

Multi-stage Decision Modeling for Optimizing Multi-process Production Systems

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Abstract. Against the backdrop of intensifying competition within the electronics manufacturing sector and rapid advancements in artificial intelligence, this paper conducts an in-depth analysis of production processes within a case study enterprise. By quantifying critical operational parameters - including component inspection costs, defect rates, and equipment reconfiguration expenditures - into measurable expected profit margins for qualified product outputs, this paper establish a comprehensive evaluation framework through a 0-1 integer programming model. Through rigorous application of exhaustive search algorithms, this paper systematically identify optimal decision-making protocols across distinct production phases. The resultant strategic recommendations provide actionable solutions for manufacturing operations, enabling enterprises to achieve concurrent optimization of production efficiency and quality assurance in practical industrial environments. This research contributes both methodologically and practically to operational decision-making under the dual pressures of market competition and technological disruption.

Keywords: 0-1 integer programming, decision strategies, exhaustive search algorithm.

1. Introduction

In the contemporary technology-driven era, the proliferation of electronic product categories has engendered ubiquitous integration across human productive activities, daily life, and professional domains, delivering substantial societal conveniences. This paradigm shift has precipitated intense market competition as manufacturers proliferate within the sector, now reaching a state of acute saturation. To strategically enhance market efficacy and ensure the commercial viability of premium products, manufacturers are compelled to establish rigorous quality assessment frameworks for finished goods. Such protocols systematically evaluate product specifications against predefined market benchmarks, thereby aligning technical performance with consumer expectations while mitigating quality-related risks in hypercompetitive environments. This operational imperative reflects both technological advancement and market evolution in modern manufacturing ecosystems.

2. Formulation and solution of multi-stage decision-making models

Within manufacturing operations, quality assurance encompasses component inspection, finished product testing, non-conforming product disposition involving disassembly with recycling or compliant disposal, and mandatory defect replacement. Given predefined defect rates across production stages, enterprises necessitate scientific decision-making to ensure quality control while optimizing costs. The systematic framework comprises: (1)Component Processing: Components may either undergo comprehensive inspection protocols or proceed directly to the assembly phase without prior quality verification.(2)Finished Product Management: Completed products are subject to either rigorous quality assurance testing prior to market release or direct distribution without intermediate quality checks.(3)Non-conforming Product Resolution: Products failing to meet quality standards necessitate strategic disposition through either disassembly for material recovery or environmentally compliant disposal.(4)Product Replacement Policy: The enterprise maintains an unconditional obligation to replace defective products identified in the market, irrespective of fault attribution.

Table 1. Challenges encountered by enterprises in production processes

Parameter	Component 1			Component 2		
	Defect Rate	Unit Purchase Cost		Defect Rate	Unit Purchase Cost	Inspection Cost
1	10%	4	1	10%	18	3
2	20%	4	2	20%	18	3
3	10%	4	3	10%	18	3
4	20%	4	4	20%	18	1
5	10%	4	5	20%	18	1
6	5%	4	6	5%	18	3

Table 2. Challenges encountered by enterprises in production processes(continuation)

Parameter	Finished Product			Non-conforming Product		
	Defect Rate	Assembly Cost	Inspection Cost	Market Selling Price	Replacement Loss	Disassembly Cost
1	10%	6	3	56	6	5
2	20%	6	3	56	6	5
3	10%	6	3	56	30	5
4	20%	6	2	56	30	5
5	10%	6	2	56	10	5
6	5%	6	3	56	10	40

2.1. Formulation of multi-stage decision-making models

This paper analyzes inspection decision-making in corporate production by incorporating component assembly status through discrete mathematical logic. Addressing inspection requirements for components 1 and 2, finished products, and disassembly protocols for non-compliance, we establish a 0-1 integer programming model [1]. With cost minimization as the objective function, the model integrates multi-stage planning to determine decision criteria and construct a comprehensive multi-phase decision framework.

2.1.1. Status 1

Phase 1 examines component inspection decisions and classification criteria, where the decision variables indicate whether components undergo inspection, formulated as follows:

$$D_{ij} = \begin{cases} D_{i1} = 1, \text{Perform tests} \\ D_{i0} = 0, \text{No testing is performed} \end{cases}, i, j = 0, 1 \quad (1)$$

In the formula, D_{ij} denotes the inspection status of Component 1 or 2.

The finished product comprises Component 1 and Component 2. The combination of their inspection statuses yields four distinct testing scenarios, as detailed in Table 2.

