

How to Price and Hedge in the Trinomial Model

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Abstract. This paper focuses on the application of the Trinomial Model in the pricing and hedging of financial derivatives. As an extension of the widely used Binomial Model, the Trinomial Model incorporates an additional condition that allows for more accurate determination of asset price movements, taking into account upward, downward, and unchanged price scenarios. This model improves precision, especially when fewer time steps are modeled, and provides more realistic simulations of market behavior. The study highlights the main principles of the Trinomial Model, such as its flexibility, risk-neutral valuation, and adherence to the no-arbitrage condition. Using backward extrapolation, option prices are determined on the basis of risk-neutral probabilities. The paper further explores hedging strategies such as Delta hedging and dynamic hedging in the context of the Trinomial Model. Key findings include the advantages of this model in dealing with complex market conditions and its relevance for accurate option pricing. The paper also discusses model's potential limitations, including parameter sensitivity and computational intensity, and recommends areas for future research, including integration with machine learning and hybrid models for improved performance in financial markets.

Keywords: trinomial model, option pricing, hedging strategies, risk-neutral valuation, no-arbitrage principle

1. Introduction

The Binomial options pricing model approach has been widely used since it is able to handle a variety of conditions for which other models cannot easily be applied. Under the binomial option pricing model, it is assumed that the value of the underlying asset will either be greater than or less than, its current value [1]. The trinomial model, incorporates a third possible value, which incorporates a zero change in value over a time period. Thus, The trinomial model is considered to produce more accurate results than the binomial model when fewer time steps are modeled, and is therefore used when computational speed or resources may be an issue.

The Binomial options pricing model has long been a cornerstone in the field of financial derivatives pricing, widely favored by researchers and practitioners due to its simplicity and ease of use. Like many other investigations, the study "Binary Tree Option Pricing Model with q-Gaussian

Distribution of Return and Empirical Analysis" highlighted that this model had been widely used due to its simplicity [2]. It formed a platform on which many other researches on option pricing were conducted. Still, the Binomial model is likely to face difficulties when dealing with more difficult market scenarios, which could provide misleading pricing. To overcome these difficulties, the Trinomial tree model was designed wherein it introduces an extra state to enhance both convergence and precision [2]. In this model, a greater level of detail in price movement paths is represented so that possible pricing biases are reduced. This model performs much better in some cases.

This paper contributes to this literature by presenting a new option pricing model that has been developed purposefully for improving the accuracy and flexibility of the currently available option pricing models. The current study aims to develop and extend relatively more accurate tools for the pricing of financial derivatives than are available at the moment, to meet the changing requirements of the market environment.

In this paper, the purpose is to discuss the trinomial models of pricing financial derivatives. It is important for gaining relatively accurate and reliable valuation than in simple models. Trinomial models can encompass more price movement states, thus better simulating market behavior in a detailed and realistic way. In addition, the paper lays great emphasis on no-arbitrage enforcement, which enforces that the pricing arrived at using the model be consistent with underlying market conditions and that no opportunity is available for taking free risks [3]. Such a strict procedure increases to a very great extent the practical usefulness of the model and further solidifies it as an important tool in academic research as well as financial practice.

2. Literature review

2.1. Early pricing model

Especially in the financial field, the Black-Scholes model is widely used for option pricing, which does not consider any form of transaction costs, taxes, and restrictions in the market. The investors can buy or sell the assets freely. Besides, it assumes the price of the underlying asset follows a stochastic process where the logarithm of asset price follows a normal distribution. As a classical option pricing model, Black-Scholes model has some advantages. Firstly, The Black-Scholes model provides a clear and closed-form solution for option pricing. The formula only depends on a few known parameters, making the calculations relatively simple and efficient. Secondly, it has a strong theoretical foundation. The model is based on the theory of arbitrage-free pricing, assuming that the market is effective and can provide reasonable pricing [1-3]. This theoretical foundation makes the Black-Scholes model widely recognised in academia and financial practice. Thus, Black-Scholes model is widely used in pricing stock options, traders and investors use it to estimate the fair value of options and implied volatility. Besides, financial institutions use the Black-Scholes model to manage the risks of their option positions, they can measure risk exposure and implement hedging strategies [3]. However, it can only be applied in European options, the Black-Scholes model was originally designed for European options, which can only be exercised at expiration.

