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Analysis of Premium Determinants and Pricing Models for Chooser Options Using Binomial Tree, Parity Decomposition, and Monte Carlo Methods

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Abstract: Chooser options are exotic financial derivatives allowing investors to select between a call and a put option on the same underlying asset at a predefined future date, addressing the limitation that most exotic options require upfront payoff structure determination and making them valuable for investors uncertain about market trends. This study prices European and American chooser options using three core methods: the multi-step binomial tree model, the parity decomposition method and the Monte Carlo method. It compares the binomial tree and Monte Carlo methods, finding they converge to similar results with sufficient steps or simulations, while the parity decomposition method aligns with the binomial tree model. Key factors affecting chooser option premiums are clarified: strike price near the asset's initial price maximizes flexibility with premiums between single options and straddles, extreme strike prices make its value converge to a put or call option, higher volatility accelerates premium growth, European choosers benefit from longer observation periods and American ones from early exercise. This study enriches exotic option pricing literature and aids investors/financial institutions in uncertain markets.

Keywords: Chooser Option, Binomial Tree Model, Option Pricing, Monte Carlo Method

1. Introduction

An option provides holders with the right to choose between a call and a put option during an agreed period. Its worth is closely linked to underlying assets' prices. As a crucial financial derivative, it effectively enhances investors' expected returns and diminishes risk exposure, serving a vital role in hedging against spot price and financial asset risks. Since the Chicago Board Options Exchange commenced options trading in 1973, the global scope of financial derivatives trading has skyrocketed. With the transaction volume of traditional standard options on exchanges increasing dramatically, financial institutions have created novel option contract types to satisfy clients' needs and extend their options-related operations. These new types of options, derivatives featuring more intricate cash flow structures, are referred to as exotic options.

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The majority of exotic options are transacted in the over-the-counter (OTC) derivatives market. These options are noted for their customization and flexibility and are often used to mitigate specific financial market risks [1]. As per Rubinstein and Reiner, exotic options can be categorized into nine types such as compound options, binary options, and exchange options, among others [2]. Every category boasts its distinctive payoff pattern and application, assisting investors in building portfolios and developing investment tactics in complex and volatile financial markets based on their views on underlying asset price trends, risk tolerance, and specific investment needs. Nonetheless, most of the aforementioned option contracts, in spite of their flexible payoff mechanisms, typically necessitate the holder to define the payoff structure initially. This characteristic often holds little attraction for certain prospective clients, particularly those who lack a definite perspective on market trends. Under these circumstances, chooser options possess distinctive importance.

A chooser option, as an exotic option, enables holders to select whether it is a call or a put option within a specific timeframe. A standard European chooser option requires investors to pick between a European call option and a European put option on the same underlying asset and strike price on the first expiration date, and obtain the corresponding payoff on the second expiration date. For American chooser options, the holder can make the selection during a flexible time period before the first expiration date, and the relevant options may be exercised anytime prior to the second expiration date. In the 1990s, Bankers Trust issued a range of such contracts. The underlying assets of these options encompassed commodities like oil and stock indices like the DAX and BCI [3].

Research suggests that chooser options are well suited for investors who expect significant price fluctuations in the underlying asset but are unsure of the movement directions [4]. They are also helpful in betting on the volatility of the underlying asset, similar to straddles (simultaneously purchasing call and put options). However, unlike straddles, chooser options necessitate just a single premium payment, cutting down the initial investment cost.

In Section 3, when pricing European chooser options, we first adopt the traditional binomial tree model method. Additionally, according to Hull and considering the nature of chooser options, in combination with the put-call parity, it can be known that a chooser option at time T_0 is mathematically equivalent in price to a portfolio consisting of a traditional vanilla options [5]. Based on this theorem, we directly combine the pricing models of standard European options to obtain the pricing model for European chooser options. Empirical analysis shows that the pricing results based on this equivalent portfolio theorem are completely consistent with those from the binomial tree model.

The subsequent portion of Section 3 employs the Monte Carlo method to evaluate the pricing of European chooser options. Subsequently, a comparative analysis is undertaken, juxtaposing the outcomes and computational efficiency of the multi-step binomial tree pricing model against those of the Monte Carlo method. The findings indicate that, the Monte Carlo method exhibits superior performance in time efficiency compared to the binomial tree model. However, the latter is capable of approximating the accurate value with a relatively smaller number of steps. Moreover, it is observed that when the time size of each individual step is sufficiently small, and the simulation time of the Monte Carlo method is sufficiently high, the option pricing results derived from both models converge to a virtually identical value.

In Section 4, the multi-step binomial tree model is utilized to price an American chooser option. Given that American options can be exercised earlier than expiration time, the theoretical value at each node is the maximum of the immediate exercise payoff and the discounted continuation value. Backward induction is employed to calculate the option's initial price, starting from the terminal payoff. Additionally, the section briefly explores the possibility that the chooser right be exercised early.

With the pricing models established, Section 5 delves into the factors causing price differences between chooser options and their call, put, and straddle counterparts. By analyzing the sources of the chooser premiums over vanilla options, we can better determine in what conditions chooser option outperform its counterparts. Key influencers like strike price, asset volatility, and the choice date's timing are closely analyzed for their effect on the price gap.

Section 5 reveals that call and put option prices, along with chooser option premiums, show clear trends as strike prices change. When the strike price is extremely high or low, the chooser option closely resembles a put or call option, respectively. The spread between chooser and call options widens, whereas the spread between chooser and put options narrows with higher strike price. The chooser and straddle option prices also exhibit inflection points with varying strike prices. In terms of volatility, increased underlying asset volatility leads to a rapid growth in chooser option premiums, as holders anticipate profiting from higher volatility. As to the influence of the first expiration date, European chooser options benefit from longer observation periods for accurate price trend determination, while American chooser options may be exercised early on the optimal exercise date.

Conclusions and prospects are discussed in Section 6 and Section 7.

