

Robust Optimization of Multimodal Transport Routes: A Case Study of a Multimodal Transportation Service Provider

Rongfei Du^{1,a,*}

¹*Faculty of Engineering, The University of Hong Kong, Pok Fu Lam Road, Hong Kong, China*
a. rfdul4@gmail.com
**corresponding author*

Abstract: This paper employs case analysis to examine the robust optimization of multimodal transportation routes under demand uncertainty. In response to the challenges a logistics company faces in transporting heavy cargo, the study develops a hybrid robust optimization model for multimodal transportation, with the objective of optimizing transportation, transshipment, and time-related costs. The model is solved using the ant colony algorithm, and robust optimization is applied across various demand scenarios to evaluate different routing and planning solutions. Findings reveal that robust optimization effectively balances the stability and cost-efficiency of transportation strategies under uncertainty, although it can lead to increased costs. Furthermore, by adjusting the regret parameter in the optimization process, a balance between cost and robustness can be achieved, thus enhancing the overall efficiency of multimodal transport operations. This study offers theoretical guidance for multimodal transport companies, particularly on optimizing resource allocation in the face of demand variability and uncertainty.

Keywords: Multimodal Transport, Hybrid Robust Stochastic Optimization, Uncertain Demand, Ant Colony Algorithm.

1. Introduction

The global movement of goods has become increasingly frequent. With the growing intensity of international trade, a single transportation mode is no longer sufficient to meet the rising demands of cargo transport. In response to these market changes, multimodal transportation, recognized for its safety and efficiency, has been widely adopted in both international and domestic transportation sectors. Multimodal transportation integrates two or more transport modes to provide seamless end-to-end cargo transport services, offering extended supply chains, high resource utilization, and significant economic benefits. It plays a vital role in reducing logistics costs, improving efficiency, fostering green and low-carbon transportation, and enhancing modern comprehensive transportation systems. As China strengthens infrastructure connectivity with Belt and Road countries, networks of road, rail, water, air, and pipelines continue to expand. The rapid development of international logistics, exemplified by China-Europe freight trains, air express, road transit transport, and large-scale containerization of sea freight, has created a favorable environment for multimodal transportation in China[1]. The "Notice on Further Encouraging the Development of Multimodal Transportation," jointly developed by the Ministry of Transport and 18 other governmental

departments over a year, for the first time established a national strategic framework for multimodal transportation, promoted through inter-departmental cooperation[2].

China's extensive multimodal transport market and supportive government policies present substantial growth opportunities for freight forwarding companies. These companies primarily generate profits by coordinating multiple transport modes—land, rail, water, and air—based on client transportation commissions. Efficient and low-cost operations are essential, and scientific transport route planning has a direct impact on both operational efficiency and costs. When planning routes, companies take into account factors such as network structure, transport capacity, regional policies, and the requirements of stakeholders. However, practical challenges arise, particularly in route planning.

Due to limitations in capital and personnel, the transport networks owned by companies are relatively small, restricting their ability to plan optimal routes. Expanding new transport corridors requires cooperation with key transport nodes, which demands considerable resources[3]. Consequently, freight forwarders must weigh current transportation efficiency against the future potential of transport routes and markets[4].

Scholars have widely researched optimization models for multimodal transportation. Lozano et al. examined the shortest path problem in multimodal transport networks using a sequential algorithm[5]. Wang and Han proposed a solution method that transforms multimodal transport networks and applies the shortest path algorithm to minimize transportation, transshipment, and delay costs[6]. Li Menglian et al. developed a multimodal transportation optimization model for post-disaster emergency materials[7], factoring in material demand, traffic conditions, and priority levels using robust optimization. Jiang Yang et al. established a multimodal model to minimize transportation and transshipment costs, which was solved using the cross-entropy algorithm[8].

