally, the LSM method introduces substantial computational overhead. This study conducts a comprehensive empirical investigation into the application of Monte Carlo simulation for pricing both European and American options, with a focus on the LSM algorithm. The main objectives are: (1) To implement a standard Monte Carlo simulator for pricing European call options and analyze its convergence behavior and computational efficiency against the Black-Scholes benchmark. (2) To implement the LSM algorithm for pricing American call options and evaluate its accuracy. (3) To compare the computational costs of standard Monte Carlo and LSM methods.

2. Theoretical background

2.1. European options and American options

An option is a financial contract that grants the holder the right to buy or sell an underlying asset at a specified price on or before a certain future date. The theoretical foundation of option pricing is built upon the groundbreaking work of Black and Scholes, who derived the analytical pricing formula for European options [1].

European Options: Can only be exercised at the expiration date T

American Options: Can be exercised at any time on or before the expiration date T.

Pricing American options is more complex because it requires determining the optimal exercise time, which is known as the optimal stopping problem.

2.2. Risk-neutral pricing principle

This is a core concept in derivative pricing. The principle states that in an efficient market, the fair price of an option equals the present value of its expected future payoff. This approach simplifies calculations by eliminating the need to estimate the actual expected return of the underlying asset. As emphasized by Hull in derivatives textbooks, risk-neutral valuation is a core concept in modern financial engineering, allowing derivative pricing through expectation calculations [2].

The option pricing formula is given by:

$$V_0 = e^{-rT} \cdot \mathbb{E}\left[\text{Payoff}\right] \quad (1)$$

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2.3. Geometric Brownian Motion

The GBM model, as a standard model for financial asset prices, has its mathematical properties and statistical characteristics detailed in Glasserman's work [3].

Geometric Brownian Motion (GBM) is used to model the random fluctuation of stock prices. The mathematical expression of GBM is:

$$dS_t = rS_t dt + \sigma S_t dW_t \quad (2)$$

R: risk-free interest rate

Σ: volatility

dWt: random Wiener process increment

The discrete form of this model is:

$$S_t = S_{t-1} \cdot \exp\left(\left(r - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t} \cdot Z\right) \quad (3)$$

2.4. Monte Carlo simulation

According to The application of Monte Carlo simulation in finance has matured, with Investopedia, this method is particularly suitable for handling path-dependent and complex payoff structured derivatives [4].

Monte Carlo simulation estimates the value of an option through random sampling and statistical averaging:

Generate Paths: Simulate a large number (e.g., 100,000) of possible stock price paths.

Compute Payoffs: For each path, calculate the option's final payoff.

Discount and Average: Discount all payoffs to the present value and take the average.

Mathematically, the option price is approximated as:

$$\text{Option} \approx e^{-rT} \cdot \frac{1}{N} \sum_{i=1}^{N} \text{Payoff}_i \quad (4)$$

2.5. Least Squares Monte Carlo (LSM) algorithm for american options

For American options, standard Monte Carlo simulation is insufficient due to the need to determine the optimal exercise time. The LSM method addresses this issue through the following steps:

Simulate Price Paths: Generate a large number of stock price paths.

Backward Analysis: Analyze each time point backward, starting from the expiration date.

Regression Analysis: At each time point, use regression analysis to estimate the value of continuing to hold the option.

Independent variable: current stock price.

Dependent variable: expected value of continuing to hold the option.

Exercise Decision: Compare the immediate exercise value with the value of continuing to hold the option, and choose the higher value.

Pricing: Compute the option value based on the optimal exercise strategy.

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3. Methodology

3.1. Basic settings

This study first sets up several models with fundamental assumptions:

Asset Model: The underlying stock price (St) follows a geometric Brownian motion (GBM).

Option Types: Pricing of European call options and American call options.

Parameter Settings: We employ a fixed set of parameters consistent with market realities:

S_0 = 188 (initial stock price); K = 200 (strike price)

r = 0.05 (5% annualized risk-free rate); σ = 0.2 (20% annualized volatility)

T = 1 (maturity: 1 year); M = 100 (Number of time steps)

N = 100000 (Number of simulated paths)

3.2. European call option pricing (benchmark)

This method uses Monte Carlo simulation to price European options.

Only the final price needs to be simulated: Generate N final stock prices (ST) using the GBM formula.

S_T = S_0 * np.exp((r - 0.5*σ**2)*T + σ * np.sqrt(T) * Z), and Z ~ N(0, 1).

Calculate payoff: For each terminal price: max(S_T - K, 0).

Calculate average price: The option price is the average of all these discounted payoffs to today.

3.3. Pricing American call options using LSM

The LSM algorithm is a solution capable of handling early exercise for American options.

