

Statistical Arbitrage with Pairs Trading: Empirical Analysis in Chinese Market

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Abstract. Our analysis of statistical arbitrage is based on the assumption that the spread of two assets follows a mean-reversing Ornstein–Uhlenbeck process when two assets are paired. By incorporating the cointegration test, Hurst component, and optimization into this framework, we develop a technique to identify the best pair in the market and verify the profitability of the pair trading strategy in China’s A-share Market. Although individual pairs may exhibit periods of inconsistency, constructing a diversified portfolio of multiple pairs significantly enhances performance.

Keywords: Pairs Trading, OU Process, Statistical Arbitrage, Cointegration.

1. Introduction

Based on statistical arbitrage, the pair trading strategy exploits deviations from long-term equilibrium by identifying pairs of assets whose historical price movements exhibit a strong co-movement. China’s A-share market presents both challenges and opportunities for effective pair trading strategies due to its high volatility and unique regulatory dynamics.

Traditional pairs trading approaches often rely on simple correlation measures, which may not adequately capture the long-term equilibrium relationships necessary for successful arbitrage. To tackle this challenge, the Engle-Granger cointegration method is used to pinpoint stock pairs in the CSI 300 Index exhibiting robust long-term bonds. The Hurst exponent is employed in our study to gauge the degree of average reversion and enduring memory across the spread series, enhancing the chance that any variance in their spread could be short-lived, returning to the mean, and potentially resulting in lucrative trading chances.

Utilizing the Ornstein-Uhlenbeck (OU) process, we measure the variance in these selected pairs, an outstanding method for simulating behaviors that revert to the mean. Calculating key factors such as reversion speed, average value, and volatility is vital for establishing active trading limits via this mechanism.

Dynamic updates to the model’s parameters are conducted within an extended window framework in our practical use. Analyzing the trading algorithm with pairs from the CSI 300 Index, it was found that despite occasional inconsistencies in individual pairs, building a varied collection

of several pairs greatly enhances performance. The combined pair trading approach surpasses the standard CSI 300 Index and yields elevated overall gains with reduced volatility.

This research introduces a new perspective on paired trading within China's A-share market by merging the methods of cointegration testing with stochastic modeling. The research indicates that dynamic, statistically precise strategies for two trading pairs can proficiently utilize mean-reversion chances in the A-share market of China, providing essential knowledge for traders and researchers focused on quantitative trading methods in burgeoning markets.

2. Specification

2.1. Cointegration test

We apply the Engle-Granger [1] method to identify the cointegration relation of stock pairs. Firstly, we perform Augmented Dickey-Fuller (ADF) test [2], on both series to check whether the logarithmic price series of the two stocks, $\ln S_A(t)$ and $\ln S_B(t)$, are cointegrated in the same order. The ADF test is based on the following regression equation:

$$\Delta X_t = \alpha + \beta t + \gamma X_{t-1} + \sum_{i=1}^p \Delta X_{t-i} \delta_i + \epsilon_t \quad (1)$$

Where $\Delta X_t = X_t - X_{t-1}$ represents the difference of the series, α is a constant, βt represents the trend term, γ is the coefficient on the lagged level of the series, δ_i represents the coefficients of the lagged differences, and ϵ_t is the error term. The null hypothesis of the ADF test is that $\gamma = 0$ indicates the presence of a unit root (non-stationarity). If both price series are found to be non-stationary but share the same integration order, say I(1), we can proceed to the next step.

In the next step, we estimate the long-run relationship between the two price series by regressing one on the other. For example, we can run the following linear regression:

$$\ln(S_A(t)) = \alpha + \beta \ln(S_B(t)) + \epsilon_t \quad (2)$$

where α and β are the parameters to be estimated, and $\epsilon(t)$ represents the residuals of the regression. The goal is to test whether the residuals $\epsilon(t)$ are stationary by applying another unit root test, such as the ADF test. If the residuals show stationarity, i.e., I(0), the two stock prices are cointegrated.

2.2. Spread model

The logarithm spread [3] at time t is defined to be:

$$X_t = \ln(S_A(t)) - \beta \ln(S_B(t)) \quad (3)$$

With $\ln S_A(t)$, $\ln S_B(t)$ denoting the I(1)-nonstationary processes of stocks A and B. The hedge ratio β suggests that if you have 1 dollar invested in asset A, then you should have β dollar shorting asset

B. The spread X_t , as linear combination of $\ln S_A(t)$ and $\ln S_B(t)$ is I(0)-stationary, indicating mean-reverting properties [4]. The spread can be further modeled by the Ornstein-Uhlenbeck (OU) process:

$$dX_t = \rho (X_t - \mu)dt + \sigma dW_t \quad (4)$$

where dW_t denotes a Wiener process. Ornstein-Uhlenbeck process is an ideal framework for modeling stochastic spread with the mean-reversion property. When X_t deviates from long run equilibrium $E[X_t] = \mu$, it will be pulled back to mean value at a speed rate of ρ [5].