In most multimodal transport models, cargo volume is assumed to be a fixed value[9], simplifying the research process but neglecting real-world challenges such as seasonal fluctuations and sudden replenishments. As a result, transport volume often remains uncertain during scheduling[10]. Current research on multimodal route optimization under demand uncertainty predominantly utilizes stochastic programming, fuzzy programming, and robust optimization[11]. Given the complexity of the multimodal transport process and the volatile external environment, robust optimization is a particularly effective method when there is insufficient information to ascertain demand distribution[12].

2. Problem description and model construction

2.1. Problem description and research hypothesis

In the domain of multimodal transportation, a logistics company is facing challenges in developing transportation strategies for heavy cargo with a weight-to-volume ratio exceeding 200, under conditions of uncertain demand. Such cargo must be transported from the origin O to the destination D via a combination of road, waterway, and rail networks, often involving several transshipment nodes. Each node presents different transportation options, which vary significantly in terms of cost, time, and efficiency.

Because multimodal transport plans need to be made in advance, while demand remains uncertain, the company must devise transport strategies that account for this uncertainty. Furthermore, market forces can cause transportation costs and times to fluctuate depending on the time and location.

The objective of this research is to identify and optimize a transportation route and mode that meets the company's needs in terms of both cost and time efficiency, while maintaining the stability of the transport plan amidst demand fluctuations. This study will offer scientific decision-making support

to multimodal transport operators, enabling the efficient allocation of resources and the optimization of transport operations.

The study is based on several theoretical assumptions during its exploration of multimodal transportation route optimization:

- 1) Cargo indivisibility assumption: The research assumes that goods must be transported as a single, indivisible unit, prohibiting the division of the cargo for separate shipments.
- 2) Adequate transshipment capacity assumption: It is assumed that all transshipment nodes possess sufficient capacity, allowing for the neglect of potential waiting times and costs at these nodes.
- 3) Mode-switch limitation assumption: The study assumes that changes in transport mode are restricted to specific transshipment nodes, with each batch of goods undergoing at most one mode change per node.
- 4) Unlimited freight capacity assumption: The study assumes that each transport mode has unlimited freight capacity, satisfying all transportation needs without considering vehicle weight or volume limitations.
- 5) Exclusion of external disturbances assumption: The study does not account for external factors, such as weather conditions, equipment malfunctions, or cargo damage, in the transportation process.

2.2. Parameter Description

Table 1: Parameters.

T	Total transportation time, including both transport time and transshipment time.
N	Set of transportation modes.
K	Set of transportation nodes.
T^a	Lower limit of the time window required for joint transportation operations.
T^b	Upper limit of the time window required for joint transportation operations.
P_1	Unit storage cost incurred due to early cargo arrival.
P_2	Unit penalty cost incurred due to delayed cargo arrival.
q_{ij}	Freight volume between nodes i and j .
d_{ij}^n	Distance between nodes i and j using transportation mode n .
v_n	Average speed of transportation mode n .
t_{ij}^n	Transportation time between nodes i and j using transportation mode n .
$t_i^{n_1n_2}$	Transshipment time required at node i for switching from transportation mode n_1 to n_2 .
$ut_i^{n_1n_2}$	Unit transshipment time required at node i for switching from transportation mode n_1 to n_2 .
ck_{ijn}	Unit transportation price for transporting cargo using mode n between nodes i and j .
$cn_i^{n_1n_2}$	Unit transshipment cost incurred at node i for switching from mode n_1 to n_2 .
c_{ij}^n	Transportation cost incurred when using mode n between nodes i and j .
$c_i^{n_1n_2}$	Transshipment cost incurred at node i when switching from mode n_1 to n_2 .

Table 2: Decision Variables.

X_{ij}^n	Whether transportation mode n is used between nodes i and j . If it is used, $X_{ij}^n = 1$; otherwise, $X_{ij}^n = 0$.
$Y_i^{n_1 n_2}$	Whether a switch from transportation mode n_1 to n_2 occurs at node i . If the switch occurs, $Y_i^{n_1 n_2} = 1$; otherwise, $Y_i^{n_1 n_2} = 0$.

