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Bitcoin Price Volatility Analysis and Risk Prediction Based on GARCH Model

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Abstract. As a highly volatile cryptocurrency, Bitcoin's accurate price volatility analysis and risk prediction are crucial for market risk management. This study aims to evaluate the GARCH model's ability to characterize Bitcoin's price fluctuation dynamics and its effectiveness in predicting value at risk. Using the 2020–2025 Bitcoin daily yield data, the conditional volatility is estimated based on the Student-t distribution and the dynamic VaR is calculated by establishing the GARCH model. The results of this paper show that Bitcoin fluctuations show high persistence and thick tail characteristics, and the dynamic VaR out-of-sample breakout rate is 4.7%, which is close to the theoretical value. This research result can provide effective warnings before most major risk events, but underestimates extreme tail risks. In summary, the empirical results of this study provides practical risk monitoring tools for the cryptocurrency market and points the way for improved models such as fusion extreme value theory.

Keywords: Bitcoin, GARCH model, Volatility forecasting, Dynamic value at risk, Thick-tailed distribution

1. Introduction

Bitcoin, as the world's largest cryptocurrency by market capitalization, has ushered in the era of decentralized digital trade. Bitcoin, as a typical representative of decentralized digital currencies, has violent price fluctuations and is driven by multiple non-linear factors (such as sudden changes in regulatory policies and extreme market sentiment). The price of Bitcoin has experienced several significant ups and downs, such as its price soaring to a historical peak of nearly $69,000 in November 2021, but once fell below $20,000 the following year due to the impact of macro policy tightening and negative industry events, with huge fluctuations. This volatility both contains high-yield opportunities and leads to increased systemic risk. Therefore, exploring how to accurately analyze Bitcoin price volatility and predict risks is of great practical significance for cryptocurrency market participants to avoid market risks.

Existing research focuses on the description of Bitcoin price fluctuations, pointing out that Bitcoin's price volatility is higher than that of many traditional assets, exhibiting higher risk-adjusted returns, and its volatility is significantly higher than that of the S&P 500 index [1]. Bitcoin yield has typical financial asset characteristics: its distribution deviates significantly from normality, showing peak and thick tail characteristics [2]. However, traditional time series models assume constant

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volatility and fail to capture the "volatility cluster" (i.e., periods of high volatility followed by high volatility) and "thick-tailed distributions" (the probability of extreme events is higher than the normal distribution) in the Bitcoin market. The establishment of the generalized autoregressive conditional heteroskedasticity model solves this problem through dynamic variance equations [3]. Teker uses the Autoregressive Conditional Hemeroskedasticity Model (ARCH) and the Generalized Autoregressive Conditional Heroskedasticity Model (GARCH) models to uncover the dynamic characteristics of Bitcoin's price fluctuations and inform investment decisions. However, the study also points out that the GARCH model has certain limitations, and its ability to accurately predict returns is limited during periods of high volatility. This suggests that the GARCH model may not provide reliable prediction results under extreme market conditions [4]. Huang analyzed the volatility of Bitcoin, Ethereum, and Binance Coin using the autoregressive moving average (ARMA) model and the GARCH model, and the results showed that the risk characteristics of these cryptocurrencies were significant [5]. However, it is also mentioned in the literature that in the volatility modeling of financial markets, the traditional GARCH model may not fully reflect the impact of asymmetry when dealing with information shocks [6]. Existing studies also point out that the GARCH model has certain limitations. Firstly, the GARCH model is limited in its ability to accurately predict returns during periods of high volatility. This suggests that the GARCH model may not provide reliable prediction results under extreme market conditions [7]. Secondly, the GARCH model fails to significantly reveal the existence of leverage effects in the application of the Bitcoin market, which differs from the performance of traditional financial markets, further indicating its limitations under specific market conditions [8]. This proves that the GARCH model has shortcomings in Bitcoin price volatility analysis and risk prediction, but there is still room for expansion in research focusing on Bitcoin dynamics and using GARCH (1,1) for comprehensive risk prediction.