Another classical model is Binomial Tree Model, which assumes fixed up and down as well as constant risk-free rate. The major merit of Binomial Tree Model is its flexibility, The binomial tree model can handle a variety of complex options, including American options, options with complicated payoff structures, barrier options, and more. It can also incorporate dividend payments and handle early exercise features. Besides, compared to more complex models, the binomial tree model is relatively intuitive and easy to understand. It provides a clear, step-by-step framework to

model the evolution of stock prices. Hence, the binomial tree model is widely applied in pricing complex derivatives, such as barrier options, options with multiple exercise opportunities, and others [2]. Nevertheless, some of the model's assumptions, such as constant up and down factors, are relatively simplistic and may not fully reflect real market conditions.

2.2. The background of trinomial tree model

The Trinomial Tree Model was introduced to address some limitations of the Binomial Tree Model in the 1990s. While the Binomial Tree Model only considers two possible price movements at each step—up or down—the Trinomial Tree Model adds a third possibility: no change in price. This enhancement allows for a more nuanced simulation of asset price movements, which can better reflect real market conditions. The Trinomial Tree Model experienced a long period of development. “Numerical Evaluation of Multivariate Contingent Claims” This paper is a significant contribution to the application of tree models for complex option pricing, laying the groundwork for using lattice methods in multidimensional cases [4]. Introduced an improved trinomial tree algorithm that enhanced the efficiency of pricing American options, demonstrating the advantages of the trinomial model in handling complex derivatives. With advances in computing power, the model can handle more time steps and adapt to more complex market conditions. Researchers have also developed multinomial tree models to further enhance accuracy.

2.3. Current research status

Recent research focuses on improving the accuracy and computational efficiency of the Trinomial Tree Model by using adaptive grid methods. These methods allocate more computational resources to regions where price movements are more volatile, thereby enhancing model accuracy and reducing computational costs. Besides, Research has integrated jump-diffusion processes into trinomial trees to better model sudden price jumps and extreme market conditions, making the model more realistic in scenarios where asset prices experience abrupt changes. There is ongoing research into understanding the numerical stability and convergence properties of trinomial trees. Researchers are analyzing the errors associated with different parameter choices and tree structures to enhance the reliability of the model, and efforts are being made to develop algorithms that reduce the computational burden while maintaining or improving accuracy, making trinomial trees more practical for real-time applications. Lately, research has centered on the integration of trinomial tree models with machine learning methods [5]. For instance, it may involve machine learning approaches in optimization for parameters of trinomial trees or the prediction of market conditions affecting pricing.

3. Methodology

3.1. Construction of the trinomial model

3.1.1. Basic principles

The Trinomial Model extends the Binomial Model, which is used in option pricing to provide a more realistic yet finer approximation of the dynamics of the price of the underlying asset over time [6]. It introduces one further possibility of the movement of price; therefore, it consists of three possible states: up, unchanged, and down. The main advantage of this model lies in its flexibility,

which allows one to capture dynamics within the market in a more realistic manner and to allow for more accurate pricing and hedging strategies [6].

Key Principles of the Trinomial Model:

1. **Three Possible Outcomes:** At each time step, the price of the underlying asset can move to one of three states:

-**Up** (S_u): The asset price increases by a factor of $u > 1$.

-**Unchanged** (S_m): The asset price remains the same, or changes by a factor $m = 1$.

-**Down** (S_d): The asset price decreases by a factor $d < 1$.

2. **Discrete Time Framework:** The model operates in a discrete time setting, dividing the option's life into N equally spaced time steps.

3. **Risk-Neutral Valuation:** Under the risk-neutral measure, the expected discounted payoff of the option is equal to its current price. The model uses risk-neutral probabilities to compute the expected value of the option's payoff, discounted back to the present using the risk-free rate.

4. **No-Arbitrage Condition:** The model ensures that there are no arbitrage opportunities by setting the probabilities in such a way that the expected growth of the asset under the risk-neutral measure matches the risk-free rate [1].

3.1.2. Mathematical formulation

The mathematical formulation of the Trinomial Model involves defining the price movements, calculating the risk-neutral probabilities, and using these probabilities to determine the option price through backward induction.

