

Predictive Research Based on Monte-Carlo Simulation for Autocallable Barrier Reverse Convertible

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Abstract. Structured notes are complex derivatives that blend features of bonds, options and equity linked securities. The study contributes to academic literature by demonstrating how to apply Monte Carlo simulation to model their payoffs under diverse market scenarios, addressing a gap between theoretical pricing frameworks and real-world product complexity. Based on the closing price of Tesla stock between January 12, 2022 and January 17, 2023, Black Sholes model and Jump Diffusion model are developed to explore the initial and mid risk-neutral price of autocallable barrier reverse convertible structure note. Besides, this paper examines the posterior distribution and convergence of the Jump Diffusion model. Black Sholes model is appropriate for stable market without extreme shocks, while Jump Diffusion model is appropriate for extreme shocks concerned. This study carries some implications for improving pricing and risk management for structured notes, informing trading and hedging strategies and advancing hybrid modeling in quantitative finance for investors.

Keywords: Monte Carlo Simulation, Autocallable Barrier Reverse Convertible, Black Sholes model, Jump Diffusion model.

1. Introduction

Monte Carlo simulation (MCS) is an algorithm that uses repeated random sampling. It is used to estimate the likelihood of various results occurring in different circumstances. This model provides the ability to assess the impact of risk across a wide range of real-life scenarios, such as AI, stock prices, sales, forecasting, project management, and pricing, among others. Based on an estimated range of values and compared to a fixed set of input parameters, MCS is used to predict the outcomes of a set of input parameters [1].

MCS is an important research method in option pricing. A Monte-Carlo pricing scheme was used by Fries and Joshi, to address the path-dependent pricing of financial instruments using MCS coupled with conditional analytical techniques to tackle path-dependent pricing [2]. Hyong-Chol et al. transformed pricing model of corporate bond options with credit risk to a terminal-boundary value of the Black Scholes equations, providing a theoretical basis for the pricing of puttable and callable bonds [3]. Zakaria et al. created two-dimensional fractional Black Scholes partial differential equations to explore the dynamic changes of option prices [4]. Bujok et al. developed structural Jump Diffusion model (JDM) to capture the abrupt fluctuations of asset values and

extreme default risks to solve the curse of dimensionality in basket derivative pricing [5]. Wang et al. studied the pricing problem of swaps within the framework of double Henson JDM with approximative fractional stochastic volatility, explored the effectiveness of pricing formula [6]. The JDM has been used by Wang, Liu to price the bivariate option on the basis of the Bayesian approach [7]. Adinya made up for deficiencies of pure diffusion model, optimized the JDM for 10 financial assets, providing a standardized process for model parameter calibration and validity verification [8].

In summary, previous studies on the Monte Carlo method have focused on autocallable products; however, the pricing of reverse convertible notes and the combination of autocallable products and reverse convertible notes has not been addressed. In this study, a MCS approach is used to develop a dynamic autocallable barrier reverse convertible price. Among the objectives of this study are to provide new insights into the autocallable barrier reverse convertible (ABRC), which can be utilized to price ABRC accurately, as well as hedging strategies to mitigate exposure to underlying asset volatility, interest rate fluctuations, and barrier breaches.

2. Methodology

2.1. Data source

This paper incorporates data from Credit Suisse, a global financial services institution that provides financial services and maintains the creditworthiness of investors. MCS is conducted using the USD ABRC, which is provided by Yahoo Finance, for the period between January 12, 2022, and January 17, 2023 [9]. Assume risk free rate is 1 year Treasury Bill rate on January 18, 2022, there are 252 trading days in a year [10].

2.2. Data processing method

In this study, a payoff function is developed to calculate the reverse convertible payoff of an autocallable barrier. The payoff equations (1) is as follows:

$$Payoff = \begin{cases} P^*(1 + c^*t), & \text{if autocalled at time } t \\ P^*(1 + c^*T), & \text{if } S_T \geq B \text{ and not autocalled} \\ P^*\left(1 + c^*T + \beta^*\frac{S_T - S_0}{S_T}\right), & \text{if } S_T < B \text{ and not autocalled} \end{cases} \quad (1)$$

Where P is principal, B refers to maturity barrier, S_0 represents initial underlying price, c indicates coupon rate, T refers to tenor and β is participation. To estimate the theoretical value of the ABRC, Black Sholes model (BSM) and JDM are used. With continuous and smooth price change, BSM is developed, and the corresponding equation (2) is as follows:

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2S^2\frac{\partial^2 V}{\partial S^2} = rV \quad (2)$$

Where V is the price of the derivative at time t when the underlying asset price is S , t is current time, S is current price of the ABRC, r is risk-free rate, σ is volatility of the ABRC price. Assume the underlying asset price follows Geometric Brownian motion (GBM), the equation (3) is introduced:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad (3)$$

Where S_t is stock price at time t , μ is expected return of the ABRC, σ is the volatility of the stock price, W_t is Wiener process.