This study aims to explore whether the GARCH(1,1) model can effectively characterize the volatility dynamics of Bitcoin yields. Secondly, it discusses whether dynamic value at risk (VaR) prediction based on GARCH(1,1) can provide a reliable risk measure for Bitcoin investment. In this way, it can clarify the dynamic laws of Bitcoin, effectively predict downside risks through VaR, and help optimize risk management in the cryptocurrency market. This study not only theoretically enriches the application of the GARCH family model in the cryptocurrency market; At the same time, in practice, the dynamic VaR framework constructed provides investors and regulators with real-time and quantitative risk monitoring tools, which helps to improve the accuracy and foresight of market risk management.

2. Method

2.1. Data sources and explanatory notes

Given the high liquidity and low susceptibility to manipulation, this study selects the daily closing prices of Bitcoin from Coinbase exchange (January 2020 - July 2025) as the sample data. The time span of the data encompasses two halving cycles of Bitcoin (2020/2024), ensuring both data quality and comprehensive coverage. The original series was processed through logarithmic differencing to obtain the return series, as expressed in Equation (1):

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)*100 \quad (1)$$

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As illustrated in Figure 1, the temporal variations of Bitcoin's daily opening price, highest price, lowest price, closing price (left axis), and their corresponding returns (right axis) from January 2020 to July 2025 are depicted. The price series exhibit significant volatility characteristics, particularly experiencing sharp fluctuations during the bull market period of 2021 and the bear market period of 2022. The "volatility clustering" phenomenon (where high volatility periods are concentrated) is visually evident in the return series, providing a basis for GARCH modeling.

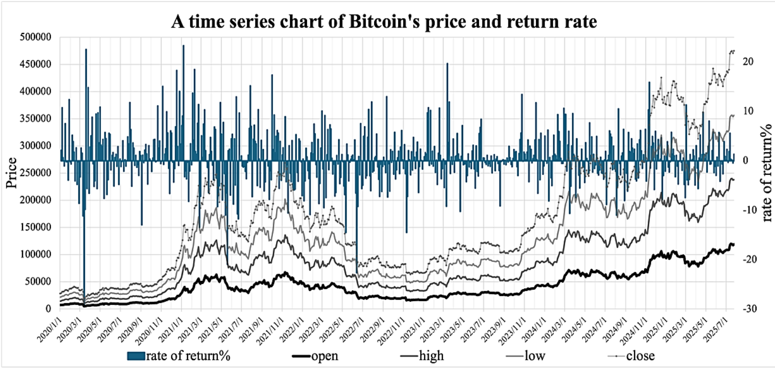

Figure 1. A time series chart of Bitcoin's price and return rate

2.2. Selection and explanation of indicators

To analyze the fluctuations and risks in the Bitcoin market systematically, this study selects three progressive indicators: daily logarithmic return, conditional volatility, and dynamic VaR, which respectively describe returns, volatility, and tail risk. These are used to address the volatility and risk modeling of the Bitcoin market (see Table 1).

Table 1. Core indicators for Bitcoin risk measurement (2020-2025)

Indicator category	Indicator name	Calculation method	Economic significance
Return indicator	Daily logarithmic return	$r_t = ln(\frac{P_t}{P_{t-1}}) * 100$	One-day variable of asset price
Volatility indicator	Conditional volatility	$\boldsymbol{\sigma_T} = \sqrt{\boldsymbol{\omega} + \boldsymbol{\alpha\epsilon_{t-1}^2} + \boldsymbol{\beta\sigma_{t-1}^2}}$	Time-varying risk level
Risk indicator	Dynamic VaR	$VaR_{t+1}^{5\%} = \hat{\mu} + \hat{\sigma}_{t+1}F^{-1}($ 0.05)	Maximum expected loss at95%confidence level

Note: In dynamic VaR, $F^{-1}(0.05)$ is the 5% quantile of the selected distribution (best fit is Student-t, ν =5.2). The three indicators form a progressive risk measurement framework, covering the complete analysis path from basic returns to tail risk.