2. Literature Review

Early research on option pricing was based on the Black-Scholes framework proposed in 1973, which laid a foundation for option pricing methods [6]. Merton introduced stochastic processes and martingale theory, establishing a preliminary pricing model that subsequent scholars continuously refined [7]. Engle proposed the ARCH model, which effectively captured heteroskedasticity in time series data by modeling the variance of error terms [8]. Bollerslev extended this with the GARCH model, incorporating the impact of past conditional variances to enhance the characterization of volatility in financial time series [9]. Boyle utilized the Monte Carlo simulation to model multiple future asset paths, offering a broad analytical approach for pricing high-dimensional and complex options [10].

As research progressed, scholars attempted to move beyond traditional models. Wang and Xue built a model under a bi-fractional Brownian motion framework and derived a pricing formula for chooser options in a bi-fractional Vasicek interest rate environment [11] . Li priced chooser options under a fast mean-reverting stochastic volatility model and conducted Greek letter analyses [12], later he proposed zeroth-order and first-order approximate analytical formulas [13]. Dong and He established a volatility DBM-ANN model based on a deep Boltzmann machine to extract volatility factors, partially overcoming the limitations of traditional volatility models [14]. Lian and Chen integrated a Markov regime switching jump diffusion model with a state dependent HJM model to propose a novel pricing framework for European chooser options in incomplete markets,offering a more realistic theoretical basis for pricing complex derivatives [15]. Buchen discovered that the payoffs of dual-expiry options could be perfectly replicated by specific first-order and second-order binary option combinations, avoiding arbitrage and offering a more concise and universal framework for pricing chooser options [16]. Miao and Lee improved the computational efficiency and accuracy of option pricing through the forward Monte Carlo method [17].

Chooser options, as financial products with delayed investment options, are suitable for portfolio investments in high-risk and high-volatility environments, offering higher present values of average returns than traditional call or put options. However, most studies have merely treated chooser options as combinations of ordinary options, ignoring the dynamic risk management ideas and strategic value embedded in the choice rights. Detemple and Emmerling analyzed the impact of model parameters on option prices and exercise boundaries [3]. Nevertheless, regarding flexible adjustments to portfolio strategies in practical investments, such as dynamically adjusting investment decisions in different market trends and combining chooser options with other financial products, only a few articles have mentioned comparisons between chooser options and straddle options [18], as well as cases of water resource managers purchasing chooser options to reduce risks [19]. The book Chooser Options: Understanding, Examples, and Strategies suggests that if chooser options seem overly complex or too risky, alternative strategies can be considered, such as gaining diversified investment exposure through exchange-traded funds (ETFs), or making long-term investments in individual stocks to align with one's risk tolerance and investment objectives.

3. Pricing Methods for European Chooser Options

3.1. Binomial tree model setup

In 1979, Cox, Ross, and Rubinstein mathematically developed a simplified option pricing model, known as the binomial tree model. [7] Employing risk neutral probabilities, they built a binomial framework for the evolution of the stock price in a risk neutral realm. The binomial pricing model posits that stock price fluctuations occur in merely two directions with the probability and extent of each movement holding unchanged throughout the whole observation period. The model splits the length of the observation period into multiple phases, simulates all potential development trajectories of the underlying stock over the entire period based on the stock price's historical volatility. By working in reverse from the end to the start, they computed the option's value in each discrete time interval through discounting the future expected value, ultimately ascertaining the option value at the initial moment.

We first examine a standard European chooser option granting holders choosing right between a European call option on an underlying stock with price S_t, exercise price K, and expires at T or a European put with other parameters holding the same. Let T_0 denote the expiration time of the choosing right(in this case the type of option should be chosen on this particular day), where $T \leq T_0$. Periods of time are split into uniform intervals, with each time step lasting t. Correspondingly, N time steps are gone through before expiry date T and n_0 steps before the choosing date.

3.2. Direct Pricing Method

Since investors don't need to immediately determine the option type when buying a chooser option, their decisions are made based on future market conditions until the choosing date. When the underlying asset's price rises over time, investors will opt for the call option. Conversely, if the underlying asset's price continues to decline, they will choose the put option. It is evident that at the selection time, the chooser option value equals the maximum of the prices of corresponding call and put options. Therefore the payoff function of the chosen option at time T_0 is:

$$Ch_{T_0} = \max\left[C(T - T_0, S_{T_0}, K), P(T - T_0, S_{T_0}, K)\right]$$

3.2.1. Multiple-step binomial tree model

At time T_0, if the stock experiences k times of declines and $n_0 - k$ times of rises, the asset price is given by:

$$S_{T_0}^{(k)} = S_0 \cdot u^{n_0-k} \cdot d^k$$

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From the choosing time $n_0 t$ to expiration Nt, there are $N - n_0$ time steps left. The expected payoff of the call option C_{T_0} is the expected discounted payoff on all binomial paths:

$$C_{T_0}^{(k)} = e^{-r(T-T_0)} \sum_{m=0}^{N-n_0} C_{N-n_0}^{m} p^{N-n_0-m} (1-p)^m \left(S_{T_0}^{(k)} u^{N-n_0-m} d^m - K\right)^+$$

Similarly, the expected payoff of the put option P_{T_0} is calculated as:

$$P_{T_0}^{(k)} = e^{-r(T-T_0)} \sum_{m=0}^{N-n_0} C_{N-n_0}^{m} p^{N-n_0-m} (1-p)^m \left(K - S_{T_0}^{(k)} u^{N-n_0-m} d^m\right)^+$$

The derivation of the payoff formula is elucidated as follows. At the designated time T_0, the underlying asset price is posited to be $S_{T_0}^{(k)}$, signifying that the asset has experienced k instances of depreciation and $n_0 - k$ instances of appreciation. With this premise, the call option's value is ascertained by initially enumerating all potential future trajectories over the subsequent $N - n_0$ time intervals. The summation $\sum_{m=0}^{N-n_0}$ iterates through all permissible quantities of downward movements m. For each m, the binomial coefficient $C_{N-n_0}^{m}$ quantifies the number of distinct trajectories that result in exactly m downward movements, while the risk-neutral probability $p^{N-n_0-m}(1-p)^m$ denotes the likelihood of attaining any particular trajectory. By multiplying these three components by the payoff function $(S_{T_0}^{(k)} u^{N-n_0-m} d^m - K)^+$ and aggregating across all trajectories, the expected terminal payoff is obtained; discounting this expected value at the risk-free rate via $e^{-r(T-T_0)}$ yields the no-arbitrage price of the call option at T_0.