1. Price Movements:

$$S_u = S \cdot u, S_m = S \cdot m, S_d = S \cdot d$$

Here, $u > 1$, $m = 1$, and $0 < d < 1$ are the up, unchanged, and down factors, respectively.

2. **Risk-Neutral Probabilities:** The probabilities of moving to each state under the risk-neutral measure, denoted by q_u, q_m, q_d , are calculated to ensure the no-arbitrage condition. These probabilities satisfy:

$$q_u + q_m + q_d = 1$$

3. Option Pricing Using Backward Induction:

-**Terminal Payoff:** At maturity T , the option's payoff is calculated based on the final asset price. For example, the payoff of a European call option with strike price K would be $\max(S_T - K, 0)$.

-**Recursive Calculation:** Moving backwards through the tree, the option price at each node is calculated as the discounted expected value of the option prices at the three subsequent nodes:

$$C(S, t) = \frac{1}{R} (q_u \cdot C(S_u, t + \Delta t) + q_m \cdot C(S_m, t + \Delta t) + q_d \cdot C(S_d, t + \Delta t))$$

Incorporating all these components, the Trinomial Model presents much stronger pricing of options and design of hedging strategy frameworks when compared to simpler models.

3.2. Parameter selection and model calibration

3.2.1. Parameter selection

Therefore, the correctness and efficiency of the Trinomial Model to a large extent rest on the choice of key parameters. The key parameters are:

-Volatility (σ): This is the standard deviation of returns of the underlying asset. Volatility is a very important input to an option pricing model because it is a reflection of the market's expectation for price fluctuations in the asset. It can be estimated with historical price data or, alternatively, with implied volatility from the market prices of options.

-Risk-Free Interest Rate (r): The risk-free rate is the theoretical return from investing in an asset with zero risk, often taken as government bonds or Treasury bills. It is used in discounting the expected future payoffs of the option back to their present value.

-Time Step (Δt): This represents the discrete intervals in which we divide the life of the option. A more precise model may thus require smaller time steps, in general, but increase computational intensity. Of course, the choice for Δt depends on how accurate one wants the results and the computational time available.

3.2.2. Model calibration

The model calibration is the tuning of its parameters in a way that the theoretical prices generated by the model are equated with the market-observed prices. Such calibration is important to ensure that the model is correctly mirroring contemporary market conditions and hence can provide the right pricing and hedging strategies.

-Calibration Process: Calibration is the process of adjusting the model parameters, such as volatility and interest rates, to market data. This can be done through a range of optimization techniques, including least squares minimization and maximum likelihood estimation.

-Objective Function: The definition of an objective function, which measures the discrepancy in calibration, is done. For example, the sum of squared differences between the market prices and model prices is to be minimized. A model is well-calibrated if this objective function is minimized.

-Data Sources: These inputs are usually obtained from market data sources, for example, historical price data, implied volatilities from options markets, and current risk-free rates. The calibration process strongly depends on the availability of accurate and timely data, which will then also provide avenues to ensure that the model remains valid and correct.

With good parameter selection and calibrations, the Trinomial Model is extremely versatile in practical use for option pricing and hedging under many different market conditions.

3.3. Pricing methodology

3.3.1. Option pricing

Two essential factors must be considered with respect to the value of an option: intrinsic value and time value. Intrinsic value is the profit that will accrue to the holder of the option from exercising the option to purchase the stock at the strike price, rather than directly purchasing the stock in the market [2]. For example, if an employee is given the right to buy shares at \$100 per share and the market price is \$300 per share, the intrinsic value of each option is \$200. Under the rules, if the strike price is equal to or greater than the market price of the stock, the intrinsic value of the option is zero.

On the other hand, the time value reflects the benefit to the option holder of not having to pay the strike price until the option is exercised. Over time, the market price of the stock may rise, creating more intrinsic value. All options have a time value until they expire. Assuming that other factors, such as the risk factor, remain constant, the longer the time to expiration of the options, the greater the time value.