2.3. Model Construction

In the context of multimodal transportation route optimization under demand uncertainty, the central challenge is integrating stochastic demand factors into the traditional route optimization framework while ensuring that the model maintains adaptability and robustness across varying demand scenarios. This study's approach begins with the development of a deterministic multimodal transportation route optimization model. Subsequently, the model is extended to account for demand uncertainty, transforming it into a hybrid robust optimization model. Such a model is designed to better accommodate fluctuations in demand, offering valuable decision support for multimodal transportation companies navigating uncertain market environments.

2.3.1. Basic Model

This study begins by developing a deterministic multimodal transportation route optimization model, where demand is considered fixed and known.

$$\begin{aligned} \min C(x) = & \sum_{i \in K} \sum_{j \in K} \sum_{n \in N} q_{ij} d_{ij}^n c k_{ijn} X_{ij}^n + \sum_{i, j \in K} \sum_{n_1 \in N} \sum_{n_2 \in N} q_{ij} c n_i^{n_1 n_2} Y_{n_1 n_2}^i \\ & + \sum_{i \in K} \sum_{j \in K} P_1 \max[(T^a - T), 0] \cdot q_{ij} + \sum_{i \in K} \sum_{j \in K} P_2 \max[(T - T^b), 0] \cdot q_{ij} \end{aligned} \quad (1)$$

s.t.

$$\sum_{n \in N} X_{ij}^n \leq 1, \quad \forall i, j \in K, \forall n \in N \quad (2)$$

$$\sum_{n_1 \in N} \sum_{n_2 \in N} Y_{n_1 n_2}^i \leq 1, \quad \forall i \in K, \forall n_1, n_2 \in N \quad (3)$$

$$X_{ij}^{n_1} \cdot X_{jm}^{n_2} = Y_{n_1 n_2}^j, \quad \forall i, j, m \in K, \forall n_1, n_2 \in N \quad (4)$$

$$X_{ij}^n \in \{0, 1\}, \quad Y_{n_1 n_2}^i \in \{0, 1\} \quad (5)$$

The objective function (1) comprises four main components: The first is transportation cost, calculated as the product of the cargo volume q_{ij} , transportation distance d_{ij}^n , and the unit cost $c k_{ijn}$. The second component is transshipment cost, determined by multiplying the unit transshipment cost $c n_i^{n_1 n_2}$ by the transshipment volume q_{ij} . The third is time cost, where the total time T includes both transportation and transshipment times, with transportation time derived from the transportation distance divided by speed. If the cargo arrives before the lower bound T^a of the time window, storage costs arise; if it exceeds the upper bound T^b , penalty costs are incurred. The time cost is also directly proportional to the transportation time.

Four key constraints are established: First, equation (2) limits the transportation between any two nodes to only one mode of transportation, simplifying the route structure. Second, equation (3) allows only one transshipment per node to minimize costs and time. Third, equation (4) ensures that mode changes at any node must be compatible with the modes used before and after to maintain smooth

operations. Lastly, equation (5) sets the decision variables as binary (0-1), where 0 or 1 indicates whether a mode is selected, streamlining the solution process.

2.3.2. Hybrid Robust Stochastic Optimization Model

As an effective method for studying optimization problems under uncertainty, the concept of robust optimization has been widely applied to research in areas such as production scheduling, transportation planning, and supply chain management. To analyze uncertain demand for goods, this study applies the concept of robust optimization to the multimodal transportation route optimization model.

