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Research on Portfolio Investment in the Risky U.S. Stock Market Based on the Mean-Variance Model

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Abstract: Against the backdrop of a thriving securities market and economy, there is a growing demand among investors for stock portfolio models in risky markets. This paper explores the application of the mean-variance model in this context, aiming to assist investors in making more informed investment decisions in an uncertain market. Through an in-depth analysis of the mean-variance model, the main conclusion of this paper is that certain stocks, such as AAPL, receive a higher investment proportion due to their lower risk and stable expected returns. In contrast, some high-risk or low-expected-return stocks, such as SBUX, MSFT, and QCOM, hold a smaller or zero proportion in the portfolio. This conclusion highlights the importance of balancing risk and return when constructing a portfolio and provides investors with specific asset allocation recommendations. It can help them effectively control risk while pursuing maximum returns, thereby achieving long-term stable investment gains.

Keywords: Mean-variance model, Risky market stock portfolio model, Efficient frontier portfolio.

1. Introduction

As China's securities and economic markets continue to develop, an increasing number of investors are emerging in society. With the continuous purchase of stocks, funds, and other financial products by investors, the question of how to construct an appropriate stock portfolio model for risky markets to maximize investor returns has become an increasingly prominent topic. According to statistics, the Nasdaq Stock Market (NASDAQ) and the New York Stock Exchange (NYSE) are among the world's leading stock exchanges. As of February 2024, the two exchanges had 3,411 and 2,256 listed companies, respectively, with domestic market capitalizations of $24.97 trillion and $24.87 trillion, respectively. Globally, there are hundreds of millions of shareholders, making the study of stock portfolio models for risky markets highly relevant to assisting investors in making informed choices during investments.

This paper will employ the mean-variance model to explore stock portfolio construction from the perspective of risky markets. The mean-variance model can maximize investment returns under a certain level of investment risk, providing a basis for investors to select an efficient portfolio of securities. The model considers not only the expected returns of the portfolio but also its risk level. In this way, it can help investors make better investment decisions in uncertain environments.

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Therefore, using the mean-variance model to construct an appropriate stock portfolio can assist investors in identifying opportunities in the capital market that offer superior returns with lower risk.

This paper is broadly divided into six parts. The second part reviews classic literature, discussing the conceptual theories and previous research on the mean-variance model. The third part focuses on the application of the mean-variance model in constructing stock portfolios in risky markets. The fourth part presents the calculation results in the form of formulas, data, and charts. The fifth part offers subjective reflections on the research results, analyzing the reasons behind the findings. Finally, the sixth part concludes by discussing the limitations of this study and suggesting directions for future research.

2. Literature Review

In today's increasingly complex financial markets, the selection and application of stock portfolio models for risky markets have become a focal point for investors. This section provides a comprehensive overview of recent research achievements in the field of portfolio management from various perspectives.

Xu Yunhui and Li Zhongfei's research integrates the dynamic mean-variance model into the quadratic utility model, revealing the significant impact of serial correlation on the optimal strategy and the slope of the efficient frontier [1]. He Chaolin studied the application of the mean-variance model in optimal asset portfolio selection under a general uncertainty framework, finding that investors with a higher degree of risk aversion tend to adopt more aggressive investment strategies when faced with larger memory coefficients and smaller indifference coefficients [2]. Liu Tengjiao emphasized the unique advantages of the PBIL algorithm in solving portfolio problems, combining it with the mean-variance model to provide decision support for investors on the efficient frontier of portfolios [3]. The research by Qi Yue and Liu Tongyang revealed the performance of socially responsible investing in China's capital markets, proposing a method to transform the relationships among finance, business, and social responsibility into a multi-objective portfolio selection model [4]. Wang Xiaoqin and Gao Yuelin established a mean-variance lower semi-variance portfolio model incorporating a typical transaction cost function, revealing the influence of transaction costs on the risk value [5]. Zola reexamined the relationship between risk and return based on Australian stock market data, emphasizing that investors with different levels of risk aversion can obtain optimal investment strategies that align with their risk tolerance in markets where risk-free assets are available [6]. The research of Huang Lu and Zhou Tingse explored specific methods for constructing portfolios from different angles [7-8] . Huang Lu, based on factor analysis and the mean-variance model combined with tax indicators of listed companies, proposed the optimal investment portfolio proportions for sample stocks in the CSI 100 Index. Zhou Tingsen utilized cluster analysis and principal component analysis to select high-quality stocks, then constructed a portfolio using the mean-variance model, providing specific capital allocation proportions. The studies by Chen Long and Cao Hongbo examined fund and stock portfolio management strategies from different market scopes and time spans [9-10]. Chen Long focused on the SSE 50 constituent stocks and significantly improved portfolio performance by refining the mean-variance model. Cao Hongbo provided essential references for product allocation in fund advisory services by screening and managing funds in the market. The research of Li Shuli and Ma Tian focused on the expansion and application of the mean-variance model [11-12]. Li Shuli analyzed the impact of changes in mean and variance on the efficient frontier curve, providing theoretical support for investors to adjust their investment strategies in dynamic market environments. Ma Tian used Excel to implement quadratic programming, deeply investigating the relationship between portfolio returns and risk, offering strategy support to investors.