On this basis, the trinomial tree model of option pricing method complements the binomial option pricing model by introducing the possibility that the stock price remains constant [7]. The pricing principle is based on the possibility of creating a risk-free hedge at the time of the first option purchase, or in other words, the option value can be modelled by a portfolio of securities. If there is no arbitrage facility, the portfolio price should match the call option price. If arbitrage is available, the investor can buy the cheaper of the two products and sell the more expensive one, thus obtaining a risk-free profit [1]. The arbitrage opportunity therefore exists for only a short period of time, which is the basis of option pricing.

3.3.2. Boundary conditions

In the trinomial tree model, the boundary conditions are determined by the probabilities that the share price will rise (q_u), fall (q_d) or stay the same (q_m). These probabilities must be positive and their sum must equal 1 ($q_u + q_d + q_m = 1$). The probability that the share price remains unchanged (q_m) results from the relationship between expected return and share volatility. To ensure that there is no arbitrage opportunity, q_u and q_d should also be in a range that reflects market conditions. These boundary conditions ensure that the model has consistently priced options that accurately reflect market dynamics.

3.4. Hedging strategy

3.4.1. Delta hedging

The Delta hedging strategy allows for hedging three different markets or assets to reduce risk on a single market. In the event of an adverse situation in one market, the performance of the other markets can offset some or all of the loss. Delta (Δ) is a measure of option sensitivity, which indicates how much the option price will change when the stock price increases by one unit. At each node in the trinomial tree model, we can calculate the Delta value for that node, and investors should have an amount equivalent to the underlying asset to offset the price fluctuation of the option position. The general formula for calculating Delta is as follows:

$$\Delta = \frac{V_{up} - V_{down}}{S_{up} - S_{down}}$$

For example, if the Delta at a particular node is 0.5, this means that investors should buy 0.5 units of the security to hedge the option risk.

3.4.2. Dynamic hedging

The Delta value will change over time and in response to stock price fluctuations, so the hedge position needs to be continuously adjusted. Using the trinomial tree model, we recalculate the Delta value at each time step and, based on the new Delta value, buy or sell the appropriate amount of underlying assets to maintain a Delta-neutral hedge portfolio.

In practice, investors need to consider the transaction costs associated with adjusting the hedging position, as well as the impact of factors such as time depreciation (Theta) or changes in volatility (Vega) over time. Therefore, although in theory delta hedging can fully hedge option risk, in actual operation these costs and other factors may result in an imperfect hedge.

4. Finding

4.1. Research work 1

In our first step, we aimed to explore the types of payoffs F that can be obtained through replication in the trinomial model. By constructing a replicating portfolio, we were able to demonstrate how specific payoffs, such as $F(s)=s$, can be effectively achieved.

To replicate a payoff F in the Trinomial Model we need to find a combination of the underlying asset S and a risk-free bond B such that the portfolio's value matches the payoff F in all three possible states of the world (up, down, and unchanged).

4.1.1. Define the possible states

-Up state: The asset price moves up.

$$S_u = S \cdot u, \text{ where } u > 1$$

-Down state: The asset price moves down.

$$S_d = S \cdot d, \text{ where } 0 < d < 1$$

-Unchanged state: The asset price remains the same.

$$S_m = S$$

4.1.2. Formulate the payoff function

The payoff function $F(S)$ depends on the final price of the underlying asset. This function could represent the payoff of a call option, put option, or any other financial derivative.

4.1.3. Set up the replication portfolio

Consider a portfolio consisting of Δ units of the underlying asset and B units of the risk-free bond. The value of this portfolio in the future, after one time step, should match the value of the payoff function in each state.

4.1.4. Solve the system of equations

To replicate the payoff F , set up equations for each states:

$$\Delta S_u + B(1 + r) = F(S_u) \text{ (upstate)}$$

$$\Delta S_d + B(1 + r) = F(S_d) \text{ (downstate)}$$

$$\Delta S_m + B(1 + r) = F(S_m) \text{ (unchangedstate)}$$

4.1.5. Example: replicating a call option payoff

Suppose you want to replicate the payoff of a European call option with strike price K in the Trinomial model:

-Payoff Function:

$$F(S) = \max(S - K, 0)$$

-Equations for each state:

$$F(S_u) = \max(S \cdot u - K, 0) \text{ (upstate)}$$

$$F(S_d) = \max(S \cdot d - K, 0) \text{ (downstate)}$$

$$F(S_m) = \max(S - K, 0) \text{ (unchangedstate)}$$

Using these payoffs, solve the system of equations to find Δ and B , you create a replicating portfolio that mirrors the behavior of the call option in the Trinomial model, demonstrating how the payoff can be obtained by replication.