To model demand uncertainty, the study adopts a scenario-based approach, where S demand scenarios are considered. Each scenario s is characterized by an uncertain demand q_{ijs} and a probability p_s of occurrence.

$$\begin{aligned}
 \min C'(x) &= \sum_{s=1}^S p_s \cdot C_s(x) \\
 &= \sum_{s=1}^S p_s \left(\sum_{i \in K} \sum_{j \in K} \sum_{n \in N} q_{ijs} d_{ij}^n c m_{ijn} X_{ij}^n + \sum_{i, j \in K} \sum_{n_1 \in N} \sum_{n_2 \in N} q_{ijs} c n_i^{n_1 n_2} Y_{n_1 n_2}^i \right. \\
 &\quad \left. + \sum_{i \in K} \sum_{j \in K} P_1 \max[(T^a - T), 0] \cdot q_{ijs} + \sum_{i \in K} \sum_{j \in K} P_2 \max[(T - T^b), 0] \cdot q_{ijs} \right)
 \end{aligned} \tag{6}$$

s.t.

$$C_s(x) \leq (1 + \alpha) C_s^* \tag{7}$$

$$\sum_{s=1}^S p_s = 1 \tag{8}$$

At the same time, constraints (2) to (5) hold.

The objective function under scenario-based robust optimization is formulated in equation (6), where it accounts for transportation, transshipment, and time costs under each scenario s . Equation (7) introduces a constraint that limits the difference between the solution for each scenario and the optimal solution, where $C_s(x)$ represents the objective function value for scenario s , C_s^* is the optimal objective value for the deterministic case, and $C_s^* > 0$ holds. The parameter α defines the maximum regret, which limits the allowable deviation from the optimal solution in scenario s . When $\alpha = 0$, the model assumes deterministic demand. Finally, equation (8) ensures that the total probability across all scenarios equals 1.