In summary, the selection and application of stock portfolio models are complex and variable processes, requiring investors to continuously monitor market dynamics and technological

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advancements. By comprehensively applying various algorithms and models, this paper aims to better optimize portfolios and manage risks, ultimately creating greater value for investors.

3. Methodology

When the N-dimensional vector portfolio weights wp for portfolio p is the solution to the quadratic programming problem, portfolio p is an efficient portfolio:

$$\min_{\{w\}} \frac{1}{2} w^T V w \tag{1}$$

s.t.

$$w^T e = E[r_p] \text{ and} \tag{2}$$

where e represents the N-dimensional vector of expected returns of the N risky assets, E[rp] denotes the expected return of portfolio p, and 1 is an N-dimensional vector of ones.

$$A = 1^T V^{-1} e \tag{3}$$

$$B = e^T V^{-1} e \tag{4}$$

$$C = 1^T V^{-1} 1 \tag{5}$$

$$D = BC - A^2 \tag{6}$$

The unique set of efficient frontier portfolio weights for a portfolio with an expected return E[rp] is:

$$w_p = g + hE[r_p] \tag{7}$$

where

$$g = \frac{1}{D}\left[B\left(V^{-1}1\right) - A\left(V^{-1}e\right)\right] \tag{8}$$

and

$$h = \frac{1}{D}\left[C\left(V^{-1}e\right) - A\left(V^{-1}1\right)\right] \tag{9}$$

Here $E[r_p] > \frac{A}{C}$, g corresponds to the efficient frontier portfolio with an expected return of 0, and g+h corresponds to the efficient frontier portfolio with an expected return of 1.

4. Results

This study utilizes data from the “Stock Market: Historical Data of 10 Companies” dataset available on the Kaggle website, focusing on the daily closing prices of ten stocks—Apple, Starbucks, Microsoft, Cisco Systems, Qualcomm, Meta, Amazon.com, Tesla, Advanced Micro Devices, and Netflix—between January and June 2023. Through a three-round screening process, the final investable stocks were identified. The first round of screening is detailed as follows.

First, the return rate of each stock was calculated, and from these return rates, the expected return rate for each stock was derived, as shown in Table 1.

Table 1: Expected Return Rate of Each Stock (‰).

Stock	AAPL	SBUX	MSFT	CSCO	QCOM	META	AMZN	TSLA	AMD	NFLX
Expected Return Rate (‰)	3.66	-0.04	3.02	0.70	1.08	7.22	3.65	7.84	5.22	3.54

Next, the covariance of the return rates was calculated, and the covariance matrix of the return rates (‰) was obtained, as shown in Table 2.

Table 2: Covariance Matrix of Return Rates (‰).

0.17	0.07	0.14	0.08	0.15	0.23	0.16	0.23	0.17	0.15
0.07	0.21	0.07	0.06	0.11	0.09	0.10	0.11	0.12	0.10
0.14	0.07	0.32	0.08	0.15	0.29	0.26	0.19	0.33	0.14
0.08	0.06	0.08	0.16	0.10	0.11	0.08	0.12	0.09	0.10
0.15	0.11	0.15	0.10	0.46	0.15	0.18	0.30	0.37	0.25
0.23	0.09	0.29	0.11	0.15	0.93	0.39	0.38	0.36	0.24
0.16	0.10	0.26	0.08	0.18	0.39	0.50	0.31	0.32	0.23
0.23	0.11	0.19	0.12	0.30	0.38	0.31	1.27	0.40	0.32
0.17	0.12	0.33	0.09	0.37	0.36	0.32	0.40	1.07	0.28
0.15	0.10	0.14	0.10	0.25	0.24	0.23	0.32	0.28	0.57

By calculating, the values of A, B, C, D, and A/C were obtained, as shown in Table 3.

Table 3: Calculated Values of A, B, C, D, and A/C in the First Round.

A	B	C	D	A/C
11.49	0.13	9957.72	1186.10	0.0012

Based on the formulas, the values of g and h were calculated, as shown in Table 4.

