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Impact of Narrow Framing on Portfolio Allocation: A Numerical Simulation-Based Approach

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Abstract: Many phenomena observed in financial markets remain unexplained by the Expected Utility Theorem (EUT), a framework which is commonly used to analyze decision-making behavior. In recent years, several theories related to behavioral economics have been proposed to address the limitations of this model and provide solutions to the problems that the EUT cannot explain. While this has significant theoretical importance, there have been few studies that have used real-world dynamics to validate the new findings. This study explores the behavioral patterns explained by a model that includes the narrow framing effect using three products in financial markets. Specifically, the study uses a multivariate GARCH model and asset simulations to validate theoretical predictions and investigates the theoretical impact of the narrow framing on portfolio allocation decisions. The results show that the theory is consistent with the real-world financial markets. This study provides new insights into understanding the changes in financial markets.

Keywords: Financial market, EUT, GARCH

1. Introduction

Numerous phenomena that appeared in financial markets remain unexplained by the expected utility theorem (EUT); a framework commonly employed to analyze decision-making behavior. Several theories, which relate to behavioral economics, have been suggested in recent years to address the limitations of this model, offering solutions to puzzles that EUT cannot explain. Narrow framing, one of cognitive bias, provides insight into the lower-than-expected participation rates in the stock market and has been incorporated into traditional utility models. Although it is theoretically significant, few studies have utilized real-world dynamics to validate the new findings. This research utilizes the pattern of three products in financial markets to explore the behavioral patterns illustrated by model incorporating narrow framing effect.

This thesis is structured as follows. Section 2 presents an extensive literature review on the relationship between financial choices and narrow framing. Section 3 explains the methodology and Section 4 presents results from the model using real-world financial data. Finally, Section 5 provides the conclusion and limitations.

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2. Literature Review

EUT is widely used in economics to analyze how individuals make choices to allocate financial assets. However, this theory fails to account for certain behaviors, such as individuals' heightened sensitivity to losses compared to gains [1]. Additionally, a significant number of individuals tend to reject small, independent gambles that are actuarially favorable [2]. Utility functions without first-order risk aversion struggle to explain this behavior, and even those incorporating such factors fail to fully account for the rejection of such favorable gambles [3-4].

Behavioral economics theories have gained prominence in recent decades, with researchers adopting its theories to address previously unexplained phenomena. For example, portfolio choices can be modeled under cumulative prospect theory frameworks, which highlights the tendency of individuals to overweight events with low probability to occur [5]. Narrow framing, a behavioral economics theory that explains how investor's view investments in isolation, is applied to address non-participation puzzle in stock markets [6-7]. Researchers suggest that investors' tendency to assign equal value to simultaneous investments in multiple assets can be attributed to a disregard for diversification, with framing effects playing a significant role [8]. Furthermore, survey data shows that most Chinese households exhibit narrow framing, and this trait is negatively associated with diversification, offering valuable insight into household portfolio choices [9].

To explore how narrow framing influences investment decisions, one study presents a tractable preference model that integrates narrow framing into the standard economic framework and demonstrates its usefulness in portfolio choice and equilibrium contexts [7]. A follow-up study expands this model by introducing probability weighting and a convex–concave value function, providing further insights into the stock market non-participation and equity premium puzzles through the lens of narrow framing and cumulative prospect theory [10]. Another study further refined the model by assuming proportional effects of gains and losses on an agent's total utility [11].

However, most models are tested using manually constructed, arbitrary datasets to assess the impact of narrow framing on decision-making. Few studies apply these theoretical models to real-world data. This research addresses this gap by utilizing real financial market data and numerical analysis techniques to validate the narrow framing model.

3. Modeling Framework

This section revisits and generalizes the framing model proposed in a previous study [7]. Following this, the procedure used to assess the effect of narrow framing in real-world financial markets is outlined, as well as the methodology for selecting time series models.

3.1. Baseline Model

The investor selects a consumption level Ct and allocate remaining wealth after consumption across n assets at time t, with respective gross returns ranging from R1, t+1 to Rn, t+1 over the period from t to t + 1. The wealth level at t + 1 could be formulated as follows.

$$W_{t+1} = (W_t - C_t)\left(\sum_{i=1}^{n} \theta_{i,t}\, R_{i,t+1}\right) \tag{1}$$

In this equation, θi,t represents the percentage of investment in asset i. Additionally, it is assumed that a proportion of assets invested (m+1 to n) are narrowly framed. Following this assumption, the utility of such investor at time t could be expressed as follows.