Using Equation (4), we can express the OU process in discrete form:

$$X_{t+\Delta t} = (1 - \rho\Delta t)X_t + \rho\mu\Delta t + \sigma\sqrt{\Delta t}\epsilon_t \quad (5)$$

We can estimate its parameters via least squares (LS) regression [6]:

$$X_t = a + bX_{t-1} + \epsilon_t, \quad \epsilon_t \sim \text{i.i.d. } (0, \sigma_\epsilon) \quad (6)$$

where the coefficients a and b are defined as:

$$a = \mu (1 - e^{\rho\Delta t})$$

$$b = e^{-\rho\Delta t}$$

$$\sigma_\epsilon^2 = \frac{\sigma^2}{2\rho} (1 - e^{(-2\rho\Delta t)})$$

The model parameters can then be estimated as follows:

$$\hat{\rho} = \frac{-\ln(\hat{b})}{\Delta t}$$

$$\hat{\mu} = \frac{\hat{a}}{1 - \hat{b}}$$

$$\hat{\sigma} = \frac{\hat{\sigma}_{\sigma_\epsilon} \sqrt{2\hat{\rho}}}{\sqrt{1 - e^{-2\hat{\rho}\Delta t}}}$$

2.3. Hurst exponent

The Hurst exponent (H) is an analytical measurement used to determine long-term memory and self-similarity of time series data. It helps determine whether a time series exhibits persistent, anti-persistent, or random behavior. We compute H in the following steps [7]:

Given a time series $\{X_t\}_{t=1}^N$ of length N, first calculate the mean X:

$$\bar{X} = \frac{1}{N} \sum_{t=1}^N X_t \quad (7)$$

We construct the cumulative deviation series $\{Y_t\}_{t=1}^N$, where:

$$Y_t = \sum_{i=1}^t (X_i - \bar{X}), \quad t = 1, 2, \dots, N \quad (8)$$

For the series $\{Y_t\}$, define the range $R(n)$ as:

$$R(n) = \max_{1 \leq t \leq n} Y_t - \min_{1 \leq t \leq n} Y_t \quad (9)$$

Standard deviation $S(n)$ of the original series $\{X_t\}^n$ can be calculated:

$$S(n) = \sqrt{\frac{1}{n} \sum_{t=1}^n (X_t - \bar{X})^2} \quad (10)$$

For each segment length n , compute the R/S ratio:

$$\left(\frac{R}{S}\right)(n) = \frac{R(n)}{S(n)} \quad (11)$$

Estimate the Hurst Exponent via Linear Regression

We estimate the Hurst exponent H by fitting a linear regression model between $\log\frac{R}{S}(n)$ and $\log(n)$ for various segment lengths n :

$$\log\frac{R}{S}(n) = H \cdot \log(n) + \log(c) \quad (12)$$

where c is a constant and H is the slope of the fitted line, representing the Hurst exponent.

Analysing a given set of time series we can differentiate between three cases:

- When $H = 0.5$, the time series exhibits a random walk behavior.
- When $H > 0.5$, the time series exhibits persistence, meaning future trends are likely to follow past trends.
- When $H < 0.5$, the time series exhibits anti-persistence, meaning future trends are likely to reverse past trends.

3. Empirical implementation

3.1. Pair selection

First, We implemented the Engle-Granger two-step cointegration test on 125 stocks that have been constituents of the CSI 300 Index during the sample period. A total of 125 stocks remain in the CSI 300 index during the sample period, allowing 7750 unique combinations. Our analysis indicates that

out of the 125 stocks tested, 311 pairs exhibit a p-value below the significance level of 0.5%. This suggests that an OU process may well describe these stocks due to their mean reversion properties. The Hurst exponent provides another measure of how fast pairs' spread revert to the mean value. A small Hurst exponent can guarantee a small likelihood of the spread's consistent deviation from long-term equilibrium. Second, We compute the Hurst exponent of the 311 spread series and choose those with a Hurst exponent smaller than 0.5, which demonstrates a mean reversion property. Third, we select the pair consisting of two stocks from the same industry. These stocks are driven by similar macroeconomic factors, providing an internal connection for cointegration relations. The statistics of the selected four pairs are as follows:

Table 1. P-values, hurst exponents and industry for pairs

Pair	p-value	Hurst Exponent	Industry
sh.600018-sh.601006	0.0035	0.464	Transportation
sz.000728-sh.601555	0.0002	0.495	Non-bank finance
sh.600887-sz.002304	0.0006	0.486	Food and beverage
sh.601901-sz.000728	0.0021	0.487	Non-bank finance

3.2. Trading algorithm

The trading strategy begins by setting the initial length of the window for constructing the spread model and selecting an appropriate trading threshold parameter k . Within this window, the model parameters μ , ρ , σ are estimated, and upper and lower trading thresholds are calculated to be [8]

$X_t < \mu_t - k\sqrt{\frac{\sigma t}{2\rho}}$ and $X_t > \mu_t + k\sqrt{\frac{\sigma t}{2\rho}}$..A short (long) position is initiated when the spread reaches the upper (lower) threshold, and the position is closed when it hits the lower (upper) threshold [9].