2.4. Ant Colony Algorithm

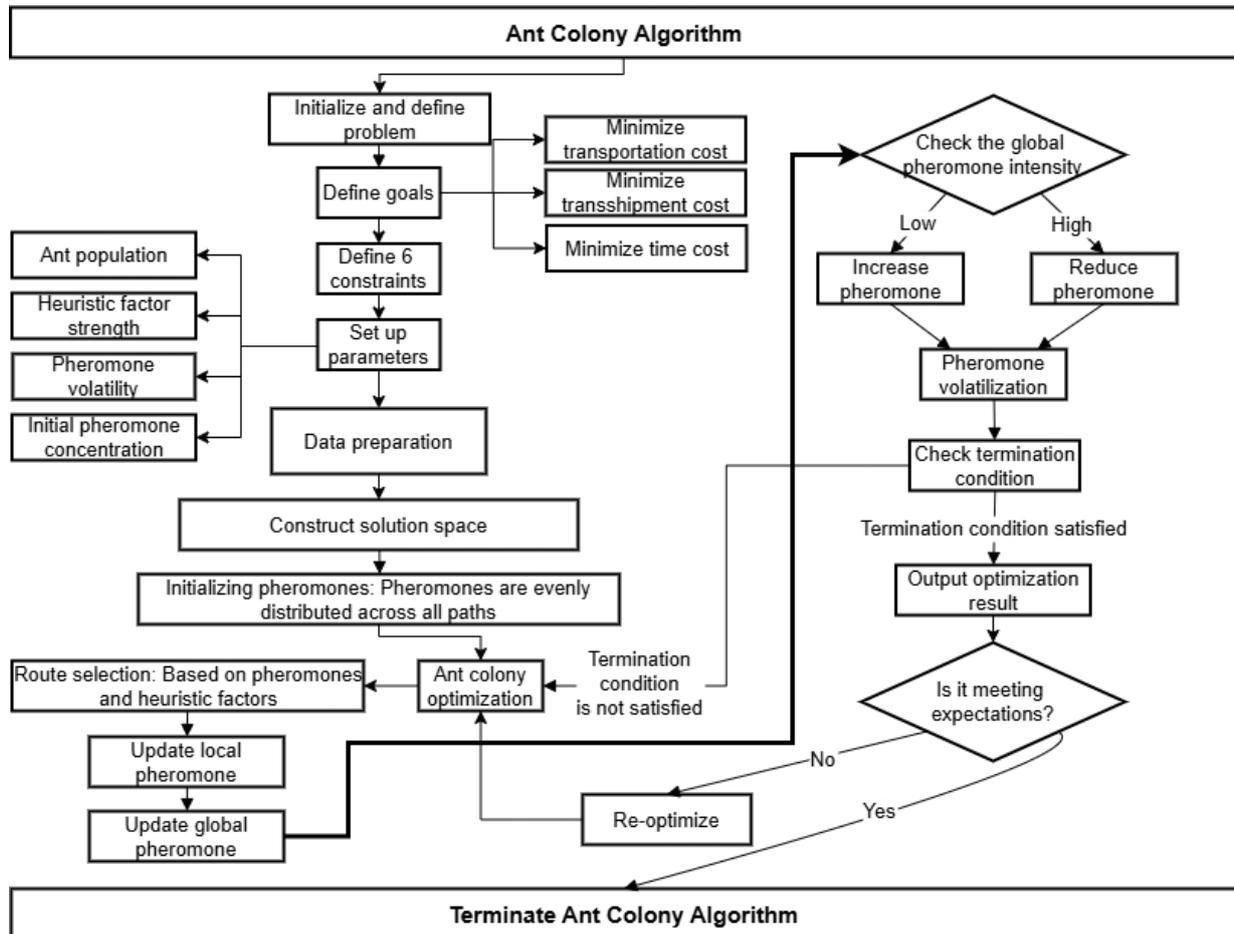


Figure 1: Ant colony algorithm flow chart.

3. Algorithm Example Analysis

3.1. Algorithm Example Data

A logistics company has received a multimodal transport order involving 15 nodes, requiring the company to plan ahead to address the uncertainty of cargo demand and ensure the goods arrive at the destination (D) from the starting point (O) within a 55-65 hour time window. If the goods arrive earlier than scheduled, the company will incur a storage cost of 15 yuan per ton per hour, while late arrivals will result in a penalty of 30 yuan per ton per hour. Therefore, the company needs to develop a transportation plan that balances flexibility and precision to optimize both time and cost. Additionally, real-time monitoring of the shipment status is crucial to ensure timely delivery and minimize total costs while fulfilling customer expectations.

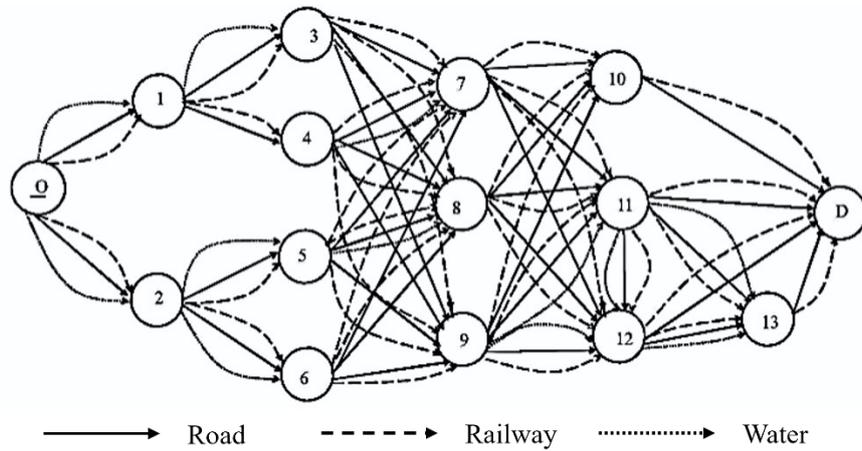


Figure 2: Transportation network.

For the multimodal transport route optimization, the transportation network between the 15 nodes from O to D has been established, with the distances between nodes recorded in Table 1. The three columns in Table 1 indicate the distances via road, rail, and waterway transport; if there is no direct route or transport mode available between two nodes, it is denoted by a "-".

Using data from literature and pricing structures from freight companies, this paper derives the transportation parameters for different modes, presented in Table 2. The base rates (a, b, c) correspond to transport prices for distances in the ranges of (0, 500] kilometers, (500, 1000] kilometers, and over 1000 kilometers, respectively.

