

Research on the Comparison of Option Pricing Models

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Abstract: Option pricing is essential for market participants as it facilitates risk management and price discovery. This study provides a comprehensive elucidation of diverse option pricing models. The author delineates the development of option pricing models from the fundamental Black-Scholes-Merton model (BSM) to more intricate methodologies. The importance of comprehending option pricing for hedging strategies and risk management is crucial in financial mathematics. The study analyses several models, including the BS model, Jump Diffusion model (JDM), Binomial Tree model (BTM), Monte Carlo simulation (MCS), and stochastic volatility models (SVM), while addressing their applications, limitations, advantages, and disadvantages. This work delineates the history and distinctions between other models and the Black-Scholes-Merton model. The Black-Scholes-Merton model exerts the most substantial influence on financial markets. The alternative models exhibit greater flexibility and enhanced accuracy when confronting complicated and uncertain market trends. The trend of future development is bound to be the continuous updating of models to adapt to more complex situations.

Keywords: Option Pricing, Black-Scholes-Merton Model, Jump Diffusion model, Monte Carlo Simulation, Binomial Tree Model.

1. Introduction

An option give buyer a right, which is also a contract between buyer and seller. For participants to buy or sell an underlying asset under the contract. In the return for the agreement to the option from the buyer. Options in the financial markets are primarily significant as tools for risk management, speculation, and assisting investors in designing complex trading strategies. So, knowing how to price option help buyer make better hedging strategy and help arbitrage in financial market. Option pricing is a critical area of study in financial mathematics and economics with a long history. The seminal Black-Scholes-Merton model, introduced in 1973, revolutionized the way options are priced by providing a closed-form solution for European-style options, based on the assumption that asset prices follow a geometric Brownian motion [1].

Merton also raised another model for the circumstances that underlying stock returns are discontinuous in 1976. It is a stochastic process that involves jumps and diffusion, so it called Jump diffusion model.

Later in 1979, Cox, Ross and Rubinstein invented a simpler model called Cox-Ross-Rubinstein model (CRRM). It also called Binomial Tree model because the image of the model is a tree with

many double branches directly perceived through the senses. It offered a flexible numerical approach to price both European and American options.

At the same period, a technique to obtain numerical solution for option pricing issues based on MCS was raised by Boyle [2]. While MCS first was used for nuclear weapon method and it was advanced by Ulam, Neumann. Nowadays, MCS was used in financial engineering, macroeconomics and computational physics.

There were lots of model emerged from 1980s' to now. Stochastic volatility model, which indicated a series model. The variance of a stochastic process is distributed by random in the series model [3]. It involves a lot of models with different characteristic. These models reflect real-world conditions by introducing randomness in volatility or incorporating discrete jumps in asset prices in a better way.

As time progresses and the financial sector evolves, comprehending the advantages and disadvantages, along with the characteristics of option pricing models, is essential for analyzing real-world scenarios. Furthermore, it will more effectively stimulate the advancement of academic finance. This work seeks to elucidate the historical evolution of option pricing models and to delineate their limitations, thereby enhancing the comprehension of these models for novices.

2. Option Pricing models

2.1. Black-Scholes-Merton Model

Black-Scholes-Merton model known as well as BS model. It's origin from Brownian motion raised by Brown in 1800's. Later in 1900, Bachelier suggests use Brownian motion to describe the rise and fall in stock market. And then Einstein mathematically formalized Brownian motion in 1905 and 1906. Einstein contributions provided the theoretical basis for modelling asset price movements as a stochastic process, particularly in finance. Wiener developed the mathematical concept of Wiener processes (also known as standard Brownian motion) in the 1920s. Lévy extended the theory of Brownian motion and made significant contributions to probability theory, including random processes, which influenced financial modelling. Itô introduced Itô calculus in the 1940s, which made it possible to handle complex non-linear problems in stochastic processes. Itô's calculus is fundamental in deriving the BSM and other financial models based on geometric Brownian motion. Samuelson applied geometric Brownian motion to financial markets, further developing asset price modelling in the 1960s. Merton, Black, and Scholes developed the famous Black-Scholes-Merton model in the 1970s, providing a closed-form solution for option pricing.

BSM make a few assumptions. The first one is the price of the underlying asset follows a geometric Brownian motion with constant volatility and interest rates. It provides a continuous-time, closed-form solution for pricing European options. This model uses stochastic calculus to derive a formula that gives the option price directly.

The BSM has profoundly impacted the finance industry. It established a precedent in the business by offering investors, hedgers, and speculators a uniform methodology for option valuation. This model has improved the sector's use of financial engineering. Currently, mathematicians are formulating models to enhance portfolio returns and reduce risk. Furthermore, they are devising intricate computational methods to detect market inefficiencies. The Black-Scholes model initiated a mathematical revolution that presently supports the financial industry [4].