Table 3. Detection status classification

Status	Inspection Description
Status1	Both Component 1 and 2 inspected
Status2	Component 1 only inspected
Status3	Component 2 only inspected
Status4	Neither Component 1 nor 2 inspected

State 1: Both components inspected and qualified; ready for assembly.

States 2 and 3: Only one component inspected. Product defect rate equals undetected component's defect rate.

State 4: Both components uninspected; pre-inspection defect rates derived from q_3 and assumed q_1/q_2 .

Assume pre-assembly pass rate ω_1 and defect rate ω_2 . Components 1 and 2: n units each. Defect rates: q_1, q_2, q_3 .

Pre-assembly defect rates across scenarios are detailed in the following table:

Table 4. Pre-inspection defect rates of components across scenarios

Status	Pre-assembly defective units	Pre-assembly non-defective units
Status1	0	n
Status2	$n \cdot q_3$	$n(1 - q_3)$
Status3	$n \cdot q_3$	$n(1 - q_3)$
Status4	$n - n(1 - q_1)(1 - q_2)$	$n(1 - q_1)(1 - q_2)$

Inspection costs per state are denoted as:

$$W_1 = \sum_{i=1}^4 COST_1 D_{1i} n + COST_2 D_{2i} n \quad (2)$$

In the formula, W_1 denotes the inspection cost per state, $cost_1$ the inspection cost of Component 1, and $cost_2$ the inspection cost of Component 2.

2.1.2. Status 2

Decision-making for finished product inspection and disposition employs a 0-1 integer programming model.

$$D_{pi} = \begin{cases} D_{p1} = 1, & \text{Perform tests} \\ D_{p2} = 0, & \text{No testing is performed} \end{cases}, \quad i = 1, 2 \quad (3)$$

A successfully inspected finished product confirms all components meet qualification standards.

$$p_1' = 1, p_2' = 0 \quad (4)$$

In the formula, p_1' denotes the pass rate of inspected components, p_2' and the defect rate of inspected components.

For an uninspected finished product, the component pass and defect rates are expressed as:

$$p_1' = p_1, p_2' = p_2 \quad (5)$$

Assume post-assembly pass rate ζ_1 and defect rate ζ_2 . Let G_{1j} denote pre-assembly compliant units, and G_{2j} denote post-assembly compliant units in state j . Applying the inclusion-exclusion principle, the non-inspected product defect probability is calculated as shown in the table below:

Table 5. Post-assembly defect and pass rates of components

Status	Post-Assembly Defect Rate	Post-Assembly Pass Rate
Status1	$G_{1j} \cdot q_3$	$G_{1j} \cdot (1 - q_3)$
Status2	$G_{1j} \cdot (q_2 + q_3 - q_2 q_3)$	$G_{1j} \cdot (1 - q_1)(1 - q_3)$
Status3	$G_{1j} \cdot (q_1 + q_3 - q_1 q_3)$	$G_{1j} \cdot (1 - q_2)(1 - q_3)$
Status4	$G_{1j} \cdot (q_1 q_3 - q_1 - q_2 + 1)(q_2 - 1) - G_{1j}$	$G_{1j} \cdot (1 - q_1)(1 - q_2)(1 - q_3)$

Post-Phase I inspection screening defective parts, two validated component types each retain m units in inventory, with S finished products available. As established, four distinct inspection scenarios exist. For each, we compute the residual quantities of both component types.

In Scenario 1 where both component types have undergone full inspection with defective units eliminated, the validated quantities available for final assembly are quantified as:

$$m_1 = \min \{n(1 - q_1), n(1 - q_2)\} \quad (6)$$

For Status 2, the quantity of remaining components that can be assembled into finished products after undergoing inspection to filter out defective parts is denoted as:

$$m_2 = \min \{n \cdot (1 - q_1), n\} \quad (7)$$

In Status 3, the validated component yield (mirroring the inspection workflow of Status 2) is quantified as:

$$m_3 = \min \{n \cdot (1 - q_2), n\} \quad (8)$$

In Status 4, the co-validated component throughput (via direct-to-assembly of dual inspected parts) is quantified as:

$$m_4 = n \quad (9)$$

Therefore, under non-inspected final output conditions, the post-Phase 2 production batch contains S validated assemblies with ($m_j - S$) non-conforming residuals.

$$S = m_j \cdot p_1, j = 1, 2, 3, 4 \quad (10)$$

$$m_j - S = m_j \cdot p_2, j = 1, 2, 3, 4 \quad (11)$$

The quality verification expenditures per case are quantified with the following cost mapping:

$$W_2 = D_{pi} \cdot COST_3 \cdot m_j \quad (12)$$

In the formula, $COST_3$ denotes the end-item quality assurance cost.