Table 4: Calculated Values of g and h in the First Round.

g	h
0.208	157.650
0.360	-83.134
0.136	-22.553
0.486	-82.336
0.037	-64.657
-0.096	26.157
0.001	-11.216
-0.068	30.698
-0.053	30.980
-0.011	18.410

Finally, the risk volatility (%) was calculated, as shown in Table 5, and the efficient frontier chart was plotted based on these values, as shown in Figure 1.

Table 5: Risk Volatility.

Risk Volatility (%)	E(%)
1.00	0.12
1.01	0.15
1.03	0.20
1.08	0.25
1.14	0.30
1.21	0.35
1.30	0.40
1.39	0.45
1.50	0.50
1.61	0.55
1.73	0.60
1.84	0.65
1.97	0.70

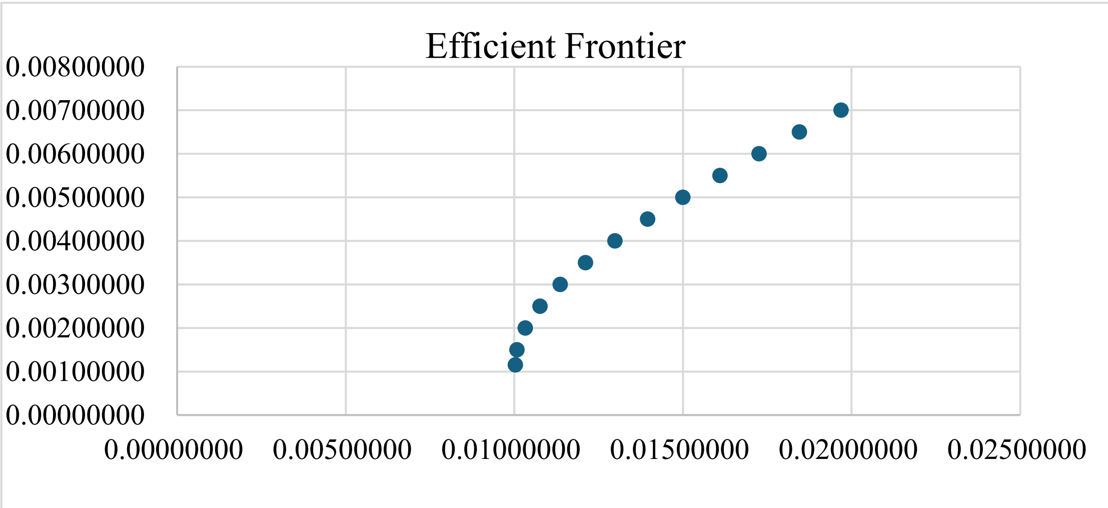

Figure 1: Efficient Frontier Chart.

This is a hyperbola where the horizontal axis represents risk and the vertical axis represents return. Points on the hyperbola represent frontier solutions, while points to the right of the hyperbola are non-frontier solutions. The upper right portion of the hyperbola is the efficient frontier. Points near MVP (0.01, 0.0012) represent risk aversion, while those further away represent risk preference. As seen from the graph, when E is 0.04, the return increases more rapidly while the risk increases less, thus E = 0.04 is selected.

Through calculation, the weights w for the first round are given in Table 6.

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Table 6: Weights win the First Round.

w
0.84
0.03
0.05
0.16
-0.22
0.01
-0.04
0.05
0.07
0.06

From the results, it is evident that the fifth stock (QCOM) and the seventh stock (AMZN) performed poorly, so these two stocks will not be included in the second round of calculations.

The second round of screening is as follows:

Since the fifth stock (QCOM) and the seventh stock (AMZN) are not invested in, their weights are set to zero. Based on the expected return rates of the remaining eight stocks, the values of A, B, C, D, and A/C are calculated and presented in Table 7.

Table 7: Values of A, B, C, D, and A/C in the Second Round.

A	B	C	D	A/C
10.73	0.12	9921.27	1039.03	0.0011

Based on the formulas, the values of g, h, and w are calculated and given in Table 8.

Table 8: Values of g, h, and w in the Second Round.

g	h	w
0.214	148.083	0.81
0.371	-104.798	-0.05
0.136	-27.868	0.02
0.500	-105.094	0.08
0	0	0
-0.103	36.698	0.04
0	0	0
-0.068	29.983	0.05
-0.045	18.152	0.03
-0.004	4.845	0.02

The results show that the second stock (SBUX) also performed poorly, so it will not be included in the third round of calculations.

The third round of screening is as follows:

Since the second stock (SBUX), the fifth stock (QCOM), and the seventh stock (AMZN) are not invested in, their weights are set to zero. Based on the expected return rates of the remaining seven stocks, the values of A, B, C, D, and A/C are calculated and presented in Table 9.