This formula has been widely used. Many empirical tests show that the formula closely approximates market prices. However, there are instances where discrepancies arise, such as the well-known phenomenon of the "volatility smile".

As an origin option pricing model, it possesses numerous inherent restrictions stemming from its fundamental assumptions. The initial aspect is the constant volatility. Volatility may be stabilised

during a brief period, but it cannot remain consistent over an extended duration. Furthermore, in accordance with the lever rule, volatility will escalate during trading activities. The alternative perspective posits that the interest rate is a fixed number, with the possibility of it being set to zero. It never occurs in the actual world. Furthermore, there would be no commissions or transaction expenses incurred.

The premise that markets are entirely liquid, and that trading is random is clearly untenable. Nonetheless, all hypotheses in this model are designed for ease of calculation, resulting in certain restrictions. It is essential to create models that consider the assumptions overlooked by Black-Scholes models, as most of these limitations pertain to fundamental market characteristics. Numerous models have been proposed over time to comprehensively recreate market aspects. No singular model can encompass all facets of the market, analogous to how no solitary mathematical expression can encapsulate every variable affecting the price of a financial product. Our objective is to encapsulate the majority of the parts [5].

2.2. Jump Diffusion Model

After Merton improved and perfected BS model, Merton found that some situation cannot be analysis by BS model. In real financial market, the stock price often varies by a continuous-time diffusion process. So, Merton raised a new stochastic process called Jump diffusion model. This model combined the continuous-trading option price change and discrete-trading price "jump" so that it can better describe the behaviours of stock price in real financial markets.

Mixing a jump process and a diffusion process, he main goal of the jump diffusion model is to account for abrupt fluctuations in asset values, or "jumps," which are difficult for continuous models such as the Black-Scholes model or SVM to depict.

Some of the problems with the BSM are addressed as part of the additional thinking behind the JDM. Give an example, the size of the leaps in Kou's double exponential model has a double exponential distribution. The goal of this model is to explain two phenomena that the BSM is unable to describe: (1) the volatility smile; and (2) the left-skewed, fat-tailed return distribution [6].

JDM has many advantages. The first one is internally self-consistent, JDM gives no arbitrage opportunities. The second one is that the JDM is able to deal with the real-world stock price. And the last but not least one is that the model is simple enough to calculate. In the comparison of BSM, JDM improved the empirical implications while retaining the analytical tractability of the BSM [7].

However, the primary issue with JDM is their inability to account for volatility clustering effects, which can be addressed by alternative models such SVM. So, the JDM is suitable for some short-period options such as European option [8].

2.3. Binomial Tree Model

Binomial Tree model also called Cox-Ross-Rubinstein model. After Black and Scholes advanced BSM, which pricing the price of underlying-asset follows lognormal distribution, Ross start to analysis the price of underlying-asset following abnormal distribution. Ross and Cox raised risk neutral pricing theory in 1976. Ross, Cox and Rubinstein raised a simple way for pricing discrete-trading options. This way called CRRM as well as BTM.

It assumes that there is only two direction which they are upwards and downwards in share price movements. Additionally, the probability of the price moving up or moving down are the same. The model split the option's duration into several intervals, using the stock's historical volatility to simulate all possible price paths over the option's life. At each node along these paths, the model calculates the option's payoff at exercise and the option's price, discounted to the present. For

American-style options, which can be exercised early, the theoretical price at each node is the greater of the option's exercise payoff and the discounted option price.

The BTM and the BSM are two complementary methodologies. The BTM is comparatively straightforward to derive and is more effective in elucidating the fundamental principles of option pricing. This model is predicted on a fundamental assumption: over a specified time frame, the price of a security might fluctuate in two potential directions—upward or downward. Despite the simplicity of this assumption, the BTM can accommodate more intricate options by subdividing a specified time frame into smaller intervals.

The BTM's distribution eventually converges to a normal distribution as the number of price changes considered rises, matching the BSM. Due to its ability to streamline option pricing calculations and improve intuition, the binomial model has gained widespread acceptance as the standard for pricing on major international stock exchanges.

The BTM offers numerous advantages, notably its capacity to compute both American and European option prices via numerical methods, rendering it especially appropriate for situations requiring early exercise. Moreover, it is extensively utilised in high-risk and uncertain industries, such as pharmaceuticals and telecommunications, for real option valuation, assisting corporations in making more informed investment decisions. In contrast to other models, the Binomial Tree Model effectively accommodates path-dependent options and adeptly addresses early exercise complexities by evaluating cash flows at each time interval. The Binomial Tree Model exhibits limits in addressing more intricate scenarios and occasionally neglects to adequately consider the benefits of managerial flexibility in projects. Although it can approximate the range of stock price fluctuations, it finds it challenging to precisely forecast the price at the conclusion of each period. The model exhibits sluggish computational speed, particularly in complex cases, and its simplifying assumptions may not consistently correspond with real-world scenarios, thereby constraining its applicability in specific contexts [8].