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$$V_t = H(C_t, \mu(\tilde{V}_{t+1}|I_t) + b_0 \sum_{i=m+1}^{n} E_t(\bar{v}(\tilde{G}_{i,t+1})))) \tag{2}$$

Where $\mu(x)$ is a standard utility function satisfying the homogeneity condition in EUT frameworks. Specifically, H(C,x) is calculated as follows.

$$H(C,x) = ((1-\beta)C^{\rho} + \beta x^{\rho})^{1/\rho} \; 0 < \beta < 1, 0 \neq \rho < 1 \tag{3}$$

$$G_{i,t+1} = \theta_{i,t}(W_t - C_t)(R_{i,t+1} - R_{i,z}), \tag{4}$$

$$\bar{v}(x) = \begin{cases} x, & x \geq 0, \\ \lambda x, & x < 0, \end{cases} \; \lambda > 1. \tag{5}$$

H(·,·) is a weighting function which incorporates two terms representing consumption level of the investor and the utility that is expected to be perceived after investment (details of the function could be found in a previous study [12]). Function G(·) and $\bar{v}$ (·) are constructed based on cumulative prospect theory, which indicates that financial decisions are evaluated against another portfolio that serves as a benchmark, and loss aversion is incorporated alongside narrow framing [13]. In this theory, the benchmark portfolio is usually chosen to be the bond, which corresponds to Ri,z in the equation that represents the risk-free rate. If the narrowly framed portfolio generates expected return lower than the risk-free rate, the investment decision is perceived as a loss. Function $\bar{v}$(·) illustrates that losses impact investor utility more significantly than gains. Consequently, the consumption-portfolio problem is formulated as follows.

$$V_t = \max_{\theta_t, C_t} H(C_t, \mu(\tilde{V}_{t+1}|I_t) + b_0 \sum_{i=m+1}^{n} E_t(\bar{v}(\tilde{G}_{i,t+1})))) \tag{6}$$

Since $\bar{v}$ (·) is piece wisely linear, the optimization of consumption and the optimization of portfolio allocation can be decoupled, represented as follows.

$$B_t^* = \max_{\theta_t} [\mu((1-\beta)^{1/\rho} \alpha_{t+1}^{1-1/\rho} \theta_t' \tilde{R}_{t+1}|I_t) + b_0 \sum_{i=m+1}^{n} E_t(\bar{v}(\theta_{i,t}(\tilde{R}_{i,t+1} - R_{i,z})))] \tag{7}$$

Where αt is the proportion of consumption to the wealth of the investor at time t. The optimal αt should satisfy the following condition.

$$(1-\beta)(\alpha_t^*)^{\rho-1} = \beta(1-\alpha_t^*)^{\rho-1}(B_t^*)^{\rho} \tag{8}$$

It is feasible to numerically solve the optimization problem of asset allocation by incorporating real world financial data. The following section presents the details of datasets used and the procedure to incorporate the data in the optimization problem.

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3.2. Procedures

The model discussed in the previous section is applied to solve a straightforward portfolio problem, similar to the study in, but it should be extended to allow analysis on real-world financial data. Similar to the settings in the previous research, it is assumed that the investor faces investment choices across three assets: bond, domestic and foreign stocks, represented by Asset 1 to 3 respectively. It is also assumed that Asset 3 is the only portfolio subject to narrow framing. For this analysis, the indices for Australian bond and stock market and US stock market retrieved from S&P Global are used to estimate the return of these three assets. The period of data utilized is between December 14, 2017, and January 31, 2024, with daily observations.

The investor is assumed to adjust the portfolio allocation on a weekly basis. Additionally, the proportion invested in Asset 2 remains constant, with θ2,t set to 0.5 throughout. These assumptions align with those in the previous study, which suggests that using non-financial assets and domestic stock to simulate returns of Asset 2 is reasonable. The objective of the analysis is to simulate how a narrow framed investor could optimize the portfolio allocation between the bond and foreign stock. The initial step involves estimating the returns for each asset and forecasting returns for the next seven days. Return series could be transformed from the price index using the following formula.