The European put option's value is derived in an analogous manner, with the sole alteration being the payoff function $(K - S_{T_0}^{(k)} u^{N-n_0-m} d^m)^+$, while all other steps, including path enumeration, probability weighting, and risk-neutral discounting, remain consistent.

Consequently, the price of the chooser option Ch_0 is determined as the expected discounted payoff at time $t = 0$:

$$Ch_0 = e^{-rT_0} \cdot \sum_{k=0}^{n_0} C_{n_0}^{k} \left[p^{n_0-k}(1-p)^k \cdot \max\left(C_{T_0}^{(k)}, P_{T_0}^{(k)}\right)\right]$$

3.3. Parity Decomposition Method

It's introduced in Hull's book(1997) [5] that a European chooser option is equivalent in price to the combination of two European options.

Theorem 1. *The price of a chooser option with the choosing date T_0 and the second expiration date T is equal to the sum of the value of a European call and put option with the same underlying asset and strike price, where the call option's strike price is K and expires at T, and the put option's strike price is $Ke^{-r(T-T_0)}$ and expires at T_0.*

Proof. Ch_t denotes the chooser option's value on time t. $C(t, S_t, K)$ and $P(t, S_t, K)$ denote the prices of a European call and put option with t days left before expiration, current stock price S_t, and strike price K. According to the definition of a chooser option, its value at time T_0 is presented in the following formula.

$$Ch_{T_0} = \max\left[C(T - T_0, S_{T_0}, K), P(T - T_0, S_{T_0}, K)\right]$$

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According to the put-call parity,

$$P(T-T_0, S_{T_0}, K) = C(T-T_0, S_{T_0}, K) - S_{T_0} + Ke^{-r(T-T_0)}$$

Substituting the equation above into the expression for Ch_{T_0}, we get:

$$Ch_{T_0} = \max\left[C(T-T_0, S_{T_0}, K), C(T-T_0, S_{T_0}, K) - S_{T_0} + Ke^{-r(T-T_0)}\right]$$

The above formula can be rewritten as:

$$Ch_{T_0} = C(T-T_0, S_{T_0}, K) + (-S + Ke^{-r(T-T_0)})^{+}$$

Which is equivalent to

$$Ch_{T_0} = C(T-T_0, S_{T_0}, K) + P(0, S_{T_0}, Ke^{-r(T-T_0)})$$

By no-arbitrage theory, this equivalence must hold at all prior times, including the initial time.

$$Ch_0 = C(T, S_0, K) + P(T_0, S_0, Ke^{-r(T-T_0)})$$

Therefore, in this pricing model, we decompose the European chooser option into a combination of two European options. Specifically, we calculate the price of a standard European call option and strike price K), and the price of a European put option (with maturity n_0 and a strike price adjusted by $K\exp(-r(T-T_0))$, where $T = Nt$ and $T_0 = n_0 t$). Finally, summing these two option prices gives the price of the chooser option.

This method circumvents the enumeration of all asset price states at the moment of choice, directly calculating the combined value through the European option pricing formula, significantly reducing computational complexity and improving computational efficiency.

Several numerical examples are ran to verify the pricing model, and to determine whether the two methods are equivalent. The verification results show that the European chooser option prices obtained by methods 3.2 and 3.3 are the same, with only negligible rounding errors.

3.4. Monte Carlo method

The Monte Carlo method, a computational technique rooted in mid-20th-century mathematics, has become a cornerstone in financial engineering. The method was initially introduced in the 1940s by Stanislaw Ulam and John von Neumann [20] to simulate neutron behavior in nuclear reactions. It was first applied to finance in 1977 by Phelim Boyle [10], who proposed using it for pricing European options. It works by simulating numerous random scenarios of asset price movements, allowing for the estimation of an option's value through the average of these simulated payoffs, all while adhering to the principles of risk-neutral valuation. This approach is particularly valuable for exotic options that lack straightforward pricing formulas, offering a flexible and powerful alternative to analytical models. At its core, the Monte Carlo method assumes that the movement of the stock price is a stochastic process, following geometric Brownian motion. Additional assumptions include constant volatility of stock prices, known and constant interest rates, and the absence of dividends and transaction costs, among others.

Asset prices are simulated using geometric Brownian motion:

$$S_T = S_0 \cdot e^{\left(r-\frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}\cdot Z}$$

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where Z is a standard normal random variable and σ is the annualized volatility of the underlying asset.

For a European call option:

$$\text{Payoff}_{\text{Call}} = (S_T - K)^+$$

For a European put option:

$$\text{Payoff}_{\text{Put}} = (K - S_T, 0)^+$$

The estimated price of the option is the discounted average payoff:

$$\text{Option Price} = e^{-rT} \cdot \text{Average}(\text{Payoff})$$

Directly utilizing the above pricing formula and Theorem 3.1, we price the European chooser option via the Monte Carlo method. This involves first simulating n paths of the underlying asset's price at maturity T using geometric Brownian motion. For each simulated S_T, we compute the payoff of European options mentioned in Theorem 3.1. These payoffs are then averaged across all n simulations, and the average is discounted back to the present at the risk-free rate $r$to obtain the chooser option's value.

3.5. Comparison between binomial tree model and Monte Carlo method

Using a consistent set of parameters, we executed the Monte Carlo model with n simulations and the binomial tree model with n steps to determine the option prices and measured the time taken by each method.