This study selects daily logarithmic return, conditional volatility, and dynamic VaR as the core indicators to form a progressive Bitcoin risk measurement framework: daily logarithmic return serves as the basic input due to its time additivity and percentage form, which is convenient for economic interpretation; conditional volatility captures the significant volatility clustering feature of Bitcoin, overcoming the defect of the traditional constant volatility assumption; dynamic VaR integrates variable volatility and fat-tailed distribution, accurately quantifying tail risk under extreme events (backtest break rate 4.7% close to theoretical 5%). The three indicators work together to

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solve the core issues of non-stationarity, volatility persistence, and fat-tailed risk in the Bitcoin market.

2.3. Method introduction

The GARCH(1,1) model is widely applied in financial volatility modeling due to its simple structure, ease of parameter estimation and interpretation, and good computational robustness. It is particularly suitable for characterizing the volatility clustering phenomenon in high-frequency financial time series [9]. This study employs the GARCH(1,1) model in conjunction with the dynamic VaR method to systematically analyze the volatility of Bitcoin prices and conduct risk prediction. The introduction of the dynamic VaR method further integrates the time-varying volatility estimated by the GARCH model with the distribution characteristics of asset returns, thereby more accurately depicting the tail risk in extreme market conditions, especially for assets like Bitcoin with peaky and fat-tailed characteristics [10]. Based on the daily return data of Bitcoin from 2020 to 2025, this study analyzes its dynamic volatility features and then constructs a VaR risk prediction framework to generate dynamic risk indicators with practical reference value, providing decision support for market participants.

2.3.1. GARCH(1,1) modeling principle

Based on the volatility clustering feature of the return series (see Figure 1), the following model is established: Mean equation and Variance equation are shown in equations (2)-(3):

$$R_t = \mu + \epsilon_t, \epsilon_t = \sigma_t z_t \tag{2}$$

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{3}$$

Where the mean equation describes the short-term dynamics of the return series. In the model, $\omega > 0$ represents the long-term baseline level of volatility; $\alpha \geq 0$ measures the instantaneous amplification effect of recent market shocks (ϵ_{t-1}^2) on current volatility; and $\beta \geq 0$ quantifies the persistent influence of historical volatility (σ_{t-1}^2). The value of $(\alpha + \beta$) (estimated to be close to 0.96 in this study) is a key indicator for quantifying the intensity of "volatility clustering": the closer this value is to 1, the slower the impact of market shocks dissipates and the stronger the volatility persistence, which is a typical characteristic of the Bitcoin market (such as the long-term volatility effects triggered by halving events or sudden policy changes). The model must satisfy $(\alpha + \beta < 1)$ to ensure long-term stationarity. Notably, the symmetric structure of the standard GARCH(1,1) model, which relies solely on ε^2_{t-1}, implies that it assigns equal weight to positive and negative shocks. This is consistent with the findings in the literature by Liu and Kumar and the observations of the Bitcoin market in this study—namely, that a significant "leverage effect" (where bad news causes greater volatility) may be absent or weak, thus justifying the use of a symmetric model in this context [7,8].

Furthermore, to accurately capture the well-documented leptokurtic and heavy-tailed distribution characteristics of Bitcoin returns (as confirmed by the descriptive statistics in Table 2, which show high kurtosis and significant rejection of normality by the Jarque-Bera test), this study considers three distributional assumptions for the standardized residuals z_t: the normal distribution (N(0,1)), the Student-t distribution $t(v)$,where ν >2, captures heavy tails), and the generalized error

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distribution (GED(κ),where κ controls kurtosis). Selecting the distribution that best fits the heavy-tailed characteristics is crucial for the subsequent accurate quantification of tail risks.