Modeling the log return rate using JDM involves mixing normal distributions, estimating posterior distributions using Bayesian inference. In the JDM, equation (4) is as follows:

$$dS_t = \mu S_t dt + \sigma S_t dW_t + (J_t - 1) S_t dN_t \quad (4)$$

Where J_t is jump size, N_t is Poisson process.

Equation (5) represents the diffusion part, which is the part of the ABRC price change that does not jump:

$$S_t^{diffusion} = S_{t-1} * \exp \left[\left(\mu - \frac{1}{2} \sigma^2 \right) \Delta t + \sigma \sqrt{\Delta t} * \epsilon_t \right] \quad (5)$$

Where S_{t-1} is the ABRC price at the previous time step, Δt is time interval between $t-1$ and t , ϵ_t is standard normal random variables.

The equation (6) represents the jumping part, which illustrates the effect of superimposed jumping:

$$S_t = S_t^{diffusion} * \exp \left(\sum_{i=1}^{N_t} \ln(J_{t,i}) \right) \quad (6)$$

3. Results and discussion

3.1. Model results and discussion

The parameters of JDM can be found in Table 1.

Table 1. Parameters of JDM

	2.5%	97.5%	GRV
Lambda	0.023	0.549	1.000
Kappa	-0.188	0.194	1.000
Sigma	0.619	0.739	1.000

Lambda is 0.194, indicating a less extreme price fluctuation. It is more likely that an early structured note redemption will be influenced by continuous fluctuations. As the Kappa of the negative jump strategy is -0.001, it indicates the strategy is designed to increase the risk that the stock price at expiration will fall below maturity barrier, which increases the likelihood of the structured note experiencing a download loss and subsequently decreases its theoretical price. Sigma is 0.675, indicating an increasing likelihood of automatic redemption and maturity barrier triggering. As all Gelman Rubin values (GRV) are 1, indicating good convergence properties for the JDM. Table 2 presents the summaries of the BSM and JDM.

Table 2. Summary of BSM and JDM

	BSM	JDM
Theoretical Expected Return (TER)	0.002	-36.933
Simulated Expected Return (SER)	-0.113	-0.234
Final Price Mean (FPM)	365.875	400.436
Annualized Log Return Mean (ALRM)	-1.027	-0.234
Theoretical Standard Deviation (TSD)	0.666	11.101
Simulated Standard Deviation (SSD)	0.247	0.769
Final Price Standard Deviation (FPSD)	279.697	523.274
Annualized Log Return Standard Deviation (ALRSD)	0.673	0.773
Delta	0.535	0
Gamma	0.591	0
Theoretical ABRC Price (Jan 12, 2022) (TABRCPI)	1006.810	1214.262
Theoretical Premium (Jan 12, 2022)	-99.410	108.042
Intrinsic Value (Jan 12, 2022)	929.360	1105.659
Coupon Value (Jan 12, 2022)	77.450	108.603
Theoretical ABRC Price (July 12, 2022) (TABRCPII)	1055.360	1150.019
Theoretical Premium (July 12, 2022)	-50.860	43.799
Intrinsic Value (July 12, 2022)	1000.750	1105.934
Coupon Value (July 12, 2022)	54.610	44.085

Using risk-neutral assumptions, BSM's TER is 0.002, suggesting that the price anticipates a mild upward drift in return. The SER is -0.113 and the ALRM is -1.027, indicating compounded losses over the simulation period. In comparison with JDM's 400.436, The FPM is 365.875, indicating no deviation from realistic pricing. JDM's SSD is 0.769, indicating that prices are closely clustered around the mean. The ALRSD is 0.673, which is consistent with the BSM behavior across metrics as defined by TSD. The BSM's FPSD of 279.697 indicates that the FPM is subject to moderate dispersion around it. Considering the JDM, the TER is -36.933, which indicates the exceptionally negative theoretical return assumed catastrophic losses. Accordingly, SER and ALRM are -0.234, which indicates that losses were greater under simulated paths. Differences between TSD and SSD indicate a disconnect between theoretical and simulated behavior. There is an exaggerated dispersion in simulation paths as indicated by the FPSD of 523.274. For BSM, ABRC is expected to change by \$0.535 for a dollar change in the Tesla stock price. Accordingly, for a \$1 change in the Tesla stock price, the delta of the ABRC will change to 0.591. Considering the scenario of extreme events, the delta and gamma of JDM are zero, which suggests that the ABRC's price is completely insensitive to changes in Tesla's stock price. The BSM's TABRCI price is 1006.810, lower than issue price (\$1106.22) Therefore, in the real world, the BSM may be lacking certain key features of the ABRC. TABRC increases by 4.8% from issue date to middle date, suggesting the ABRC become less undervalued relative to principal. The rise of intrinsic value (7.7%), indicating higher simulated final prices of the ABRC increase the expected principal repayment. The coupon value declines 29.5%, because the remaining term to maturity shortens by half. The JDM's TABRCPI is 1173.356, which is greater than 1106.22. Hence, it is possible that the JDM does not fully reflect the factors that influence the actual price. TABRC fall by 5.3% from January 12, 2022, to July 12, 2022, reducing

the positive premium. The stable intrinsic value suggests JDM predicts little change in the expected principal repayment over the period. Coupon value reduces 59.4%, driven by JDM's inclusion of jump risk reducing expected coupon payments. The Table 3 presents the probability of three scenarios.