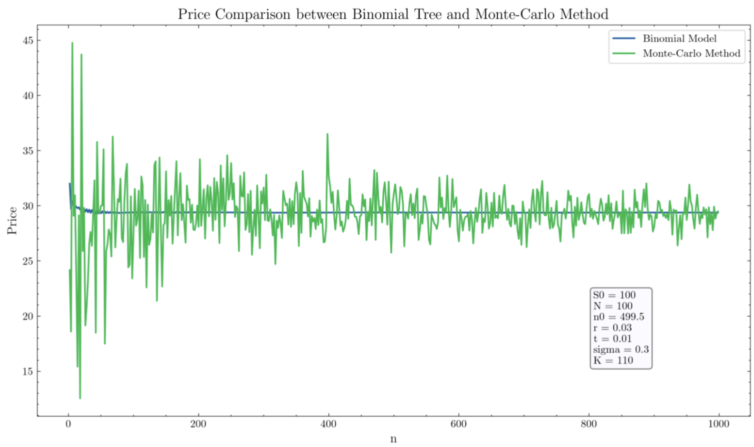

Figure 1: Price comparison of two models.

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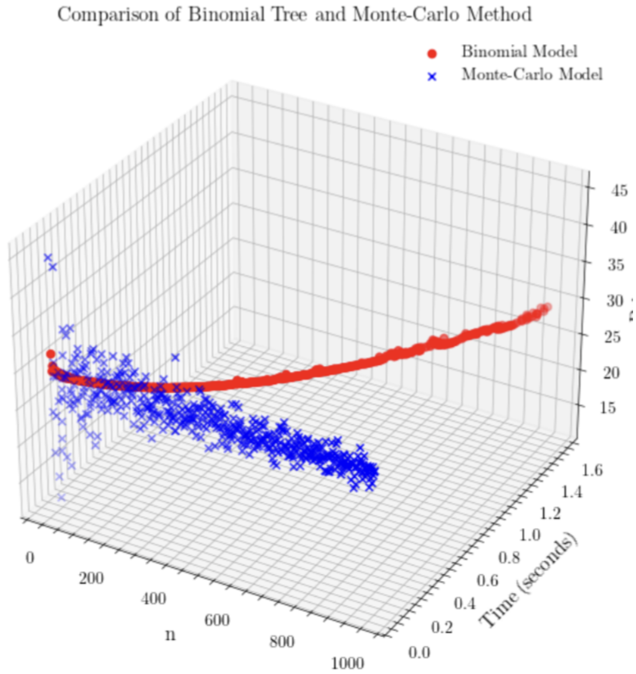

Figure 2: Price and efficiency comparison of two models.

The results (Figure 1,2) indicate that while the binomial tree model does indeed require more computation time as the number of steps increases, it converges to a stable price with fewer steps compared to the Monte Carlo method. In our specific case, we found that around 100 steps were sufficient for the binomial tree model to approximate the central tendency of the Monte Carlo results.

On the other hand, the Monte Carlo method, despite its time efficiency, did not achieve the desired level of precision with only 1,000 simulations, as the results were still subject to significant variability. Overall, if the binomial tree model is allowed a sufficiently high number of steps, it can produce option prices that are comparable to those obtained through the Monte Carlo method. This finding underscores the importance of selecting an appropriate number of steps or simulations to balance computational efficiency with the accuracy of the pricing models.

4. Pricing Methods for American Chooser Options

4.1. Binomial tree model

In the context of American chooser options, the term American bears a dual significance. Firstly, it refers to the flexibility in choice of option type, allowing the holder to select between a call and put option at any point prior to a predetermined date, rather than being restricted to the expiration date. Secondly, it pertains to the exercise style, indicating that once the option type is chosen, it functions as an American-style option, thereby permitting exercise at a flexible time prior to expiration.

Initially, the payoff attributable to the American exercise feature is assessed. For each node in the binomial tree, the option prices for both put and call options are computed. At each node, a comparison is made between the expected value if the holder continues to hold the option until the subsequent step and the value if the option is immediately exercised, with the higher value being adopted as the node's option value. This process is iterated until the nodes corresponding to the final day of the

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choice period are reached, where the higher value between the call and put options is selected as the payoff.

Subsequently, utilizing this result as the final payoff, the option prices for each node are computed iteratively, in a manner analogous to the valuation of European options. Throughout this process, comparisons are made between the larger values of adjacent layers until the initial time is reached, thereby calculating the payoff from the American choice correctly.

At the terminal time node, stock price is denoted by:

$$S_T^{(k)} = S_0 \cdot u^{N-k} \cdot d^k$$

For the American call option at time $T_i = i \cdot t$ (where i starts from $N-1$ and decrease to 0, and for each i, k ranges from i to 0):

$$C_i^{(k)} = \max\left(S_i^{(k)} - K,\, e^{-rt}\left(p \cdot C_{i+1}^{(k)} + (1-p) \cdot C_{i+1}^{(k+1)}\right)\right)$$

For the American put option:

$$P_i^{(k)} = \max\left(K - S_i^{(k)},\, e^{-rt}\left(p \cdot P_{i+1}^{(k)} + (1-p) \cdot P_{i+1}^{(k+1)}\right)\right)$$

At the last choosing date $T_0 = n_0 \cdot t$, the chooser option's payoff is the maximum of the call and put values at that node:

$$Ch_{n_0}^{(k)} = \max\left(C_{n_0}^{(k)}, P_{n_0}^{(k)}\right)$$

For nodes prior to the last choosing date ($T_i = i \cdot t$ where $i < n_0$ and $k < i + 1$):

$$Ch_i^{(k)} = \max\left(\max\left(C_i^{(k)}, P_i^{(k)}\right),\, e^{-rt}\left(p \cdot Ch_{i+1}^{(k)} + (1-p) \cdot Ch_{i+1}^{(k+1)}\right)\right)$$

Ultimately, the initial value of the American chooser option $V_0(0)$ is calculated.

4.2. Analysis on early execution

In the realm of American chooser options, the prerogative to exercise the choice option may be executed prior to the designated expiration date. This conclusion is derived from a comparative analysis of the pricing between standard American choosers and their counterparts that prohibit early selection. The resultant (Figure 3) representation elucidates that on the spectrum where the strike price K is substantial, conducive to the selection of a put option—the value is augmented when the provision for early choosing is permitted.