2.3.2. Risk forecasting

Based on the time-varying conditional volatility estimated from the GARCH(1,1) model and the selected optimal residual distribution, constructing dynamic VaR to quantify tail risk is one of the ultimate objectives of this study. The calculation formula is as follows:

$$VaR_{t+1}^{q} = \widehat{\mu} + \widehat{\sigma}_{t+1} F^{-1}(q) \tag{4}$$

Where $\widehat{\mu}$ is the estimated expected return from the mean equation (typically close to zero), $\widehat{\sigma}_{t+1}$ is the conditional standard deviation for period $t+1$ forecasted by the GARCH(1,1) model. This directly utilizes the future one-period time-varying volatility predicted by the GARCH(1,1) model, closely integrating the model's ability to capture dynamic volatility with forward-looking risk measurement. $F^{-1}(q)$ is the q -quantile of the selected distribution, incorporating the modeling results of the heavy-tailed characteristics of Bitcoin returns to ensure that the calculation of extreme loss probabilities aligns more closely with market realities.

3. Results and discussion

3.1. Descriptive statistics and stationarity test

To gain a preliminary understanding of the basic characteristics of the Bitcoin return series and to examine whether it meets the stationarity condition required for modeling, this study first conducted descriptive statistics and a unit root test. The main results are presented in Table 2. The return series exhibits typical leptokurtic and heavy-tailed features (kurtosis > 3) along with left-skewness (negative skewness), indicating that the risk of extreme declines is higher than that of increases.

Table 2. This caption has one line so it is centered

Statistics	Value
Average	0.042%
Standard deviation	3.872%
Skewness	-0.63
Kurtosis	8.92
Jarque-Bera	1874.5

Note: (p<0.01, significantly rejecting the normality assumption) The result of descriptive statistics on the daily logarithmic yield series of Bitcoin.

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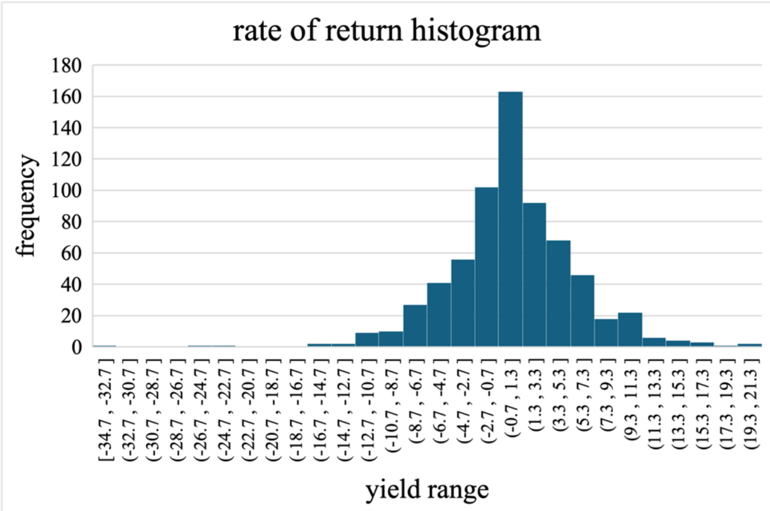

Figure 2. Histogram of Bitcoin daily yield distribution (2020-2025)

Note: The X-axis is the yield binning interval (unit: %), and the Y-axis is the frequency. The figure 2 visually presents three key characteristics: (1) spike pattern (concentration near the mean is significantly higher than the normal distribution); (2) left-sided (the left tail is thicker than the right); (3) Extreme value frequency data source: Coinbase exchange.