Table 3. Sim scenario of BSM and JDM

	Autocall	Not autocall and $S_T \geq B$	Not autocall and $S_T \leq B$
BSM	0.536	0.345	0.119
JDM	0.782	0.086	0.132

BSM indicates a 53.6% probability of triggering automatic redemption. The probability of triggering automatic redemption when ABRC does not trigger automatic redemption and the maturity is greater than maturity barrier is 34.5%, whereas maturity is less than maturity barrier is 11.9%. In comparison with the JDM, the probability of triggering automatic redemption is 78.2%. A maturity that exceeds the maturity barrier triggers automatic redemption at 8.6%, while a maturity that is less than the maturity barrier triggers automatic redemption at 13.2%. Figure 1 illustrates the dynamic autocall probability during the period when the ABRC is in effect.

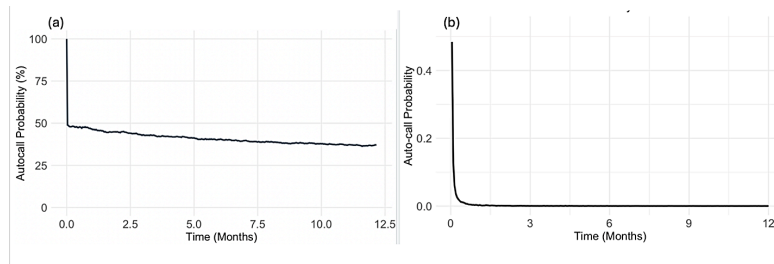


Figure 1. Evolution of autocall probability over time

Figure 1(a) illustrates the dynamic autocall probability of BSM. Initiates with a relatively high probability, around 100%, and gradually declines over time. Figure 1(b) illustrates the dynamic autocall probability for JDM. After a probability of approximately 40%, the probability plummets sharply in the first month and stabilizes at around 0%.

3.2. Model assumption and evaluation

In Figure 2, the posterior density distributions and traces of the parameters of the JDM are shown.

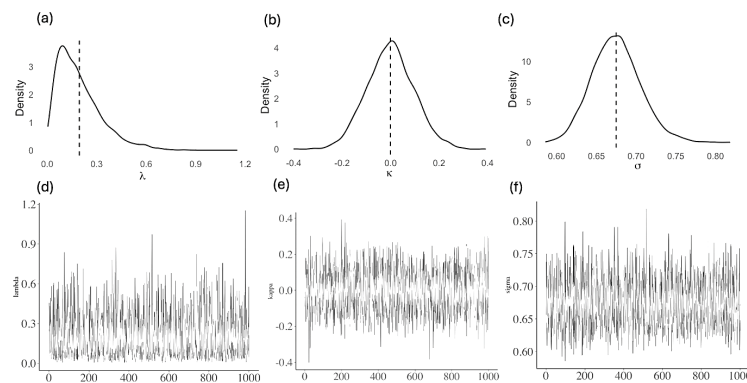


Figure 2. Plot of parameters of JDM

Figure 2(a) illustrates the jump density in the JDM. There is no irregularity in the posterior distribution of Kappa and Sigma, which is well-behaved; Figure 2(b) illustrates the jump size in the JDM around 0 and Figure 2(c) shows the volatility of the diffusion part of the JDM, which is centered around 0.68. Lambda has a right-skewed distribution with a single mode. These distributions provide a good indication that the posterior distribution has converged to a single mode. The figures 2(d), (e) and (f) show the trajectory of lambda, kappa and sigma for each iteration of the MCMC sampling procedure. The chains are intertwined and fluctuate randomly around a certain central value, without climbing, divergence, or periodicity, indicating good convergence—ensuring that the parameter estimation is accurate. As shown in Figure 3, there is a correlation between the parameter sample and the values of different lag orders.