This observation is somewhat counterintuitive, considering previous discourse indicated that the European chooser option exhibits enhanced value when the interval allocated for observation constitutes a larger fraction of the total time frame. A plausible explanation may be that, under certain circumstances, the optimal exercise date could precede the final date designated for making the choice. Consequently, should the holder defer the decision-making process, the resultant payoff may no longer be optimal.

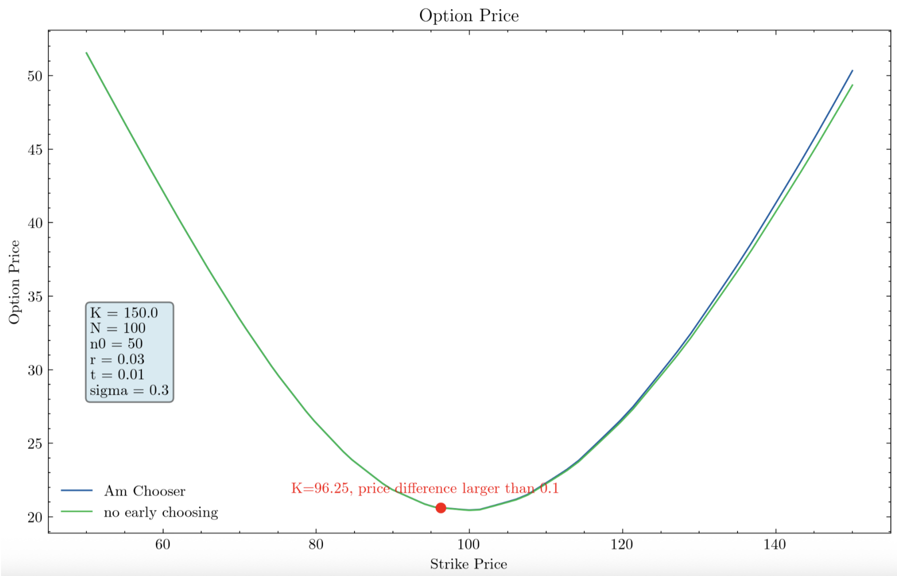

Figure 3: Analysis on early execution premium.

5. Results

The chooser option bestows upon investors the prerogative to make a selection, and we are interested in examining the potential benefits that this power of choice can confer upon investors. Consequently, our initial approach was to conduct a comparative analysis of the prices between chooser options and traditional options.

Conversely, a comparison between the chooser option and a straddle is insightful. A straddle entails the acquisition of both a put and a call option on the same underlying asset with identical strike price K and expiration T, closely mirroring the structure of a European chooser option as discussed in section 3.3. The primary distinction lies in the expiration timing and strike price of the put option within the portfolio. Consequently, the straddle profits when the asset price exhibits significant volatility beyond the cost of the straddle itself, irrespective of the direction of the movement of stock price.

Analogously, the chooser option proves advantageous when significant price fluctuations are anticipated, yet the direction remains uncertain. Similar to a straddle, the payoff of a chooser option increases with the magnitude of deviation from the strike price K. Moreover, should the chooser option be accompanied by the purchase of the underlying stock, and a distinct trend emerges during the observation period, the chooser option holder can capitalize on the more remunerative option type by making an informed choice.

While unlike a straddle, chooser option lets investors decide on option types after some time based on market information, and finally investors only need to pay for one option contract before the choice date, therefore the cost of a chooser option is between a single option and a straddle.

Drawing upon the preceding discourse, this section undertakes a comparative premium analysis focusing on the European and American variants of straddles, chooser options, and traditional vanilla put and call options. The analysis specifically examines the impact of K , the volatility of stock price, and the timing of the initial expiration date on the premium valuation.

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5.1. Influence of strike price

The exercise price K is the key variable that determines the nature of the option. As shown in the figure, when K deviates from the current stock price S_0, the images of American/European option choices exhibit significant nonlinear characteristics.

In the deep in-the-money range (i.e., $K << S_0$ or $K >> S_0$), whether it is an American option or a European option, their graphs will converge to one of the options. This is because when the stock price continuously deviates from the exercise price, the uncertainty of the option selection is reduced, the flexibility brought by the option is compressed, and the option value leans more towards one side. While in the at-the-money range ($K \approx S_0$), the option selection is neutral regarding the increase in stock price and the probability of a bearish trend. The uncertainty reaches a balanced state, and at this time, the gains brought by the choice are minimized, so the price is at the lower point and the flexibility is the strongest.

It is intuitively understood that when the strike price K is set at an exceedingly high level, the probability of the underlying asset's price surpassing K becomes negligible. Consequently, the holders are inclined to exercise the chooser option in favor of a put option. Under such circumstances, the value of the chooser option is predominantly influenced by the put option, leading to a reduction in its premium relative to that of a European put option.Conversely, when the strike price K is established at an exceedingly low level, the likelihood of the underlying asset's price plummeting below K is minimal. In this scenario, the holders are inclined to exercise the chooser option in favor of a call option. Subsequently, the value of the chooser option is primarily dictated by the call option, resulting in a decrease in its premium relative to that of a European call option.

The experimental outcomes (Figure 4) corroborate these expectations: as K increases, chooser option's price initially converges towards that of the call option. The premium of the chooser option relative to the European option is most pronounced when K is situated at intermediate levels. When K escalates to a very high level, the price of the chooser option progressively converges towards that of the put option. Besides, the inflection point of chooser option occurs at S_0.