Simulating Full Paths: Generate N stock price paths over M time steps.

Starting from the end of the product term, at final maturity, the option's value is its payoff: max(S_T - K, 0).

Then, we iterate backward from the second-to-last time step back to the initial time.

3.4. Test content

For testing, we vary two factors and observe outcomes:

Number of paths (N): Run simulations with different N values. As N increases, prices become more accurate and stable.

LSM basis functions: Test the complexity of basic functions required by our regression model:

Using only stock price (S)

Using stock price and its square (S^2)

Using stock price, its square, and cube (S^3)

4. Implementation and results

4.1. Parameter settings

This section mainly sets the parameters as follows:

S_0 = 188; K = 200; r = 0.05; sigma = 0.2; T = 1.0; M = 100; N = 100000

4.2. Price path generation

100,000 price paths were generated using the Geometric Brownian Motion (GBM) model. Path generation took approximately 0.29 seconds. The corresponding Figure 1 is shown below.

Figure 1. Monte Carlo simulated price paths of the underlying asset (GBM)

4.3. European option pricing results

The Monte Carlo pricing results for the European call option are as follows in Table 1.

Table 1. Comparison of European call option prices: Monte Carlo and theoretical

Name	Data
Monte Carlo Price	13.8332
Black-Scholes Theoretical Price	13.9682
Absolute Error	0.134990
Relative Error	0.6664%
Standard Error	0.075451
95% Confidence Interval	[13.6853, 13.9811]
Computation Time	0.01

According to this document, The Monte Carlo implementation for European options is relatively straightforward, with Corporate Finance Institute (2023) emphasizing that its core lies in estimating expectations through extensive simulations [5]. The following data regarding European options can be clearly seen.

The Monte Carlo prices closely align with those derived from Black-Scholes theory, with a relative error of only 0.67%, demonstrating the validity of Monte Carlo simulation. The standard error and confidence intervals are also very small. The extremely short computation time reflects the efficiency of pricing European options.

4.4. American option pricing results

The LSM algorithm proposed by Longstaff and Schwartz cleverly addresses the optimal stopping problem in American option pricing by combining regression analysis with dynamic programming [6]. Based on the content of this literature, we can determine that the LSM algorithm is particularly helpful for American options. Below are the results of the experimental analysis.

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American options were priced using the LSM algorithm, testing three different combinations of basis functions shown in Table 2.

Table 2. American option pricing results under different basis functions

Base Function Combination	LSM Price	Difference from European Price	Calculation Time (seconds)
[S]	13.8350	0.0018	4.38
[S, S^2]	13.8447	0.0115	4.63
[S, S^2, S^3]	13.8543	0.0211	4.88

Convergence: The American option prices obtained from all basis function combinations closely match European option prices, consistent with theoretical expectations.

Basis Function Impact: As basis function complexity increases, estimated prices rise slightly, indicating that more complex basis functions yield higher returns. This conclusion is reasonable because Research by Tsitsiklis and Van Roy demonstrates that the choice of basis functions significantly impacts the accuracy of the LSM algorithm, with simple polynomial functions often providing good approximation results [7].

Computational Efficiency: The LSM algorithm's computation time significantly exceeds that of European pricing, reflecting increased computational costs.

4.5. Convergence analysis

To validate the convergence properties of the Monte Carlo method, pricing results were tested under varying path counts (Table 3).

Table 3. Convergence testing across different path counts

Number of Paths (N)	European Price	Standard Error	95% Confidence Interval Width
1,000	10.9721	0.3812	1.377
10,000	13.8811	0.1512	0.436
50,000	13.8602	0.0897	0.195
100,000	13.8332	0.0755	0.138

Figure 2 below shows the trend of error decreasing as the number of paths increases, verifying the convergence rate of $1/\sqrt{N}$:

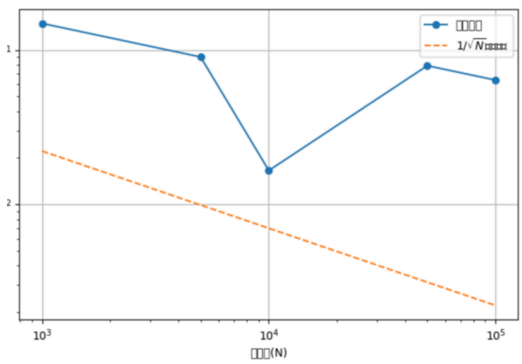

Figure 2. Monte Carlo error convergence vs. sample size (1/√N)

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Our convergence results align with the findings of Broadie and Glasserman, confirming that the error of Monte Carlo methods indeed decreases at a rate of $1/\sqrt{N}$ [8].