It is assumed that all nodes have the capacity to handle transfers across different modes of transport, and that the transfer time and costs are consistent. Based on relevant literature and real-world conditions at ports and freight transfer hubs, this paper consolidates the average transfer costs and times for each transport mode, as summarized in Table 3.

Table 3: Transport distance of different transport modes between each transport node.

Section	Road Distance	Railway Distance	Waterway Distance
O-1	124 km	101 km	81 km
O-2	143 km	151 km	102 km
1-3	212 km	194 km	124 km
1-4	266 km	197 km	-
2-5	246 km	299 km	278 km
2-6	224 km	212 km	170 km
3-7	244 km	187 km	-
3-8	276 km	249 km	-
3-9	179 km	187 km	-
4-7	156 km	143 km	198 km
4-8	206 km	197 km	-
4-9	204 km	221 km	-
5-7	178 km	198 km	-
5-8	186 km	164 km	174 km
5-9	201 km	214 km	-
6-7	256 km	179 km	-
6-8	246 km	267 km	-

Table 3: (continued).

6-9	243 km	257 km	-
7-10	226 km	237 km	-
7-11	197 km	227 km	-
7-12	211 km	223 km	-
8-10	201 km	202 km	-
8-11	199 km	200 km	-
8-12	221 km	233 km	-
9-10	200 km	203 km	-
9-11	178 km	203 km	224 km
9-12	214 km	176 km	178 km
10-D	212 km	172 km	-
11-12	238 km	247 km	190 km
11-13	203 km	214 km	143 km
12-13	179 km	216 km	227 km
12-D	209 km	212 km	-
13-D	167 km	154 km	-

Table 4: Transportation parameters of different modes of transportation.

Transportation Mode	Average Speed (km/h)	Operating Base Price (¥/(km*t))
Road	80	(0.526, 0.497, 0.361)
Railway	60	(0.392, 0.340, 0.273)
Waterway	30	(0.090, 0.073, 0.051)

Table 5: Transport distance of different transport modes between each transport node.

Transshipment Mode	Transshipment Time (h/1000t)	Transshipment Cost per Unit (¥/t)
Road-Railway	50	8
Railway-Waterway	50	10
Waterway-Road	50	9

This paper, drawing on actual freight distribution patterns and related literature, defines three distinct freight volume scenarios. In the low-demand scenario, cargo volumes are assumed to follow a normal distribution with a mean of 100 tons and a variance of 64 tons, i.e., $q_{ij1} \sim N(100,64)$. In comparison, the medium- and high-demand scenarios assume cargo volumes that are 100 and 200 tons higher than the low-demand scenario, respectively. The occurrence probabilities p_s for these scenarios are 0.14 for low demand, 0.5 for medium demand, and 0.36 for high demand.

3.2. Results Comparison

Using the ant colony algorithm, this study calculates the multimodal transportation plan and costs for both deterministic demand (Mode I, where demand is the average across three scenarios) and uncertain demand (Mode II, where the maximum regret value is 0.2), as presented in Table 4. Based on the results in Table 4, the total cost ranking is "Mode II > Mode I". A detailed comparative analysis is provided below. (Railway for short is Ra and Road for short is Ro.)

Table 6: Transportation parameters of different modes of transportation.

Item	Average Speed (km/h)	Operating Base Price (¥/(km*t))
Transportation Route	O-1-4-7-11-13-D	O-2-6-8-12-D
Transportation Mode	W-Ra-W-Ra-Ra-Ra	W-W-Ra-Ra-Ra
Total Cost (¥)	51,414.52	72,902.21

Mode I represents the base model with deterministic demand. The transportation route in this case is "O-1-4-7-11-13-D", involving three mode transfers using waterways and railways in sequence. The total cost amounts to 51,414.52 yuan.