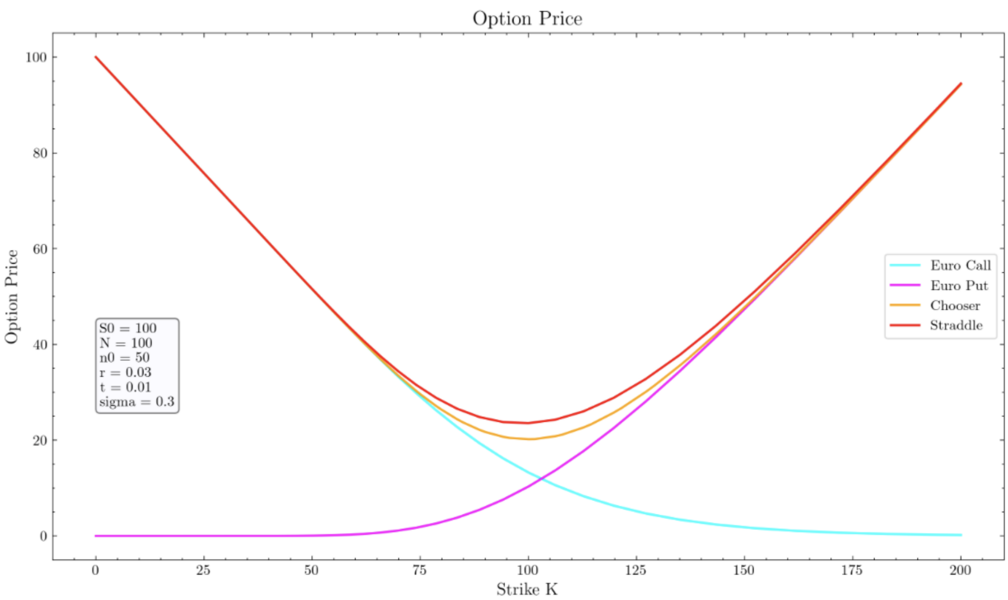

Figure 4: Influence of strike price on European option prices.

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When the option is in the at-the-money range ($K \approx S_0$), its price is lower than that of the straddle option. The core logic of the straddle option is to purchase ”two” options, while the selection option only requires the purchase of one option. Therefore, the cost is lower. When the exercise price deviates from S_0, the profit brought by one deep-in-the-money option can basically cover the loss of the other out-of-the-money option. Thus, the trends of these two options are the same.

Both chooser and straddle options exhibit an inflection point in their pricing dynamics: initially, the call component predominates, causing the option price to decrease in tandem with the call price; ultimately, the put component gains ascendancy, leading to an increase in the option price commensurate with the put price. Consequently, a pivotal turning point emerges between these two phases. However, the locations of these inflection points vary slightly between the two option types: this discrepancy can be elucidated through the lens of the equivalent portfolio theorem. In a straddle, the call and put options share the same strike price, whereas in a chooser option, the put option possesses a marginally reduced strike price. This subtle difference results in the inflection point of the chooser option being modestly retarded relative to that of the straddle.

The premium scenario for American options closely parallels that of their European (Figure 5), albeit with the caveat that American options command a higher price (Figure 6).

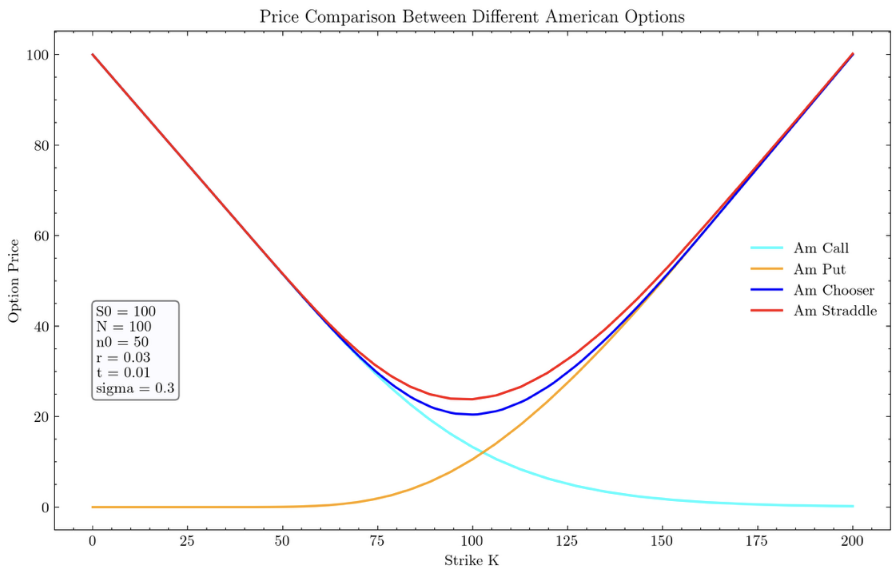

Figure 5: Influence of strike price on American option prices.

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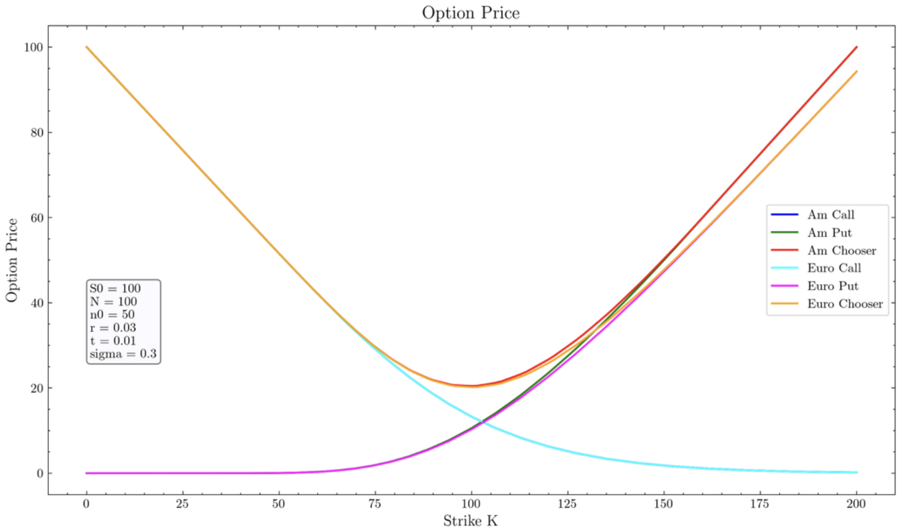

Figure 6: Price comparison between European options and American options.

5.2. Influence of stock volatility

It is well-established that the payoff of a straddle escalates significantly when the underlying asset's price deviates substantially from the strike price K. Investors typically engage in straddles when they anticipate substantial price fluctuations in the stock but are uncertain regarding the direction of the movement. Analogously, chooser option holders anticipate increased volatility to attain enhanced payoffs, thereby justifying the higher premiums associated with such options.