2.4. Monte-Carlo Simulation

MCS was first invented during WWII as part of the United States' efforts in the Manhattan Project, which aimed to simulate the random diffusion of neutrons in fissile materials. This statistical method was pioneered by the mathematicians Neumann and Ulam. Monte Carlo was a city in the small European principality of Monaco, which was famous for its casinos at the time. Given that the essence of gambling is computing probability, and the statistical method invented by Neumann and Ulam was also based on analysing probability. It was highly appropriate to name the method after a well-known gambling city. That is the reason why it is called Monte Carlo Simulation

MCS is a statistical technique that involves running a computational model thousand or even millions of times, with each iteration using randomly selected input values. These input variables are typically drawn from estimated ranges, and the computer randomly selects a value from within these ranges for each simulation. By performing these simulations on a large scale, the method generates a cumulative probability distribution, offering a comprehensive statistical understanding of potential outcomes for complex scenarios.

In the comparison with Black-Scholes model, the simulated price nearly the same under short trade period. But the Monte Carlo simulation performed better with the amount of data increased. When the stock price is uncertainty and complex, Monte Carlo simulation always gives a better result due to its flexibility and value [9].

Because simulation methods have shown to be valuable and adaptable computational tools to calculate the value of alternatives with many sources of uncertainty or with sophisticated features, they always produce better results when numerical and analytical models become unavailable. The primary feature that draws so much appeal to simulation is its straightforward approach to handling

uncertainty. Since powerful workstations are readily available and the tool's application has advanced recently, MCS is one of the most often used numerical methods for pricing financial options and other derivative securities. Because MCS is adaptable, simple to use, and adaptable, it has proven to be a very appealing technique [10].

However, MCS relies heavily on the quality of the random numbers used in the process. Recent research, such as a report by Ferrenberg, revealed that common computer programs used to generate random number sequences can introduce errors in simulations. For instance, in a model simulating atomic behaviour in magnetic crystals, five widely used programs produced faulty results because the number sequences they generated were not truly random. These sequences exhibited hidden patterns and correlations, which only became apparent when they distorted the known properties of the crystal model [11].

Thus, while MCS is a powerful tool for predicting complex trends and events, its accuracy depends on the reliability of the random number generators used in the computations.

2.5. Stochastic Volatility Models

Stochastic volatility is a set of models whereby the variance of a stochastic process is itself randomly scattered. Mathematical finance makes considerable use of stochastic volatility models to value derivative securities like options. These theories hold that the underlying asset's volatility follows a stochastic process and is hence variable. Among other factors influencing this process are the price level of the asset, the inclination for volatility to return to a long-term mean, and the fluctuation of the volatility itself.

SVM are necessary because of a BSM restriction that requires constant volatility over the complete derivative lifetime. Observations of the market show that volatility changes implied depending on the strike price and expiration date: the volatility smile and skew. These models give a more exact way to price derivatives by seeing volatility as a stochastic process rather than a fixed value.

Between the basic BSM and SVM is local volatility models. In these models, volatility is a function of the underlying asset rather than random; it is not injected by anything.

3. Conclusion

This article presents five essential option pricing models. Option pricing consistently presents a substantial challenge for all participants in financial markets. Individuals interested in options and possessing the right to exercise them are advised to acquire knowledge of option pricing. The models referenced in the article elucidate the historical development of mathematics and finance. The evolution of option pricing models from the foundational Black-Scholes model has been highlighted in response to changing trends and conditions. Numerous mathematicians sought to create option pricing models to enhance precision. Merton, who enhanced and refined the BSM, developed a JDM to address its limitations. Nonetheless, fundamental reasoning and recent historical events after various market occurrences indicate that no model can encompass every facet of the market, as not all factors influencing the price of a financial instrument can be scientifically quantified.

The BTM, which offers greater flexibility than the BSM, can analyse American options but has several downsides. The Monte Carlo simulation, grounded in probability theory and employing random sampling, is an innovative approach to option pricing. In the research conducted by Martinkutė-Kaulienė and Jabbour, the performance of the MCS is roughly equivalent to that of the BSM. However, with a substantial volume of data, MCS exhibit superior accuracy. Furthermore, simulation methods have proven to be effective and versatile computational tools for assessing the value of options with intricate characteristics or many sources of uncertainty, continuously producing superior outcomes. The primary allure of simulation lies in its direct method of addressing uncertainty.

SVM address the limitation of constant volatility in the BSM. It derives a variety of models. Nevertheless, individuals must select the optimal model based on the real-world scenario. Each individual stochastic volatility model possesses distinct utility for various types of underlying assets.

Ultimately, the BSM remains the most effective method for individuals to price options and forecast financial markets, owing to its precision and efficiency. Each model possesses distinct characteristics tailored for various purposes.

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