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Table 9: Values of A, B, C, D, and A/C in the Third Round.

A	B	C	D	A/C
14.33	0.10	8800.93	716.85	0.0016

Based on the formulas, the values of g, h, and w are calculated and given in Table 10.

Table 10: Values of g, h, and w in the Third Round.

g	h	w
0.30	123.98	0.79
0	0	0
0.20	-46.34	0.02
0.81	-191.64	0.04
0	0	0
-0.17	56.91	0.05
0	0	0
-0.11	40.81	0.06
-0.05	18.84	0.03
0.02	-2.55	0.01

The investment proportions for each stock are summarized in Table 11.

Table 11: Investment Proportions for Each Stock.

Stock	AAPL	SBUX	MSFT	CSCO	QCOM	META	AMZN	TSLA	AMD	NFLX
Investment Proportion	79.480%	0	1.608%	3.936%	0	5.279%	0	5.698%	2.771%	1.228%

5. Discussion

Firstly, the selection of the ten stocks mentioned above as the research subjects was the result of a multi-factor consideration and filtering process. Given the numerous companies in the market, this study opted to select well-known enterprises as the initial sample to ensure both data representativeness and public attention, as well as to facilitate the applicability of subsequent analyses.

During the experimental design phase, a detailed screening process was implemented, removing stocks with poor performance, high volatility, or potential risks, leaving behind those with relative stability in the market. This process ensures the quality of the data and the reliability of the analysis results.

Since the popularity of stocks is not always directly related to their returns, a more scientific and efficient evaluation system was constructed after the preliminary screening of stocks, focusing on individual stocks more likely to produce meaningful conclusions. After further eliminating underperforming stocks, the remaining ten stocks were used for calculation and analysis.

Secondly, in exploring investment strategies, this study found significant differences between models that purely pursue return maximization and those that balance risk and return. These differences are reflected in the rationality of investment decisions and directly impact the long-term stability of the investment portfolio.

Examining stock returns in isolation often neglects the crucial factor of risk. Investors might prefer stocks that perform well in the short term, ignoring the high risks associated with them. In contrast, the mean-variance model optimizes the portfolio by considering both the expected return and risk of assets, thereby effectively controlling risk while pursuing returns.

For example, the first stock (AAPL) has an expected return rate (3.66‰) that is noticeably lower than that of the eighth stock (TSLA) (7.84‰). However, since its risk (0.17‰) is significantly lower compared to the eighth stock (1.27‰), it receives a higher investment weight (79.480%) under the mean-variance model optimization, despite not having the highest expected return. This reflects the model's unique advantage in balancing risk and return.

Therefore, the difference between considering return alone and a comprehensive consideration of risk is that the latter can more thoroughly and scientifically assess the value of investment targets, achieving the best balance between risk and return when constructing an investment portfolio.

Finally, when discussing investment dimensions, this study uses six months of data for prediction. Short-term predictions often face cyclical issues because they effectively cover only the immediately following day or short period.

In multi-period prediction and optimization, although trading costs are a non-negligible negative factor, their relative impact is low in this study. This is because, with a higher set return level (0.4%), the trading costs associated with frequent transactions are relatively mitigated. Further analysis shows that an increase in the return level means a rise in potential risk. However, as trading costs are fixed and risk-free, the reduction due to these costs only affects the return level, while the risk level remains unchanged. This indicates that a higher return target can extend the effective prediction period, reducing constraints and negative impacts of non-ideal factors in multi-period prediction and optimization.

6. Conclusion

Based on the mean-variance model, this paper primarily discusses how to construct an appropriate stock investment portfolio model for risky markets. The problem is divided into two aspects: maximizing investment returns and controlling risk, with the aim of helping investors find a balance between expected returns and risk to achieve stable returns in the capital market.

Through the optimization of the mean-variance model, this paper identifies the seven optimal stocks from the "Stock Market: Historical Data of 10 Companies" dataset as Apple, Microsoft, Cisco Systems, Meta, Tesla, Advanced Micro Devices, and Netflix, with respective investment proportions of 79.480%, 1.608%, 3.936%, 5.279%, 5.698%, 2.771%, and 1.228%. Stocks such as Starbucks, Qualcomm, and Amazon.com are not recommended for investment due to their poor returns.

The limitations of this study mainly concern the cyclical issues that may arise from using a six-month dataset for predictions, which may not cover longer-term trends and changes. To address this issue, future research could implement multi-period forecasting strategies to optimize and maximize returns over a week or even longer periods.

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