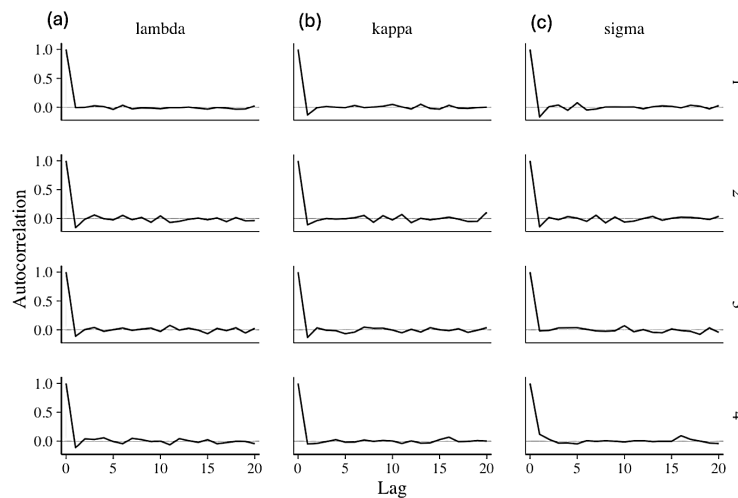


Figure 3. Autocorrelation function of parameters of JDM

Figure 3(a) presents the autocorrelations for different lambda lags. The figure 3(b) displays the autocorrelation plots of different lag kappa's. According to Figure 3(c), the autocorrelations of different lag orders of sigma are shown. As the Lag increases to 1, the autocorrelation coefficients between lambda, kappa, and sigma drop to 0 and are close to the horizontal axis, indicating that the lambda, kappa, and sigma are highly dependent on each other.

This research provides a basis for pricing ABRC, and mitigating the risks of the underlying assets by applying it in the financial and insurance industries. In BSM, risk associated with changes in asset prices is quantified. To offset changes in the value of derivatives such as ABRCs, hedgers use delta to calculate how much of the underlying asset must be held. As well as assessing derivative risks, BSM could price financial derivatives. By capturing certain market phenomena, such as sudden jumps in asset prices, JDM provides a perspective on risk quantification. Moreover, the JDM is able to assist in setting premium rates and reinsurance strategies based on a model which simulates the occurrence and impact of catastrophes. The limitations of this study include the absence of historical data on extreme events, which may restrict the calibration of JDM's jump parameters. The model assumptions may be simplified (e.g., constant jump intensity, which may not fully align with time-varying market conditions); and the cross-asset dependencies may not be fully captured, as ABRC performance can be influenced by interconnected market factors in addition to the underlying assets and interest rates. Pricing and hedging effectiveness may be affected by these factors. The accuracy of the results will be affected by a lack of data, incorrect interpretation of the model, time constraints, and some deviations from the model. Further research may be conducted to

incorporate more time effects, add independent variables, and introduce applicable models to this research result. The BSM can be improved by refining factors such as market friction, continuous price changes, and constant parameters. To adjust for changing frictions and parameter dynamics, BSM can incorporate real-time market data. Further, JDM could be enhanced by estimating the assumptions underlying ABRC's features, while at the same time employing machine learning algorithms to update volatility and drift estimates continuously. For JDM, stress testing under extreme market conditions, as well as expanding the model framework to include multi-factor JDM that integrate correlations among underlying asset volatility, interest rates, and macroeconomic indicators. Furthermore, exploring the application of machine learning algorithms to predict jump occurrences and intensities could enhance the adaptability of JDM in dynamic market environments, whereas expanding the analysis to multi-asset ABRCs could provide a more comprehensive understanding of complex structured product pricing.

4. Conclusion

Based on this research, BSM and JDM are created and compared to simulate ABRC prices. Aside from their differences, both of these models have their own advantages. JDM focuses more on covering the potential market risks, which is why its pricing is conservative and more in line with the actual market situation that may experience extreme fluctuations. As a result of the BSM approach, it can be assumed that fluctuations in target asset prices will not affect ABRC value directly. Instead, other characteristics of continuous fluctuations will be taken into consideration. A sudden jump in JDM's model reflects its ability to capture complex and extreme market conditions. Thus, BSM is appropriate for scenarios where the market is stable and there are no extreme shocks, while JDM is appropriate for scenarios where black swan events are a concern. By integrating the JDM's ability to capture extreme fluctuations into practical pricing frameworks, ABRC pricing could be optimized to enhance pricing accuracy by adjusting for jump intensity and amplitude as a function of real-time market volatility. On the basis of the model and the aforementioned figures and tables, BSM's delta, gamma, and TABRCP more closely reflect the nature of the ABRC than JDM's. In comparison with JDM, BSM's TSD, SSD, FPSD, and ALRSD show a more stable and reasonable level of volatility.

Some problems have been encountered due to a lack of historical extreme events data, an incomplete capture of cross-asset dependencies, and time constraints. In this study, many influencing factors have not been considered, such as interest rates and inflation rates. Machine learning algorithms can be used in the future to capture multiple influencing factors by applying multi-factor JDM, improving jump parameter estimation, and validating the models across diverse market regimes.

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