The latent quality deficit induced by omitting end-item verification is quantified as:

$$F = \sum_{i=1}^r m_j \cdot \kappa \cdot \eta_i \cdot \chi_i, i = 1, 2 \quad (13)$$

In the formulation, m_j denotes the end-item production batch size, κ represents the latent defect probability of uninspected assemblies, η quantifies the component replacement expenditure, χ defines the decommissioning cost for disassembling non-conforming components.

2.1.3. Status 3

Phase 3 focuses on establishing a disposition decision-making framework for non-conforming products, employing a 0-1 integer programming model to optimize de-manufacturing decisions [2].

The binary decision variable indicating whether to de-manufacture non-conforming end-items is formulated as:

$$D_{ci} = \begin{cases} D_{c1} = 1, \text{Disassemble} \\ D_{c2} = 0, \text{Do not reconnect} \end{cases} \quad (14)$$

As established in Phase 2 analysis, the influx of uninspected end-items into the market results in m_j -s non-conforming units reaching consumers, where the resultant de-manufacturing cost for these defective assemblies is quantified as:

$$W_{31} = D_{ci} \cdot COST_4 \cdot (m_j - S) \quad (15)$$

In the formulation, $COST_4$ denotes the end-item de-manufacturing expenditure, W_{31} quantifies the Phase 3 decommissioning liability arising from uninspected end-items entering the market

The de-manufacturing expenditure for post-inspection non-conforming end-items is formulated as a function of defect propagation metrics:

$$W_{32} = D_{ci} \cdot COST_4 \cdot (m_j - S + S \cdot p_3) \quad (16)$$

In the formulation, W_{32} Phase 3 de-manufacturing liability for inspected end-items released to market, p_3 non-conformance rate of finished products.

The disassembly decision-making matrix for non-conforming products, integrating outcomes from Phases 1-2, yields 16 discrete operational scenarios through binary decision tree expansion.

These scenarios are systematically categorized as follows:

Table 6. De-manufacturing strategy matrix demonstration

Component 1 Inspection Mandate	Component 2 Quality Screening	End-item Verification Protocol	Non-conforming Unit De-manufacturing Decision
T	T	T	T
			F
		F	T
			F
	F	T	T
			F
		F	T
			F
F	T	T	T
			F
		F	T
			F
	F	T	T
			F
		F	T
			F

2.1.4. Status 4

Building upon the tri-phase modeling architecture, this stage focuses on quantifying replacement losses incurred by enterprises due to non-conforming product substitutions. The cost dynamics are analyzed through the lens of end-item inspection policy trade-offs [3].

The field replacement liability arising from uninspected end-items entering the market is quantified as:

$$W_4 = COST_5 \cdot (m_j - s) \quad (17)$$

In the formulation, W_4 post-market replacement liability for uninspected non-conforming end-items, $COST_5$ field replacement expenditure (FRE) for defective product substitution.

Synthesizing the cost models across all phases, the aggregate cost function is formalized as:

$$W = W_1 + W_2 + W_{3i} + W_4 + R, i = 1, 2 \quad (18)$$

$$W = n \cdot (COST_1 \cdot D_{li} + COST_2 \cdot D_{2i}) + m \cdot COST_3 \cdot D_{pi} + (m - s) \cdot (COST_4 D_{ci} + COST_5 + R) \quad (19)$$

Therefore, the objective function for total cost minimization is formalized as:

$$\min W \quad (20)$$

3. Results and discussion

This article implements an exhaustive search algorithm. Enumerating all decision scenarios from 100 to 1000 at 100-unit intervals, optimal solutions are derived via cost-minimizing comparisons of component-equivalent schemes [4]. As noted, "currently, the screening of the optimal decision-making scheme for the quantity of components being 100 has been completed" [5]. The resultant metrics are detailed in Table 6.

Table 7. The optimal decision-making plan

Statu s	Inspect Component 1	Inspect Component 2	Inspect Finished Product	Disassemble Non-conforming Products	Total Cost
1	F	F	F	F	2380
2	F	F	F	F	2560
3	F	F	T	F	2620
4	T	T	T	F	2600
5	F	T	F	F	2460
6	F	F	F	F	2310

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