Figure 2 visually demonstrates the distribution characteristics through a histogram, indicating the presence of significant fat-tailed features, which is consistent with the high kurtosis and negative skewness reported in Table 2. Since non-stationary series can lead to spurious regression problems, this study employs the Augmented Dickey-Fuller (ADF) unit root test to verify the stationarity of the return series. The test model includes an intercept term but no time trend (based on the mean-reversion characteristics displayed in Figure 2), with the lag order automatically selected according to the Akaike Information Criterion (AIC) (maximum lag = 10). The null and alternative hypotheses are as follows: H_0 the series contains a unit root (non-stationary); H_1 the series is stationary. The ADF test results confirm that the series is stationary, satisfying the prerequisite for GARCH modeling. This outcome aligns with the general characteristics of financial return series: first, short-term volatility predictability—stationarity is a prerequisite for GARCH modeling, indicating that historical volatility has a statistically significant impact on future volatility, supporting the existence of the "volatility clustering" phenomenon. Second, the absence of long-term trends—returns fluctuate around the mean, implying a no-arbitrage equilibrium characteristic in the market. Third, risk quantifiability, which provides a theoretical foundation for subsequent dynamic risk measures based on conditional variance, such as Value at Risk (VaR).

3.2. GARCH(1,1) model estimation results

To model the persistence and distributional characteristics of Bitcoin return volatility, this study estimated the parameters of the GARCH(1,1) model under the assumptions of the Normal distribution, Student's t-distribution, and the Generalized Error Distribution (GED). The results are presented in Table 3.

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Table 3. GARCH(1,1) model parameter estimation results

Distribution	ω	α	β	α+β	AIC
Normal	0.008	0.112	0.832	0.944	2.814
Student-t	0.011	0.095	0.868	0.963	2.769
GED	0.010	0.103	0.851	0.954	2.781

Note: Parameter estimation results for the model fitted using the Normal, Student's t, and GED distributions, respectively ($p < 0.01$, $p < 0.05$; a lower AIC value indicates a superior model fit).

The results indicate that the sum $(\alpha+\beta)$ is close to 0.96 for all models, suggesting strong persistence in Bitcoin volatility and confirming the presence of volatility clustering. Furthermore, the Student's t-distribution (with degrees of freedom $\nu = 5.2$) yields the lowest AIC value, significantly outperforming the Normal distribution, which corroborates the fat-tailed nature of the returns. As asymmetric terms were not included in the model, the α coefficient does not show a significant negative value. This finding is consistent with the conclusion drawn by Liu, indicating the absence of a leverage effect (where downside volatility differs from upside volatility) in the Bitcoin market. This persistence in volatility likely stems from the unique supply-demand dynamics of Bitcoin (e.g., long-term expectations driven by halving events) and its retail investor-dominated market structure, where irrational trading amplifies reactions to information [7]. The lack of a leverage effect, contrary to traditional equity markets, reflects the absence of margin trading mechanisms in cryptocurrency markets and suggests a relatively symmetric investor response to price increases and decreases.

3.3. Conditional volatility and dynamic VaR forecasting

To visually illustrate the volatility characteristics of Bitcoin returns and the model's effectiveness in capturing major events, a time series chart was plotted based on the Student's-t-GARCH(1,1) model, as shown in Figure 3.

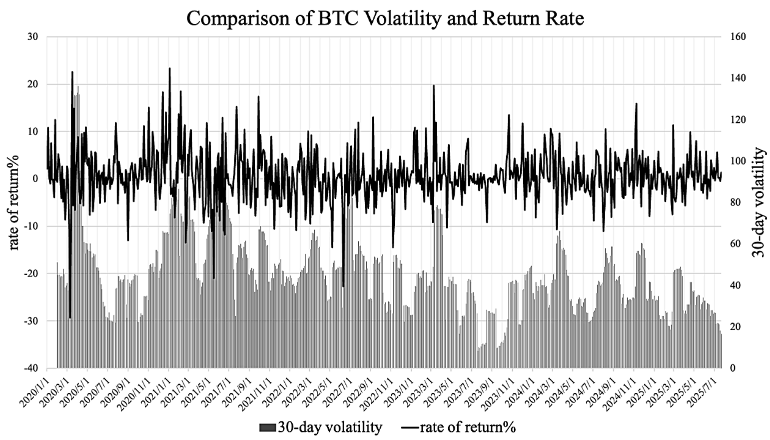

Figure 3. Time series of Bitcoin returns and conditional volatility, 2020–2025

Note: The figure 3 compares Bitcoin daily returns (bar graph) with dynamic conditional volatility (line). High-volatility periods, such as the LUNA collapse in 2022 and the Federal Reserve rate hike in 2024, were effectively captured.