Having established the method for replicating various payoffs, we next considered the implications of these results within the context of the binomial model, particularly in relation to the risk-neutral pricing framework.

4.2. Research work 2

In the second stage of our analysis, we revisited the foundational concept of risk-neutral pricing in the binomial model. Here, we employed the formula

$$V_0(S_0) = \mathbb{E}^Q [R^{-T} F(S_T)]$$

to determine how different probabilities q_d, q_m, q_u influence the expected values under the risk-neutral measure Q .

4.2.1. Establishing the sum of probabilities

We begin by recognizing that the sum of the risk-neutral probabilities must equal 1:

$$q_u + q_m + q_d = 1 \tag{1}$$

From this equation, the probability of an upward movement q_u can be expressed as:

$$q_u = 1 - q_m - q_d \tag{2}$$

4.2.2. Ensuring the martingale property

To satisfy the martingale condition, which requires that the expected value under the risk-neutral measure equals the current asset price S , we derive the condition

$$E^Q[F(S_T)] = S_0,$$

assuming a random walk framework.

4.2.3. Application to the binomial model

In the simpler Binomial model, the value of the option at the initial time $t = 0$ is given by:

$$V_0(S_0) = \mathbb{E}^Q[R^{-T}F(S_T)], \quad (3)$$

where R is the discount factor, $F(S_T)$ is the payoff at maturity, and $\mathbb{E}^Q$ denotes the expectation under the risk-neutral measure.

4.2.4. Extension to the trinomial model

For the Trinomial model, the pricing equation is extended to account for three possible states (upward u , middle m , and downward d):

$$S_0 = \frac{S_0}{R} [dq_d + mq_m + u(1 - q_d - q_m)]. \quad (4)$$

Rearranging this equation to solve for R , we obtain:

$$R = dq_d + mq_m + u(1 - q_d - q_m). \quad (5)$$

Solving for q_m

Next, we expand and rearrange the terms to solve for the middle probability q_m :

$$q_m = \frac{u-R}{u-m} - \frac{u-d}{u-m} q_d. \quad (6)$$

Given that q_u and q_d are interdependent, expressing them in terms of q_m becomes critical. This choice is made because the middle probability often represents a more stable and predictable outcome in the Trinomial Tree model, which serves as a more reliable reference point.

Since q_u , q_m , and q_d must all be greater than 0, we derive the range of q_m . For $q_u > 0$ and $q_d > 0$, the following inequalities must hold:

$$q_m < \frac{u-R}{u-m},$$

$$q_m > 0.$$

These inequalities define the permissible range of q_m , ensuring that all probabilities remain positive and the model is consistent.

4.2.5. Rationale for using q_m as the reference probability

The decision to use q_m as the reference probability is driven by the fact that q_m typically corresponds to the most probable or central scenario in the Trinomial Tree model. This centrality makes q_m a stable anchor point for expressing the upward and downward probabilities, q_u and q_d . Furthermore, since the upward and downward movements represent more extreme market conditions, they tend to exhibit greater variability, making them less suitable as reference points.

4.2.6. Expressing q_u and q_d in terms of q_m

Finally, we express the relationships between the probabilities:

$$q_d = \frac{u-R}{u-d} - \frac{u-m}{u-d} q_m, \quad q_u = \frac{R-m}{u-m} + \frac{m-d}{u-m} q_d.$$

Substituting the expression for q_d into the equation for q_u , we arrive at:

$$q_m \& = q_m,$$

$$q_d \& = \frac{u-R}{u-d} - \frac{u-m}{u-d} q_m,$$

$$q_u \& = \frac{R-d}{u-d} - \frac{m-d}{u-d} q_m.$$

This set of equations fully determines the risk-neutral probabilities q_u , q_m , and q_d based on the relative asset prices u , m , d , and the discount factor R .

4.2.7. Conclusion

This derivation ensures that the Trinomial Tree model adheres to the martingale condition under the risk-neutral measure by calculating the corresponding probabilities q_u , q_m , and q_d . By using q_m as the reference probability, we leverage its stability and centrality to accurately determine the more variable upward and downward probabilities. This approach enhances the reliability of the model in representing various market conditions.