In this instance, volatility was manipulated while other variables were held constant to examine the impact on option premiums. Consistent with expectations (Figure 7,8), the prices of all options escalate with heightened volatility. Initially, standard put and call options are more prone to yield positive payoffs under increased volatility. Here, the disparity between European call and put prices appears to be invariant, a phenomenon that can be rationalized through the put-call parity theorem: the difference between put and call prices is independent of volatility.

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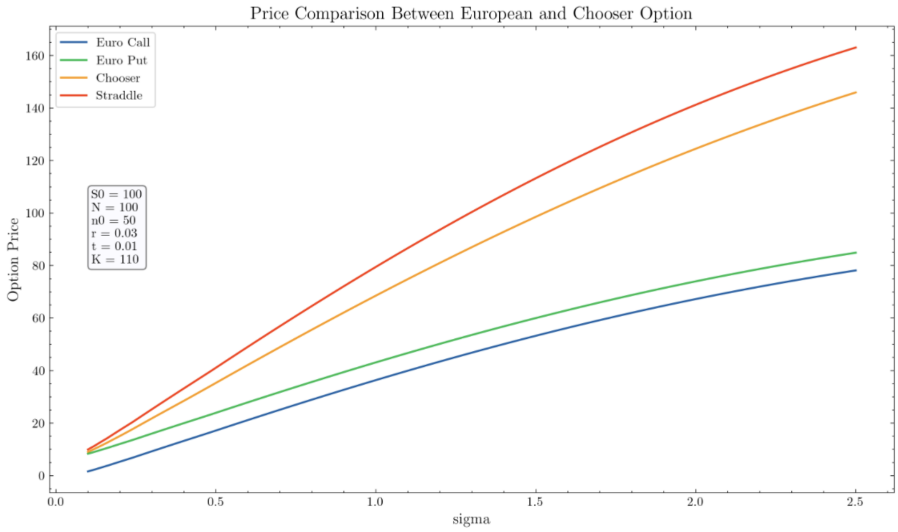

Figure 7: Influence of volatility on European option prices.

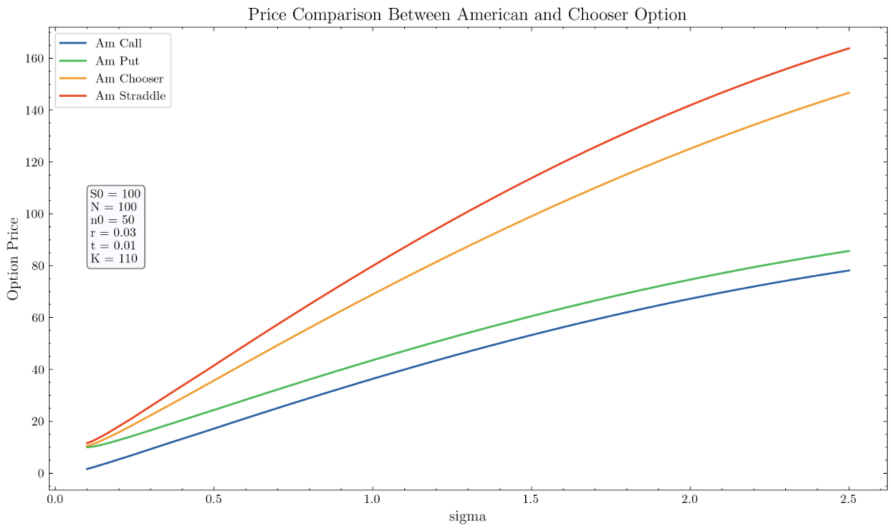

Figure 8: Influence of volatility on American option prices.

Furthermore, the prices of chooser and straddle options increase at a more rapid pace compared with plain vanilla call and put options, as greater price movements in either direction are more likely to generate superior returns.

5.3. Influence of the first expiration date

Ultimately, we have a rather straightforward observation: the longer the observation window, the greater the certainty regarding the stock price at expiration. In instances where the observation period is exceedingly brief, the chooser option should be tantamount to the option exhibiting higher anticipated returns between a conventional European call and a put. Conversely, when the observation

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period spans the entirety of the expiration date, the expected return of the chooser option should align with that of a straddle. (Figure 9,10)

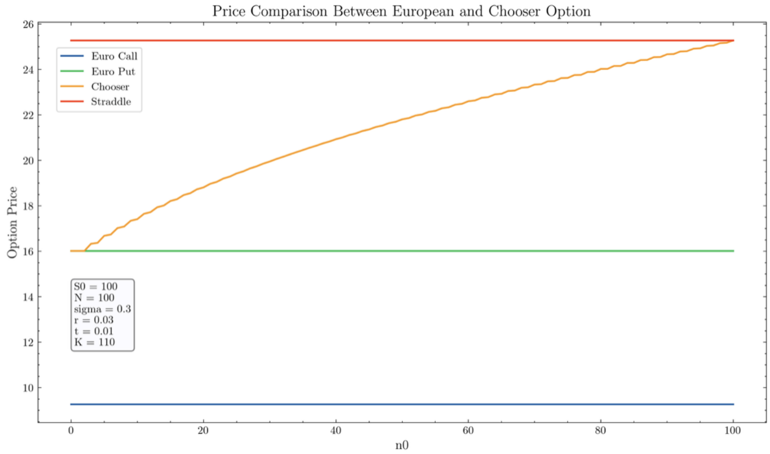

Figure 9: Influence of the first expiration date on European option prices.