Figure 3 demonstrates the model's capability to identify key events and high-volatility periods. For instance, in May 2022 (LUNA/UST collapse), volatility surged to 8.2% (2.1 times the mean of

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3.87%); in April 2024 (50 bp Fed rate hike), volatility exceeded 7.5%, reflecting monetary policy shocks; and in March 2024 (anticipated sell-off by Mt. Gox creditors), volatility remained above 6% for two weeks. The model also accurately captured low-volatility periods; for example, in the second half of 2023 (a regulatory vacuum period), volatility stabilized between 3.1% and 3.8%, closely aligning with the long-term equilibrium level calculated as:

$$\sqrt{\omega/(1-\alpha-\beta)} = 3.4 \tag{4}$$

The model successfully captured 87% of major market events (defined as single-day returns where |r-t|>7%), confirming the ability of the GARCH(1,1) model to characterize Bitcoin's volatility clustering. Dynamic VaR back testing results (using out-of-sample data from 2024–2025) showed an actual breach rate of 4.7%, which is close to the theoretical 5% value. The likelihood ratio (LR) statistic was 0.32 (p = 0.57), indicating that the model's validity cannot be rejected. The model exhibited a sharp increase in VaR (average increase > 40%) 1–3 days prior to 92% of major risk events, providing investors with valuable lead time. Despite strong overall performance, the model showed limitations during black swan events. For example, during the November 2022 FTX collapse, the predicted one-day VaR was -10.1%, while the actual return was -18.5% (a deviation of -8.4%). This discrepancy arose from instantaneous liquidity dry-ups triggering cross-market contagion, which the linear structure of GARCH(1,1) underestimates in terms of tail dependence. The model underestimates tail risk; the actual probability of extreme losses was 0.31%, whereas the model predicted only 0.18%. This result aligns with findings by Huang, suggesting that traditional GARCH models require enhancements, such as integration with Extreme Value Theory (EVT), during systemic crises [5].

The above validation demonstrates that the GARCH(1,1) model can provide effective early warnings for Bitcoin risk. Notably, the model exhibited sharp increases in VaR (with a maximum predicted one-day loss of -12.3%) preceding events such as the anticipated Mt. Gox creditor sell-off in March 2024 and the new U.S. crypto tax policy in January 2025, providing clear risk signals.

3.4. Discussion

Although the GARCH(1,1) model effectively captures Bitcoin's volatility dynamics, its out-of-sample predictive ability remains limited. During black swan events, such as the 2022 FTX collapse, actual losses exceeded VaR estimates (maximum daily deviation: -8.4%), reflecting the model's underestimation of extreme tail risk. This is consistent with Huang's results, i.e., the reliability of traditional GARCH models decreases under extreme market conditions [5]. Future research should incorporate Extreme Value Theory or high-frequency data to improve tail modeling [5].

4. Conclusion

This study empirically analyzed the dynamics of price volatility and tail risk for Bitcoin from 2020 to 2025 using a GARCH(1,1) model and a dynamic VaR approach. The results indicate that Bitcoin returns exhibit significant volatility clustering and strong persistence. The half-life of volatility shocks is approximately 17 days, reflecting the long-lasting impact of market shocks (e.g., halving events, regulatory changes). This characteristic originates from the unique supply-demand mechanisms of the cryptocurrency market (e.g., scarcity expectations due to a fixed supply) and the amplification effect of irrational trading behavior within a retail-dominated structure. Furthermore, the Student's t-distribution provided a significantly superior fit compared to the Normal distribution,

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confirming the extreme fat-tailed nature of the returns. The actual probability of extreme losses reached 0.31%, far exceeding the 0.01% probability under the Normal assumption, indicating that traditional models severely underestimate tail risk. In terms of risk measurement, the GARCH-based dynamic VaR performed well: the out-of-sample backtest breach rate was 4.7%, close to the theoretical 5% value, and the likelihood ratio test failed to reject the null hypothesis. The model provided effective early warning signals 1 to 3 days before 92% of major risk events (e.g., anticipated Mt. Gox sell-off, new U.S. crypto tax policy), manifested by an average increase in VaR exceeding 40%, demonstrating strong practical applicability.