The extended window add the latest data point iteratively. In each iteration, we aim to maximize profit in the window period and find the optimal threshold k for future transactions. We adopt Nelder-Mead optimizer, a numerical optimizer which is easily accessible in Python's scipy library [10]. In Figure 1 illustrates the spreads of the pairs and optimal thresholds are plotted. Finally, the strategy is optimized by adjusting the rolling window length to maximize the Sharpe Ratio. The transaction cost is set to 0.2%, and the initial investment in four pairs is set to 10000 dollars.

3.3. Strategy performance

Figure 2 illustrates the Profit and Loss (PNL) trajectories of the four individual pairs employed in our pair trading strategy. While each pair has generated respectable returns over the analyzed period, the consistency and robustness of these returns are insufficient. Specifically, the pair sh.600018-sh.601006 failed to exhibit sustained positive returns from 2016 to 2021, indicating potential vulnerabilities in its mean-reversion dynamics during that period. Similarly, the PNL for sz.000728-sh.601555 experienced stagnation between 2017 and 2020, suggesting extended periods where the strategy did not capitalize on price divergences.

Pair trading fundamentally relies on the assumption of mean reversion between correlated securities. However, various irrational market factors—such as speculative bubbles, structural breaks, or shifts in underlying economic conditions—can impede the speed of mean reversion or

even cause divergence, leading to significant losses in extreme cases. These anomalies highlight the importance of robust pair selection and dynamic risk management within the strategy.

In Figure 3, we present the aggregate PNL of the four pairs alongside the PNL of a CSI300 index portfolio for comparative analysis. The aggregated pair trading strategy demonstrates a slight outperformance relative to the market index, characterized by higher cumulative returns and lower volatility.

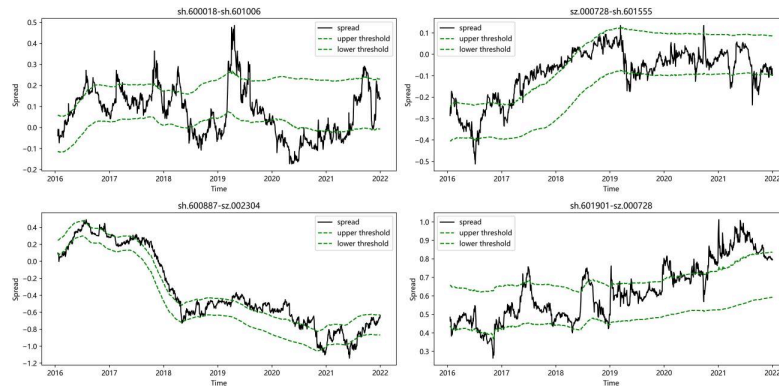


Figure 1. Optimal thresholds and spreads of four pairs



Figure 2. PNL of four pairs



Figure 3. Aggregate PNL of four pairs and PNL of CSI300 index portfolio

The detailed backtesting result is provided in Table 2. While pair trading shows potential for generating excess returns, the elevated volatility and drawdowns associated with individual pairs contribute to lower Sharpe ratios. When considering individual pairs, the Sharpe ratio highlights the inefficiency of return generation relative to the risk taken. However, by constructing a diversified portfolio of multiple pairs, we significantly mitigate idiosyncratic risks associated with single pairs. This diversification reduces overall portfolio volatility and improves the Sharpe ratio, resulting in a more favorable risk-return profile.

Table 2. Backtesting results for four pairs and aggregate portfolio

Portfolio	Annualized return	Max drawdown	Sharpe ratio
sh.600018-sh.601006	13.47%	14.59%	0.053
sz.000728-sh.601555	19.43%	12.22%	0.086
sh.600887-sz.002304	17.46%	15.67%	0.059
sh.601901-sz.000728	22.03%	19.23%	0.077
Aggregate portfolio	16.40%	5.63%	0.115

4. Conclusion

In this study, we developed a dynamic cointegration arbitrage model for pairs trading within the CSI 300 index to enhance trading effectiveness in China's A-share market. We began by selecting stock pairs using the Engle-Granger cointegration test and the Hurst exponent, identifying pairs with strong long-term equilibrium relationships and mean-reverting characteristics.

The spread between these pairs was modeled using the Ornstein-Uhlenbeck process, allowing us to estimate dynamic trading thresholds within an extended window framework. By updating parameters over time, the strategy was able to adapt to changing market conditions. To capture market volatility and generate precise trading signals, we use Bollinger Bands. Additionally, we establish the optimal entry and exit points to capitalize on temporary pricing discrepancies.

While individual pairs generated respectable returns, the trading strategy's performance revealed periods of inconsistency and elevated volatility, resulting in lower Sharpe ratios. However, the addition of multiple pairs to a diversified portfolio resulted in a significant reduction in volatility and drawdowns, leading to improved risk-adjusted returns and a higher Sharpe ratio.

Our dynamic cointegration arbitrage model is effective in capturing mean-reversion opportunities in the CSI 300 index, which provides a viable trading strategy that offers consistent excess returns with manageable risk. By integrating statistical methods and dynamic adjustments, the strategy can be adaptable and resilient in varying market conditions.

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