4.6. Computational efficiency comparison

Table 4 shows that European options exhibit high pricing efficiency due to their characteristic of being exercisable only at expiration, making calculations highly efficient. However, European options also lack flexibility and cannot maximize profits.

Table 4. Computational time comparison

Method	Average Computation Time (seconds)
European Monte Carlo	0.001
American LSM([S])	4.38
American LSM([S,S^2])	4.63
American LSM([S,S^2,S^3])	4.88

The LSM algorithm specifically addresses the unique characteristics of American options. Its primary advantages include handling high-dimensional problems and complex path-dependent features, as well as enabling the pursuit of maximum profit. However, significant drawbacks exist: computational costs are high due to the generation of numerous price paths and multiple regression analyses. Furthermore, its accuracy and stability are influenced by the choice of basic functions and the number of paths.

4.7. Analysis

Based on this code implementation, the effectiveness of Monte Carlo methods and LSM algorithms in option pricing has been validated.

Accuracy: European pricing results closely match theoretical values; American pricing aligns with theoretical expectations.

Convergence: Verified the theoretical convergence rate of $1/\sqrt{N}$.

Practicality: The LSM algorithm effectively addresses American option pricing challenges but incurs higher computational costs.

Stability: Results remain relatively stable across different basis function combinations, though more complex basis functions yield superior numerical performance.

5. Discussion

5.1. Validity of the Monte Carlo method

The empirical results of this study reaffirm the effectiveness and reliability of Monte Carlo simulation as a tool for pricing financial derivatives. The pricing results for European call options align closely with the Black–Scholes theoretical values, with a relative error of only 0.67%, and the theoretical value falls within the 95% confidence interval.

Moreover, even with a scale of 100,000 simulated paths, the method still delivers high accuracy, further demonstrating the leading role of Monte Carlo simulation in the field of financial derivatives pricing.

5.2. Pricing American options using the LSM algorithm

In this study, the LSM algorithm effectively addressed the challenge of determining the optimal exercise time for American options. The pricing results for American options are very close to those of European options, which is consistent with financial theory.

It can be observed that the choice of basic functions does have an impact. In this experiment, three basic functions were used. As the complexity of the basic functions increased, so did the estimated option value. While this represents an improvement, it also leads to an increase in computational cost. Also, consistent with the analysis by Clement et al., while the LSM algorithm is powerful, its computational cost is significantly higher than simple Monte Carlo methods [9].

5.3. Analysis of convergence properties

The convergence analysis clearly illustrates both the fundamental characteristics and the main limitations of the Monte Carlo method. The error decreases proportionally to $1/\sqrt{N}$, confirming certain inherent constraints of the method. This implies that reducing the error requires increasing the number of simulation paths quadratically, which inevitably significantly raises computational costs.

5.4. Practical implications and recommendations

Based on the above findings, the following recommendations are offered regarding the use of Monte Carlo methods:

Basis function selection: In most cases, it is advisable to use simple basis functions. Higher complexity should only be considered when high precision is required and computational resources are sufficient.

Number of paths: Generally, 50,000 to 100,000 paths are sufficient.

Algorithm selection: For European options, Monte Carlo simulation is efficient and reliable. For American options, LSM is a practical choice, though one must accept the trade-off between error and computational cost. As noted in Risk.net's 2022 practical guide, real-world applications require finding an appropriate balance between computational accuracy and efficiency, which aligns with our research findings [10].

6. Conclusion

This study explores the application of Monte Carlo simulation in pricing European and American options through data analysis. Based on the geometric Brownian motion model, the Monte Carlo method successfully calculated the pricing of European call options. The LSM algorithm was employed to resolve the unique pricing challenges associated with American options.

Data demonstrates that Monte Carlo simulation performs exceptionally well in pricing European options, yielding results only 0.67% different from Black-Scholes outcomes while exhibiting high computational efficiency. For American options, the LSM algorithm also proved highly successful, producing results closely aligned with theoretical expectations and partially resolving the challenge of exercising options at any time.

Convergence analysis confirmed the $1/\sqrt{N}$ convergence property of the Monte Carlo method, with the standard error decreasing from 0.3812 for 1,000 paths to 0.0755 for 100,000 paths. However, this precision comes at a high computational cost.

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Neither the Monte Carlo nor the LSM method is perfect. We should continue our efforts in this direction, pursuing further research on precision, computational resources, computational costs, and time efficiency to select better pricing tools for scientific advancement. Of course, Monte Carlo simulation, as a vital component in the field of financial derivatives, will continue to play an irreplaceable role.

This study has several limitations: The model is based on GBM and does not account for more complex market phenomena such as stochastic volatility. Fixed parameters were used throughout, with no sensitivity analysis conducted. Due to algorithmic constraints, no comparative analysis with other American option pricing methods was performed.

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