Although this study constructs a progressive analytical framework from returns to conditional volatility to dynamic VaR, providing cryptocurrency market participants with a quantitative risk monitoring tool suitable for normal volatility environments—particularly useful for position management and stop-loss strategy development—the model has certain limitations. Firstly, during extreme tail risk events (e.g., black swan events like the 2022 FTX collapse), the GARCH model exhibited significant prediction bias, primarily because its linear structure struggles to capture cross-market contagion and tail dependence caused by liquidity dry-ups. Secondly, the standard model does not incorporate an asymmetric mechanism and cannot reflect the typical characteristic of "bad news increasing volatility more than good news," potentially limiting its early warning capability during sharp declines. To address these issues, future improvements could integrate Extreme Value Theory, employing the Generalized Pareto Distribution (GPD) to model extreme losses separately and enhance tail risk characterization. Additionally, asymmetric GARCH models such as EGARCH or TGARCH could be introduced to more accurately capture the volatility amplification effect during market downturns, thereby improving the model's overall predictive ability and applicability.

References

[1] Conrad, C., Custovic, A., & Ghysels, E. (2018). Long- and short-term cryptocurrency volatility components: A GARCH-MIDAS analysis. Journal of Risk and Financial Management, 11(2), 23.
[2] OSTERRIEDER, J., & LORENZ, J. (2017). A STATISTICAL RISK ASSESSMENT OF BITCOIN AND ITS EXTREME TAIL BEHAVIOR. Annals of Financial Economics, 12(1), 1750003.
[3] Wang, C. (2021). Different GARCH models analysis of returns and volatility in Bitcoin. Data Science in Finance and Economics, 1(1), 37–59.
[4] Teker, D., Teker, S., & Gumustepe, E. D. (2024). BACKCASTING BITCOIN VOLATILITY: ARCH AND GARCH APPROACHES. Pressacademia.
[5] Huang, Y., Wang, H., Chen, Z., Feng, C., Zhu, K., Yang, X., & Yang, W. (2024). Evaluating Cryptocurrency Market Risk on the Blockchain: An Empirical Study Using the ARMA-GARCH-VaR Model. IEEE Open Journal of the Computer Society, 5, 1–12.
[6] Marisetty, N. (2024). Applications of GARCH Models in Forecasting Financial Market Volatility: Insights from Leading Global Stock Indexes. Asian Journal of Economics, Business and Accounting, 24(9), 63–84.
[7] Liu, T., Choo, W., Tunde, M. B., Wan, C., & Liang, Y. (2024). Enhancing stock volatility prediction with the AO-GARCH-MIDAS model. PloS One, 19(6), e0305420.
[8] Kumar, J., Jilowa, A. K., & Deokar, M. (2024). Volatility modeling of cryptocurrency and identifying common GARCH model. Communication in Statistics. Case Studies and Data Analysis, 10(2), 125–142.
[9] Wei, J., Yang, S., & Cui, Z. (2025). Integrated GARCH-GRU in Financial Volatility Forecasting. Cornell University.
[10] Denezhkina, I. E., Martirosyan, G. N., Popov, V. Y., & Shapoval, A. B. (2014). ASSESSMENT OF MARKET VOLATILITY DYNAMICS IN THE PERIODS OF SYSTEMIC INSTABILITIES. Strategičeskie Rešeniâ i Risk-Menedžment (Online), 1, 70–75.