In contrast, Mode II applies robust stochastic optimization to account for demand uncertainty, where carbon trading prices are assumed fixed, and robust optimization accommodates multiple demand scenarios. The optimized route for Mode II is "O-2-6-8-12-D", using "water-water-railway-railway-railway" transportation modes. The total cost under this model is 72,902.21 yuan, nearly 50% higher than that of Mode I. This highlights the substantial impact of demand uncertainty on multimodal transportation costs, which is largely driven by the robust optimization's focus on ensuring stability across various scenarios.

3.3. Results Comparison

By comparing the basic deterministic model (Mode I) with the robust optimization model under multiple demand scenarios (Mode II), we applied Mode II's transportation routes and methods to three different freight volume scenarios—low, medium, and high—and calculated the corresponding total costs. These were then compared with the optimal total costs derived from Mode I under each freight volume scenario. The results demonstrate that Mode II's total costs across all scenarios are never lower than the optimal costs achieved under Mode I. This finding suggests that while robust optimization tends to offer more conservative solutions aimed at ensuring stability, this emphasis on robustness often results in higher costs. Although robust optimization may not always minimize costs, it offers a more stable transportation strategy for firms dealing with demand uncertainty.

Table 7: Comparison of total cost between Mode I and Mode II.

Freight Volume Scenario	Mode I (¥)	Mode II (¥)
Low Freight Volume	52,802.29	79,002.29
Medium Freight Volume	73,881.52	90,713.33
High Freight Volume	95,729.12	95,729.12

To better understand the stability-driven nature of robust optimization, we conducted a sensitivity analysis on the maximum regret value. The analysis reveals that as the maximum regret value increases, total costs tend to decrease, indicating that allowing for greater regret enables the system to identify lower-cost solutions. Notably, when $0 \leq \alpha < 0.2$, total costs decrease more rapidly, implying that in this range, even small increases in regret can lead to significant cost reductions. However, when $\alpha > 0.2$, the rate of cost reduction slows, indicating that further increases in regret yield diminishing cost savings.

These results suggest that when a multimodal transportation system is highly robust, total costs may not increase dramatically. Therefore, in practice, decision-makers can strategically adjust the maximum regret value to strike a balance between robustness and cost. This allows for stable transportation plans while minimizing total costs, ultimately enhancing the operational efficiency of

multimodal transport. By achieving this balance, firms can optimize cost-efficiency without compromising the stability of their operations.

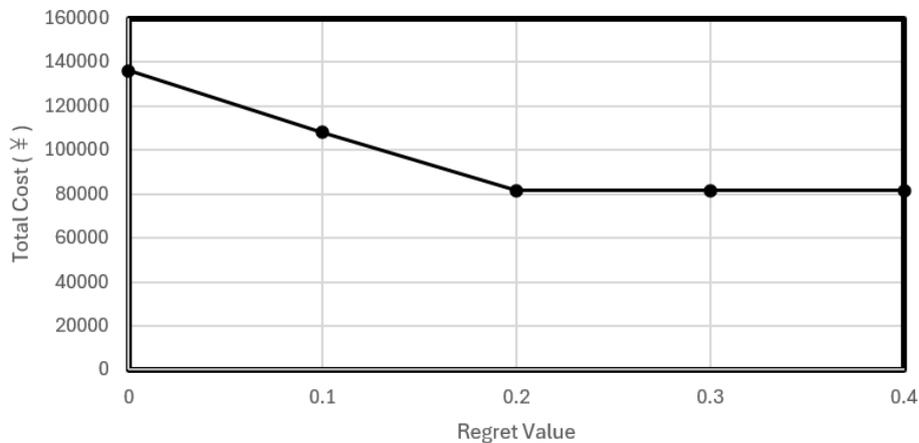


Figure 3: Transportation network.