$r_{t+1} = 100(ln(p_{t+1}) - ln(p_t))$.

The results of both the ADF and KPSS tests suggest that the return series are stationary. Given the presence of time-varying correlations between the assets, a multivariate model is deemed appropriate. However, to build a solid foundation for the multivariate analysis, each series are analyzed univariately. The mean equations for each series are determined by examining information criteria and likelihood ratio test. It is found that the most suitable model is ARMA (1,1) for Australian bond, MA(2) for Australian stock, AR(2) for US stock respectively. In addition, the residuals generated after fitting the mean equations are tested if there are ARCH effects present. Significant lags are found, though only the first two lags are considered to maintain model parsimony. Following the similar criteria to find the optimal mean equations, it is found that GARCH (1,1) depicts the residuals of the three series most appropriately.

As correlations between these three series are detected, the next step is to identify the multivariate GARCH system for the series. In this step, both the mean equations derived above, and VAR models are considered. The lag length of the VAR model is selected by examining BIC and is chosen small enough to ensure model parsimony. Based on these criteria, VAR(1) is selected. In this analysis, diagonal VECH, constant correlation and diagonal BEKK are considered. Residuals from each model are checked for correlation. The optimal model is still selected based on information criteria and likelihood. It is also worth noting that Constant Correlation model is examined to exhibit unusual significant discrepancy in the covariance between US stock and Australian bond and therefore excluded from consideration. Finally, the most suitable multivariate model is chosen to be VAR(1) CC GARCH(1,1) model. This model could be presented in the following matrix form.

$$\begin{bmatrix} R_{1,t} \\ R_{2,t} \\ R_{3,t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} + \begin{bmatrix} p_{1,1} & p_{1,2} & p_{1,3} \\ p_{2,1} & p_{2,2} & p_{2,3} \\ p_{3,1} & p_{3,2} & p_{3,3} \end{bmatrix} \begin{bmatrix} R_{1,t-1} \\ R_{2,t-1} \\ R_{3,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix} \quad (9)$$

Where

$$\begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix} \sim N(0, H_t). \quad (10)$$

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Ht is the covariance matrix in the following form.

$$H_t = \begin{bmatrix} \sigma_{1,t}^2 & \sigma_{12,t} & \sigma_{13,t} \\ \sigma_{21,t} & \sigma_{2,t}^2 & \sigma_{23,t} \\ \sigma_{31,t} & \sigma_{32,t} & \sigma_{3,t}^2 \end{bmatrix} \quad (11)$$

The terms in Ht satisfies the following equations.

$$\sigma_{i,t}^2 = \omega_i + \alpha_{ii}\varepsilon_{i,t-1}^2 + \beta_{i,i}\sigma_{i,t-1}^2, \quad (12)$$

$$\sigma_{ij,t} = \sigma_{ji,t} = \rho_i j \sigma_{i,t}\sigma_{j,t} \quad (13)$$

Table 1 presents the values of coefficient in the model. After the model estimating and forecasting portfolio returns established, several simulations are conducted to evaluate the performance of each asset. The value of the objective function could be obtained through the simulations, which will be briefly outlined in the next section.

Table 1: Coefficient values in VAR(1) CC GARCH model

c1	0.026	p2,3	-0.003	α2,2	0.070
c2	0.003	p3,1	0.025	α3,3	0.195
c3	0.086	p3,2	0.001	β1,1	0.848
p1,1	-0.006	p3,3	-0.064	β2,2	0.903
p1,2	0.050	ω_1	0.027	β3,3	0.779
p1,3	0.035	ω_2	0.003	ρ12	0.013
p2,1	-0.002	ω_3	0.039	ρ13	0.074
p2,2	-0.087	α1,1	0.121	ρ23	-0.012

4. Numerical Simulation

Once the multivariate model is established, return forecasts for each asset can be simulated. 10,000 sample paths for the returns of assets over a 7-day horizon are simulated via bootstrap. As a result, 10,000 possible outcomes for one-unit investment in each asset over this time frame are observed.