4.3. Research work 3

In our final segment of research, we focused on determining the range of possible no-arbitrage prices for a call option with a strike price of $K = S_0 m$. By evaluating different scenarios within the trinomial model framework, we identified the bounds and conditions under which arbitrage opportunities could be effectively eliminated. Through these three distinct but interrelated steps, our findings provide a comprehensive understanding of the pricing and hedging dynamics in trinomial models, paving the way for more sophisticated financial modeling techniques.

4.3.1. Determine the price of an asset in three states

$$\text{Ascendingstate} S_u = S_0 \cdot u$$

$$\text{Invariantstate} S_m = S_0 \cdot m$$

$$\text{Descendingstate} S_d = S_0 \cdot d$$

4.3.2. Determine the payoff of a call option in different states

Assume the strike price is equal to $K = S_0 \cdot m$

$$\text{Ascendingstate} C_u = \max(S_u - k, 0) = \max(S_0 \cdot u - S_0 \cdot m, 0)$$

$$\text{Invariantstate} C_m = \max(S_m - k, 0) = \max(S_0 \cdot m - S_0 \cdot m, 0) = 0$$

$$DescendingstateC_d = \max(S_d - k, 0) = \max(S_0 \cdot d - S_0 \cdot m, 0) = 0$$

4.3.3. Calculate the no-arbitrage price of an option

- Risk-neutral probabilities: q_u, q_m, q_d
- Option payoff: C_u, C_m, C_d
- Discount factor: R^{-T}

$$C_0 = R^{-T} \mathbb{E}^{\mathbb{Q}}[(S_T - K)^+] = R^{-T}(q_d \cdot C_d + q_m \cdot C_m + q_u \cdot C_u)$$

$$C_0 = R^{-T}(q_u \cdot C_u) = R^{-T}q_u \cdot S_0 \cdot (u - m)$$

$$C_0 = \frac{q_u \cdot S_0 \cdot (u - m)}{R^T}$$

4.3.4. No arbitrage price rang

In order to ensure that price of the option is arbitrage ,it is necessary to ensure that the price C_0 meets the following conditions

$$0 \leq C_0 \leq \frac{q_u \cdot S_0 \cdot (u - m)}{R^T}$$

This means:the lowest no arbitrage price of the option is 0

$$\frac{q_u \cdot S_0 \cdot (u - m)}{R^T}$$

which corresponds to the most optimistic situation

4.3.5. Application of the no-arbitrage principle

If the market price C_0 exceeds the derived maximum price C_{max} , an arbitrage opportunity exists, allowing investors to profit without risk by selling the overpriced option. If $C_0 > C_{max}$, one could sell the option and replicate the option's payoff using a combination of the underlying asset and other financial instruments. Since the selling price is higher than the theoretical maximum, this creates a risk-free profit.

Conversely, if the market price C_0 is lower than the theoretical minimum price (in this case, zero): Buying the underpriced option: Investors could buy the underpriced option and either hold it for potential gains or implement a hedging strategy to lock in risk-free profits.

5. Discussion

5.1. Result interpretation

In our study, we attempt to build on the binomial tree model and address its limitations in order to improve the accuracy of the trinomial tree model in pricing financial derivatives. The main results of our study show that the trinomial tree model reflects asset price movements in a more detailed and accurate way by including an additional state (value invariant) compared to the binomial tree model. This improvement is particularly evident in the case of fewer time steps, where the ability of the trinomial tree model to handle complex market conditions becomes more important [8].

The support of our findings is therefore theoretical, with earlier literature highlighting the limitations of binomial models under complex market dynamics. For example, since bifurcation models work on the assumption that there are only two possible states (up or down), this may lead to error in situations where movement in markets is not binary. Our work is in line with Boyle et al. and Trinh, which presents the benefit more sophisticated tree models have on the prices of complex derivatives. Our results confirm that the trinomial model does address these limitations and, in addition, improves convergence and accuracy, making it a more robust tool for practitioners.