4. Conclusion

From the standpoint of multimodal transport operators, the advanced planning required for multimodal operations, coupled with the difficulty of accurate demand forecasting—particularly in the face of seasonal fluctuations and unpredictable events—presents significant challenges. This study addresses these challenges by focusing on the low-carbon multimodal transport problem under uncertain freight demand. We propose a hybrid robust stochastic optimization model to tackle the path selection problem for multimodal transport in uncertain environments. An ant colony algorithm is designed to solve the model, and case studies are conducted to compare transport schemes and costs between deterministic and uncertain demand models. We further analyze how uncertainties affect total costs.

Our findings indicate that robust optimization, though designed to ensure solution stability under demand uncertainty, may result in higher total transport costs for multimodal operations. The regret value in robust optimization plays a crucial role in influencing these costs, and by fine-tuning this parameter, it is possible to strike a balance between cost efficiency and solution stability, thus enhancing the operational efficiency of multimodal transport under uncertainty.

Despite these insights, the complexity of the model precluded consideration of key real-world operational factors, such as carbon emissions, traffic congestion, and waiting times. Future research will aim to develop more practical, environmentally sustainable optimization models to support greener multimodal transport strategies. Moreover, analyzing historical demand data across different regions will help assess the model's effectiveness and applicability in real-world scenarios, providing deeper insights into multimodal transport under uncertain demand conditions.

References

- [1] Sun, Y. (2017). *Research on the Methodology of Optimization Modelling for the Multimodal Routing Problem Based on Transportation Scenarios*. Beijing Jiaotong University, Beijing.
- [2] Zhen, Y.D., & Yang, B., (2018). *Multi-Target Planning of Container Multimodal Transport Under Uncertainty*. *Computer Application and Software*(05), 21-26+119. doi:CNKI:SUN:JYRJ.0.2018-05-005.
- [3] Zhang, W., Wang, X., & Yang, K. (2020). *Uncertain multi-objective optimization for the water-rail-road intermodal transport system with consideration of hub operation process using a memetic algorithm*. *Soft Computing*, 24(5), 3695-3709.

- [4] Sun, Y., Liang, X., Li, X., & Zhang, C. (2019). A fuzzy programming method for modeling demand uncertainty in the capacitated road–rail multimodal routing problem with time windows. *Symmetry*, 11(1), 91.
- [5] Lozano, A., & Storchi, G. (2001). Shortest viable path algorithm in multimodal networks. *Transportation Research Part A: Policy and Practice*, 35(3), 225-241.
- [6] Qingbin, W., & Zengxia, H. (2010, November). The optimal routes and modes selection in container multimodal transportation networks. In *2010 International Conference on Optoelectronics and Image Processing (Vol. 2, pp. 573-576)*. IEEE.
- [7] Li, M.L., Wang, X.F., Sun, Q.X., & Yang, Z.N. (2017). Study on the strategy of multi-modal transport of emergency materials based on robust optimization. *Journal of the China Railway Society* (07), 1-9. doi:CNKI:SUN:TDXB.0.2017-07-027.
- [8] Jiang, Y., Zhang, X.C., & Wang, Y.L. (2017). The Cross Entropy Method for Multimodal Transport Scheme Selection. *Journal of Transportation Systems Engineering & Information Technology*(05), 20-25. doi:10.16097/j.cnki.1009-6744.2012.05.015.
- [9] Archetti, C., Peirano, L., & Speranza, M. G. (2022). Optimization in multimodal freight transportation problems: A Survey. *European Journal of Operational Research*, 299(1), 1-20.
- [10] Elbert, R., Müller, J. P., & Rentschler, J. (2020). Tactical network planning and design in multimodal transportation—A systematic literature review. *Research in Transportation Business & Management*, 35, 100462.
- [11] Delbart, T., Molenbruch, Y., Braekers, K., & Caris, A. (2021). Uncertainty in intermodal and synchromodal transport: Review and future research directions. *Sustainability*, 13(7), 3980.
- [12] Prakash, S., Kumar, S., Soni, G., Jain, V., & Rathore, A. P. S. (2020). Closed-loop supply chain network design and modelling under risks and demand uncertainty: an integrated robust optimization approach. *Annals of operations research*, 290, 837-864.