The value of objective function 3 could be determined by obtaining the mean of the return series. In the objective function, $\mu(x)$ is determined to be a power utility function that could be represented by the following equation.

$$\mu(x) = E[x^{1-\gamma}]^{1-\frac{1}{\gamma}}. \quad (14)$$

The value of γ is set to be equal to ρ in function $H(\cdot,\cdot)$ specified in section 2. It is also assumed that the α_t in the objective function 3 remains constant over time, which satisfies the assumption in previous research [7]. To calculate the optimal α, an initial value for it is selected and plugged into objective function to generate the optimal B_t^*. This value of B_t^* is then used to solve Equation 4, yielding an updated α. The process is repeated iteratively until the value of α converges. At that stage, the value of $\theta_{1,t}$ and $\theta_{3,t}$ represents the solution to the optimization problem.

The process described above is accomplished via 10,000 loops in R. The following graph illustrates how the investor allocates wealth in Asset 3 as the degree of loss aversion, narrow framing and risk aversion —represented by λ, $b_0$and γ—changes. Lines in the graph represents the value of $\frac{\theta_{3,t}}{\theta_{1,t}+\theta_{3,t}}$, with a step size of 0.05 used in the codes (See Figure 1).

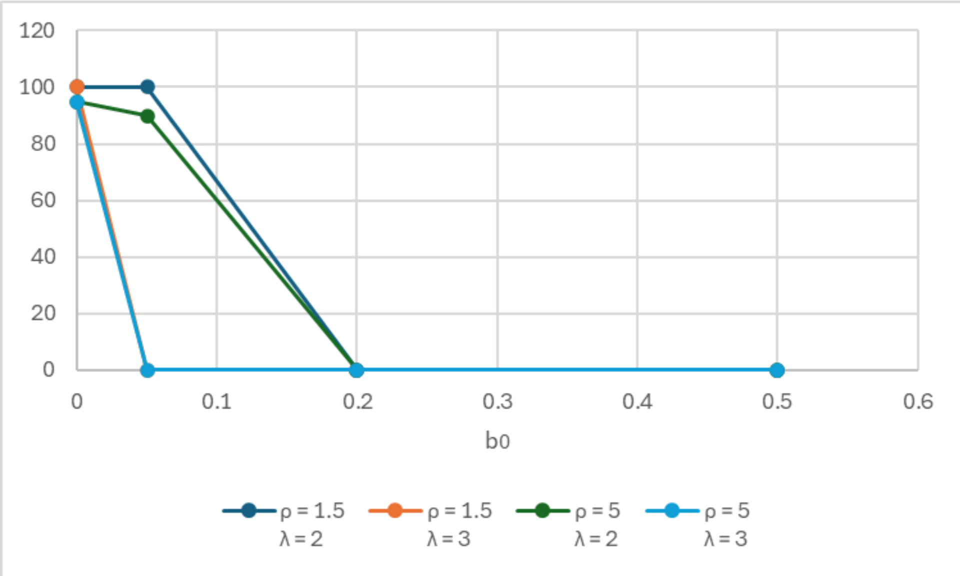

Figure 1: Optimal proportion invested in US stock

The results align with the findings from the previous study [7]. When b_0 equals zero, the investor is not narrow framed, and the value represents the decision from a risk-averse perspective. Since Australian bonds and U.S. stocks have low correlation, this type of investor tends to allocate a significant portion of post-consumption wealth to stocks, seeking higher returns and diversification benefits. However, the investor becomes more narrowly framed, stock investments are likely to be perceived in isolation and the diversification benefits could become hard to aware. This, combined with heightened sensitivity to losses, results in a more reluctant attitude to invest in riskier assets.

5. Conclusion

This research investigates the alignment between the theoretical effects of narrow framing on portfolio allocation decisions and real-world financial market dynamics, using a multivariate GARCH model and asset simulations to validate the theoretical predictions. However, the model has some limitations. One limitation is that it only provides a static forecast, capturing investor preferences at a single point in time. Additionally, the model does not account for the behavior of framed investors, who tend to be risk-averse when experiencing gains and risk-seeking when facing losses. Future research could address these limitations by allowing investors to change position in daily frequency and examine the timeline of position. In addition, an S-shaped narrow framing function could be applied.

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