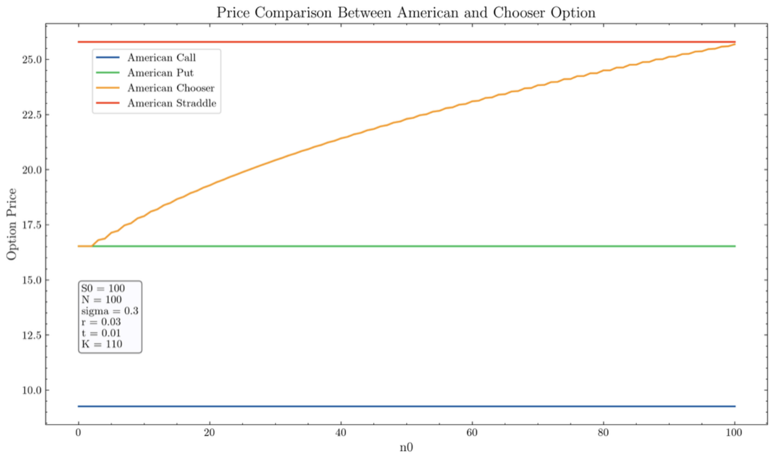

Figure 10: Influence of the first expiration date on American option prices.

This phenomenon arises because an overly early choosing date limits the accumulation of price fluctuations in the underlying asset during the initial period, restricting the release of the flexibility premium inherent in the “two-for-one choice” right. As$n_0$increases, the underlying asset accumulates more fluctuations over additional time steps. This gradual enhancement of the market’s two-directional expectations drives a convex growth in the option price. When the choosing date approaches the option’s expiration, the chooser value stabilizes, converging toward the more valuable of the two component options (call or put).

6. Conclusions

This study systematically explores the pricing mechanisms and value drivers of chooser options, offering comprehensive insights into both European and American variants through theoretical derivation, numerical simulation, and comparative analysis. By employing two core pricing methods: the binomial tree model and the Monte Carlo model, we have established a reliable framework for evaluating the value of options, and at the same time, we have adopted an insight method to enhance computational efficiency–the parity decomposition method, providing solid support for subsequent research.

Regarding pricing methodologies, the findings highlight the strengths of each approach: the parity decomposition leverages the parity relationship between puts and calls to reduce the pricing of European options to a combination of standard calls and puts, significantly lowering computational complexity without sacrificing accuracy. The binomial tree model offers high accuracy and requires a moderate number of steps—about 100 are sufficient for stable results—making it practical for pricing both European and American options. Monte Carlo methods are highly efficient for large-scale simulations but demand a greater number of iterations to converge; nevertheless, after sufficient simulations their results align with those of the binomial tree, confirming the robustness of both techniques.

Our analysis of key factors influencing chooser option premiums reveals critical patterns that guide practical application. Strike price emerges as a key factor: when the strike price is close to the underlying asset's initial price, the chooser option's flexibility is maximized, and its premium sits between that of a single option and a straddle. In contrast, extreme strike prices (either very high or very low) reduce flexibility, causing the chooser's value to converge to that of a put or call option respectively. Volatility also plays a significant role—higher volatility accelerates premium growth, as the potential for profitable price swings in either direction enhances the value of the choice right. Additionally, the timing of the first expiration date affects premiums differently across option types: European choosers benefit from longer observation periods, which allow more informed decision-making, while American choosers may capture additional value through early exercise if market conditions justify it.

Beyond pricing, this study clarifies the unique positioning of chooser options in financial markets. Compared to straddles, choosers offer similar exposure to volatility but at a lower initial cost, making them attractive for investors uncertain about market direction but cautious of high upfront premiums. Compared to vanilla options, they provide flexibility to adapt to evolving market trends, addressing the limitation of fixed payoff structures in traditional derivatives.

Overall, this research enriches the literature on exotic option pricing by focusing on the "choice value" embedded in chooser options—an aspect often overlooked in prior studies that focus mainly on pricing of chooser options under complex market conditions. The findings not only provide theoretical support for refining pricing models but also offer actionable guidance for investors and financial institutions: chooser options can be effectively integrated into risk management strategies or speculative portfolios, particularly in volatile or uncertain market environments. Future work could extend this framework to more complex chooser structures (e.g., those with variable strike prices or multiple choice dates) or relax idealized market assumptions (e.g., incorporating transaction costs or stochastic interest rates) to enhance real-world applicability.

7. Discussion

This paper has established a multi-step binomial tree model, a Monte Carlo method, and a parity decomposition approach to provide a precise pricing framework for chooser options. This framework offers crucial insights for investment decisions and advances the understanding and application of complex derivatives in financial markets. Previous studies often treated chooser options as mere

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combinations of vanilla options, falling short of exploring the unique value embedded in the choice feature. This study zooms in on the premium of chooser options, delving into its origin and providing a fresh perspective on their market worth. Through in-depth analysis of various factors such as strike prices, asset volatility, and choice dates, the study reveals that the premium of chooser options reflects the market pricing of flexibility. In summary, this study offers a comprehensive perspective on the pricing and value analysis of chooser options, and furnishes theoretical support and practical guidance for investors to utilize chooser options in risk management.

However, the study has certain limitations. The pricing models used, including the binomial tree and Monte Carlo methods, have inherent computational constraints. For instance, the binomial tree model experiences increased computational time as the number of steps grows, while the Monte Carlo method requires a substantial number of simulations to reduce result volatility and achieve a stable valuation. Additionally, the study is based on idealized market assumptions like frictionless markets and geometric Brownian motion of asset prices, which may not fully align with real-world market conditions where transaction costs and liquidity differences play significant roles. Future research could explore the application of these pricing frameworks to more complex forms of chooser options, such as those with embedded features or those traded in over-the-counter markets. Furthermore, integrating machine learning techniques to optimize the computational efficiency of the pricing models could address the technical limitations identified in this study. Specifically, machine learning algorithms could be employed to predict optimal choice dates or to more accurately model asset price movements, thereby enhancing the precision and efficiency of the pricing process. Additionally, investigating the impact of market frictions such as transaction costs and liquidity differences on chooser option pricing would provide deeper insights into the practical applicability of these instruments in real-world trading environments. Overall, this study lays a solid foundation for understanding chooser options, while also highlighting areas for further research and development to address the limitations and expand the applicability of the pricing models.

Author contributions

All authors participated in the discussion, reviewed the manuscript, and approved the final version. Xixing Zheng was responsible for writing the introduction and main body, developing the pricing code, analyzing the premium results and visualization of the results. Junrui Wu wrote the literature review and conclusion, and contributed to empirical analysis. Xinping Bao conducted numerical verification.

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