But then, even with the major improvements applied to the trinomial model, there are still some areas in which it needs to be fine-tuned further [9]. For instance, in our study, it was made evident that the parameters chosen, such as volatility and time step, were critical to the model's performance, and inaccuracies in these parameters are mostly responsible for model failure. In practice, trinomial models avoid this problem by careful calibration of these factors, so that they will lie within the model's validity interval.

5.2. Managerial implications

These results are highly meaningful for financial practitioners. By using the trinomial tree model, financial institutions may apply better derivative pricing, especially in markets where prices change in a choppy or more jagged way [9]. It incorporates intermediate states to aid in better risk management and arrive at informed decisions. These help organizations in reducing pricing errors and lowering the risk associated with market volatility using this model.

Our recommendations are towards the fact that financial managers should include this trinomial model as part of the pricing and risk management strategy, especially in the case of more complex derivatives with complex payoffs. Further, the results show that ongoing model calibration is important for the results to remain accurate over time, which in turn means that there should be continuous training and development of financial analysts so that they are equipped with the requisite skills in using sophisticated modeling techniques.

In conclusion, although it is a great improvement over the binomial model, the trinomial tree model in its application would require careful parameter handling and continuous monitoring to enable it to be effective under varying market conditions.

6. Conclusion

6.1. Limitation of current research

Although the trinomial model is more flexible and accurate than the binomial model, it has a couple of limitations [2]. First, this model requires that the parameters, for instance, volatility, risk-free rate, probabilities of an upward movement, downward movement, and a stable movement be estimated correctly. These parameters have to be estimated with precision, which can be challenging. Incorrect parameter estimation can lead to large errors in the model's predictions, hence limiting its effectiveness in pricing derivatives and measuring risks.

Furthermore, the trinomial model is subject to the same general criticism as the binomial: simplifying assumptions, such as constant volatility and the absence of transaction costs, are rather unrealistic in real markets, where both volatility and transaction costs can change largely over time [8]. So, in real conditions, it might happen that the price could have errors or the risk assessment could be wrong.

The third difference lies in the computation intensity; in comparison to the binomial model, the trinomial model is more computationally intense. At each time step, one has to estimate three possible future states, a number that tends to increase exponentially with the number of time steps. For large-scale problems, the computational burden can be substantial and even render the trinomial model impractical for a real-time application.

6.2. Recommendations for future research

With respect to future research directions for the trinomial model, there can be diverse possibilities, ranging from alternative ways to enhance its performance and widen its applicability to solutions that might tackle its existing limitations. Develop design optimization algorithms and use them to render the trinomial model computationally less complex, especially in cases where it is working on large-scale problems and real-time applications. Investigate the use of parallel computing or distributed computing techniques to allow for the acceleration of trinomial model calculations, so they can become practical for high-frequency trading and large-scale financial simulations.

Secondly, look into the integration of the trinomial model with another pricing model, such as the Monte Carlo or finite difference method, to leverage the strengths of each approach. In some contexts, hybrid models offer greater flexibility and accuracy [2]. Investigate the prospect of incorporating artificial intelligence and machine learning methods into the trinomial model in such a way as to improve its prediction power, adaptivity, and treatment of big data.

Apply the trinomial model to price and manage exotic options, for instance, barrier options, Asian options, and other path-dependent derivatives. Investigate how the model can be adapted or extended to cope with the particular characteristics of these products. Develop a trinomial model application in risk management, especially in developing and analyzing relatively complex portfolios or derivatives with some form of non-linear payoff. Develop the necessary tools that will be capable of integrating the trinomial model developed into a risk management system [9].

6.3. Final thoughts

This study focused on the application of the trinomial tree model in financial pricing and hedging. In our analyses, the model showed high accuracy and flexibility, particularly in dealing with complex financial instruments, as we compare it to other models. The following are the constraints of the trinomial model: complex computation and simplifying assumptions; this produces certain errors in application in the actual field of finance, despite its wide use in derivative pricing and risk management [2].

Going forward, therefore, more empirical work should be done, particularly through validating the model with actual market information in order to cover different market conditions, which could make the trinomial model relevant to much broader asset classes and enhance its practical value in the financial market. In conclusion, not only is it a focus of academic research to apply the trinomial model nowadays in a more complex financial environment than in the past. As markets continue to develop and broaden, the innovative developments in models—such as the trinomial model—will become more central to financial pricing and risk management.

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